Passenger demand oriented train scheduling and rolling stock circulation planning for an urban rail transit line

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Abstract

We study the integration of train scheduling and rolling stock circulation planning under time-varying passenger demand for an urban rail transit line, where the practical train operation constraints, e.g., the capacity of trains, the number of available rolling stocks, and the entering/exiting depot operations, are considered. Three solution approaches are proposed to solve the resulting multi-objective mixed-integer nonlinear programming (MINLP) problem to deliver both an irregular train schedule (i.e., departure and arrival times of all train services) and a rolling stock circulation plan (including entering/exiting depot operations of rolling stocks and connections between train services) simultaneously. We first present an iterative nonlinear programming (INP) approach, where the solutions of the original MINLP problem are obtained by solving a nonlinear programming problem and a mixed integer linear programming (MILP) problem iteratively. Moreover, an equivalent MILP formulation of the original MINLP model is developed and an approximated MILP approach is proposed to reduce the number of constraints introduced by passenger demand. A case study is conducted based on the practical data of the Beijing Yizhuang line, where the three proposed approaches are compared with a state-of-the-art approach and a practical method used by the traffic planners. This comparison shows the effectiveness and efficiency of the three proposed approaches.

Keywords: urban rail transit, train scheduling, rolling stock circulation, iterative nonlinear programming, MILP

1. Introduction

Urban rail transit systems are of crucial importance for the stability, sustainability, and efficiency of public transportation systems. The number of passengers commuting with urban rail transit systems is more than 10 million per day in big cities, such as Beijing, Shanghai, and Tokyo. During the morning and evening peak hours, especially during workdays, the departure headway between two consecutive services has been recently reduced to 2 minutes for some busy urban rail transit lines. With the increasing of passenger demand and the saturated operations of
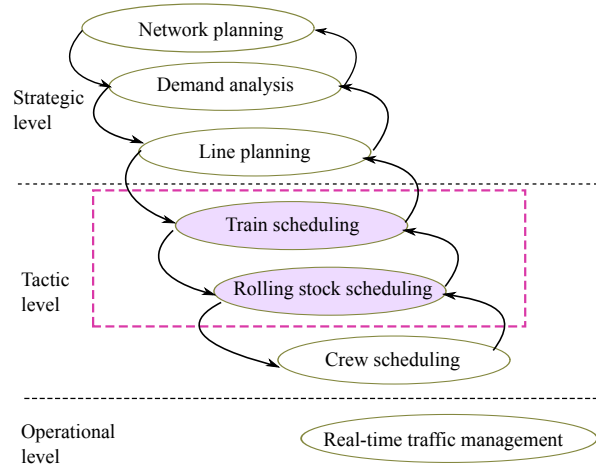
Figure 1: The hierarchical planning process for urban rail transit lines

trains in urban rail transit lines, the planning process is attracting more and more attention, because the operating costs of rail operators and the passenger satisfactions are hugely influenced by the quality of the planning process.

Traditionally, the planning process for urban rail transit lines is a sequential planning process (Ghoseiri et al., 2004; Bussieck et al., 1997), as given in Figure 1, which consists of network planning, demand analysis, line planning, train scheduling, rolling stock scheduling (or circulation planning), and crew scheduling. In general, the results of previous planning process are used as inputs or constraints for the following process, which may lead the railway system to be operated in an suboptimal way (Schöbel, 2017). The line planning determines lines, stop patterns, and frequencies of train services to satisfy passenger demand and infrastructure constraints (Caprara et al., 2005). The train schedules are determined based on the results of the line planning process, where the frequencies could be adjusted in a certain time interval to obtain a feasible train schedule. The rolling stock circulation plan is then calculated based on the feasible train schedule obtained in the train scheduling process. The rolling stock circulation planning may also need to adapt the departure/arrival times of train services or even need to cancel some train services, in order to satisfy the constraints proposed by the rolling stock circulation. The drawbacks of the sequential process are as follows: (1) the operating costs, such as the number of rolling stocks required, are considered only in the late stage of the process; (2) the adjusted line plan and train schedule may not be optimal for rail operators and passengers any more or even they may not satisfy the passenger demand.

We focus on the integrated optimization of train schedules and rolling stock circulation plans for an urban rail transit line. In most urban rail transit systems, the lines are generally separated with each other. Rolling stocks are not operated across different lines but they only belong to a particular line. In our optimization problems, we make the following assumptions: (1) trains do not meet and overtake each other due to the station layout (without siding tracks); (2) each platform of a station can only accommodate one train at a time; (3) there is only one depot which is connected with the start station; (4) all the train services are full-length services, i.e., trains go from the start station to the end station, turn around at the end station, go towards the start station, and stop at all the stations; (5) the decomposition and composition of rolling stocks (or

EMUs) are not included in the rolling stock circulation planning; (6) the available rolling stocks can operate during the whole operating period, i.e., the maximum number of services that a rolling stock can perform is not limited; (7) the passenger demand is considered as the number of passengers traveling between two consecutive stations for a certain time period (e.g., 30, 15, or 5 minutes), while the origin and destination of passengers are not considered; (8) the number of passengers is approximated by real numbers instead of integers. Detailed explanations regarding these assumptions will be given in Section 3.2.

In this paper, we propose an integrated optimization of the train scheduling and rolling stock circulation planning for an urban rail transit line, where the headways between train services, the departure/arrival times of train services, and the rolling stock circulation plan are optimized simultaneously. A multi-objective mixed integer nonlinear programming (MINLP) model is formulated, where the time-varying passenger demand, the turnaround operations, the connection of train services, and the entering/exiting depot operations are included in the model formulation. An iterative nonlinear programming (INP) approach and two mixed integer linear programming (MILP) approaches are then proposed to solve the resulting integrated optimization problem. The proposed three solution approaches will be compared with a state-of-the-art approach (i.e., the two-stage approach presented in Wang et al. (2017a)) and the current practical solution generated by traffic planners based on the practical data of the Beijing Yizhuang line.

The remainder of this paper is organized as follows. Section 2 provides a literature review on the passenger demand oriented train scheduling and on the integration of train scheduling and rolling stock circulation planning. In Section 3, a detailed problem statement and the assumptions for the model formulation are presented. In Section 4, the integrated optimization problem for passenger demand oriented train scheduling and rolling stock circulation planning is formulated. In Section 5, the INP approach is presented to solve the resulting MINLP problem. In Section 6, two MILP approaches, i.e., the accurate MILP and approximated MILP approaches, are introduced to solve the integrated MINLP problem, by transforming the nonlinear terms via mixed logical dynamic constraints. In Section 7, the proposed mathematical model and solution approaches are evaluated by real-world operation data taken from the Beijing Yizhuang line. Finally, the conclusions and future works are presented in Section 8.

2. Literature review

In this section, we review the state of the art in two directions: 1) passenger demand oriented train scheduling; 2) integration of train scheduling and rolling stock circulation planning.

2.1. Passenger demand oriented train scheduling

The passenger demand oriented train scheduling problem has attracted much attention for the past years. The research on this problem can be split into two streams based on the characteristics of the obtained train schedules: (1) periodic (or regular) train schedules and (2) non-periodic (or irregular) train schedules. Recently, Robenek et al. (2017) examined the benefits of combining the regularity of periodic train schedules and the flexibility of the non-periodic ones.

For the regular train schedules, genetic algorithms were adopted by Nachtigall and Voget (1996) for the regular train schedule optimization with the objective of minimizing the passenger waiting times. Chierici et al. (2004) proposed a demand-dependent optimization approach to maximize the total demand captured by the trains, where a branch-and-bound method and heuristic algorithms were used to solve the resulting mixed integer nonlinear problem. Ceder

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(2009) proposed a methodological framework to determine the departure times of train services with even headways and provided smooth transitions between different time periods. Liebchen (2006, 2008) formulated the train scheduling problem as a periodic event-scheduling problem based on a well-established graph model and applied integer programming methods to optimize the train schedule for Berlin subway system. Cordone and Redaelli (2011) presented a mixed integer nonlinear programming model to capture the feedback mechanism between transport offer and passenger demand, where a branch-and-bound algorithm and a heuristic algorithm were developed to optimize the regular train schedule for a regional network in Italy. Kaspi and Raviv (2012) proposed a service-oriented line planning and train scheduling model with the objective of minimizing both the user inconvenience and the operational costs, where the problem was solved by a cross-entropy metaheuristic algorithm. In addition, regular train schedules of urban rail transit systems are also optimized with consideration of energy consumption (including/excluding regenerative energy), e.g., in Su et al. (2013) and Li and Lo (2014), where the speed profiles of the trains are managed simultaneously. Moreover, Yin et al. (2017) integrated the energy efficiency into the passenger oriented train scheduling, where the train schedules and the speed profiles are collaboratively optimized by solving a mixed integer linear programming problem.

Irregular train scheduling is more appropriate when considering time-varying passenger demand since a regular train schedule may lead to longer waiting times and ineffective rolling stock utilization. Cury et al. (1980) presented a hierarchical methodology to generate optimal train schedules for urban rail transit lines based on a train movement model and a passenger behavior model. Based on the model proposed in Cury et al. (1980), Assis and Milani (2004) proposed a model predictive control approach based on linear programming to efficiently generate train schedules for a whole day. A two-level approach is proposed in Albrecht (2009) to generate the demand oriented train schedule, where the optimal frequency and capacity of trains are computed by a branch-and-bound algorithm first and a genetic algorithm is then used to determine the departure times of trains at stations. Niu and Zhou (2013) developed an optimization model for constructing demand-responsive and congestion-sensitive timetables for a heavily congested urban rail transit line. A customized genetic algorithm is designed to solve the resulting integer programming model with the objective of minimizing the number of waiting passengers and weighted remaining passengers. Niu et al. (2015) proposed nonlinear mixed integer optimization models for the train scheduling problem with the objective of minimizing the total passenger waiting time at stations under time-dependent passenger demand and predetermined stop-skipping patterns. A bi-level approach was proposed in Wang et al. (2014) to obtain the optimal train schedule for an urban rail transit line with consideration of time-varying passenger demand and stop-skipping strategy. Furthermore, the passenger demand oriented train scheduling for an urban rail transit network is considered in Wang et al. (2015b), where the train schedules of two lines are optimized simultaneously to satisfy the passenger demand, while the transfer between different lines is included in the problem formulation. Sun et al. (2014) proposed three models to design demand sensitive train schedules based on the concept of equivalent time (interval) and assessed the performance of these three models based on the data of a metro line in Singapore. It is concluded that the dynamic timetable with capacity constraints is the most advantageous. Canca et al. (2014) developed a nonlinear integer programming model to optimize the departure/arrival times of train services under a dynamic passenger demand, where a test case was performed based on the C5 line of Madrid rapid transit system. Barrena et al. (2014) proposed three formulations for the train scheduling problem with the aim of minimizing the passenger waiting time and a branch-and-cut algorithm to design train schedules under a

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dynamic demand environment. Zhu et al. (2017) proposed a bi-level model for designing a train timetable on an urban rail line, where the passenger demand and the carrying capacity of rolling stocks are jointly considered in the train timetabling stage. However, in the above-cited studies, the turnaround operations and the rolling stock circulation plan are not directly considered due to the complexity of the problem formulation. The turnaround operations are usually the bottleneck of the urban rail transit line and the constraints proposed by the rolling stock circulation plan are also critical for the train scheduling process.

2.2. *Integration of train scheduling and rolling stock circulation planning*

An integer programming model based on a transition graph is proposed by Alfieri et al. (2006) to determine the rolling stock circulation plan with coupling and decoupling constraints at the stations on a single line for daily operations, where the objective is the minimization of the number of train units required. Fioole et al. (2006) assigned the rolling stocks to train services based on the given departure/arrival times and the expected number of passengers, where the objective criteria involved operational costs, service quality and reliability of railway systems. Moreover, Peeters and Kroon (2008) proposed a circulation planning model to allocate the rolling stocks according to the given timetable and the passengers' seat demand, where a branch-and-price algorithm was applied to find the best trade-off between the passenger satisfaction, the robustness, and the cost of the circulation plan. Cadarso and Marín (2010) presented an integer programming model to shunt the rolling stocks efficiently. In addition, a mixed integer optimization model is proposed in Cadarso and Marín (2011) to explore the robustness of the rolling stock assignment, where the objective is to minimize the costs, including service trips, empty train movements, and composition changes. Overall, the papers mentioned above address the railway planning process in a sequential way, where the train schedules and the passenger volumes on each train are assumed to be known for the rolling stock circulation problem.

Since the adjustments of departure/arrival times or the cancellation of train services in the rolling stock circulation planning process may result in less optimal train operations, some researchers have recently proposed to optimize the train schedule and the rolling stock circulation plan collaboratively. Based on the periodic train scheduling model in Liebchen (2006, 2008), the rolling stock circulation is integrated into the periodic train scheduling by Liebchen and Möhring (2007) with the objective of minimizing the number of rolling stocks required to perform the operations. An integrated train scheduling and rolling stock planning model is proposed by Cadarso et al. (2012) for urban rapid transit networks, where the maximum and minimum frequencies for the scheduled train services are predefined by the line planning process. Based on their previous study, Cadarso et al. (2013) studied the recovery measures in presence of disruptions for urban rail transit networks. Recently, Chang et al. (2015) presented an integrated optimization model for train scheduling and rolling stock circulation planning, where they assume that there exists an ideal train schedule and the difference between the ideal and optimized train schedules is minimized. A path-indexed nonlinear formulation was proposed by Hassannayebi et al. (2016) to minimize the average waiting time of passengers, where a Lagrangian relaxation approach was introduced to relax some rolling stock circulation constraints. Yue et al. (2017) proposed a bi-level programming model, where the upper level model is used for the train scheduling with the objective of minimizing the trade-off between the number of waiting passengers and the number of train services and the low level model is used to schedule the rolling stocks based on the given train schedule with the aim of minimizing the number of infeasible train paths. The train schedule obtained by the upper level is the input for the lower level model and the solution process is iterated until a sufficiently good solution is found. Wang et al. (2017a) presented a

two-stage optimization approach to optimize the train schedule and the rolling stock circulation plan sequentially, where the rolling stock circulation plan is obtained by adjusting the departure and arrival times in the demand oriented train schedule. Wang et al. (2017b) integrated the train scheduling and the rolling stock circulation planning problems for an urban rail transit line, where the integrated problem is not directly based on the time-varying passenger demand, but it is based on the service patterns generated by the demand analysis and line planning, i.e., the headways in the peak and off-peak hours.

2.3. Paper contributions

Table 1 summarizes some relevant studies on the passenger demand oriented train scheduling and the integration with the rolling stock circulation planning, in terms of problem description (i.e., integration, infrastructure, travel demand, train capacity, available trains), mathematical formulations (i.e., objectives, constraints, model structure), and solution algorithms. For the summary of Table 1, studies tend to integrate more and more attributes in the train scheduling process to search for a global optimal solution. In particular, the integration of passenger demand oriented train scheduling and rolling stock circulation planning is one of these trends.

Nevertheless, the integrated optimization of train schedules and rolling stock circulation plans under time-varying passenger demand increases the modeling difficulty and computational complexity. Most of the existing studies are typically devoted to these two problems separately. For example, in rolling stock circulation models, the departure frequencies of trains are usually pre-given as constant values, while the demand oriented train scheduling models typical neglect or simplify the rolling stock circulation constraints. Papers on passenger demand oriented train scheduling and rolling stock circulation planning either solve the integrated optimization problem in a sequential way (e.g., the two-stage approach in Wang et al. (2017a)) or in an iterative way (e.g., the simulated annealing approach in Yue et al. (2017)).

Our paper focuses on the mathematical modeling and algorithmic designs required to compute optimal train schedules and rolling stock circulation plans simultaneously under the time-varying passenger demand for urban rail transit lines. This paper proposes the following contributions to the literature:

- This work combines train scheduling and rolling stock circulation planning under time-varying passenger demand and formulates the integrated optimization problem as a multi-objective mixed-integer nonlinear programming (MINLP) model. Our model rigorously considers the practical train operating constraints and passenger demand constraints, which can be used to simultaneously generate an irregular train schedule according to the passenger demand as well as the corresponding rolling stock circulation plan. The objectives of the MINLP model are to minimize the load factor variation and the headway variation of trains, in order to provide better passenger services, and to minimize the number of entering/existing depot operations, in order to reduce the operational complexity and costs from the perspective of rail operators.

- To capture the computational complexity arising from this integrated multi-objective MINLP model, we introduce three solution approaches, which involve an iterative nonlinear programming (INP) approach, an accurate mixed-integer linear programming (MILP-acc) approach and an approximated mixed-integer linear programming (MILP-app) approach. The INP and MILP-app approaches are developed by relaxing and approximating some constraints and by imposing additional penalties to the original model, while the MILP-acc

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Table 1: Summary of relevant studies on passenger demand oriented train scheduling and its integration with rolling stock circulation planning for urban rail transit systems*

Publi-cations	Inte-gration	Infra-structure	Travel demand	Train capacity	Trains available	Objective(s)	Model structure	Solution algorithms
Cadarso et al. (2012)	TS,RCP	network	dynamic	yes	yes	operating costs and passenger dissatisfaction penalty	MILP	CPLEX
Niu and Zhou (2013)	LP,TS	bi-direc	dynamic	yes	no	total waiting time	nonlinear	GA
Barrena et al. (2014)	LP, TS	uni-direc	dynamic	no	no	average waiting time	MILP	branch and cut
Barrena et al. (2014)	LP, TS	uni-direc	dynamic	no	no	average waiting time	nonlinear	ALNS
Canca et al. (2014)	LP, TS	uni-direc	dynamic	yes	no	average waiting time and average load factor	MINLP	GAMS
Wang et al. (2014)	TS,SP	uni-direc	static	yes	no	total traveling time and energy consumption	MINLP	bi-level approach
Li and Lo (2014)	TS,SP	bi-direc	–	no	no	net energy consumption	nonlinear	GA
Niu et al. (2015)	LP,TS	uni-direc	dynamic	yes	no	total waiting time	MINLP	GAMS
Wang et al. (2015b)	TS,SP	small network	dynamic	yes	no	total traveling time and energy consumption	nonlinear	SQP,GA
Chang et al. (2015)	TS,RCP	bi-direc	–	no	yes	deviations from the expected timetable and fleet size	MILP	CPLEX
Hassannayebi et al. (2016)	TS, RCP	bi-direc	static	yes	yes	average waiting time per passengers	NLP	Lagrangian relaxation
Yin et al. (2017)	TS,SP	bi-direc	dynamic	yes	no	energy consumption and total waiting time	MILP	CPLEX and heuristic algorithm
Yue et al. (2017)	TS, RCP	bi-direc	dynamic	yes	yes	numbers of waiting passengers, train services, infeasible train paths	Simulated annealing	
Wang et al. (2017b)	TS,RCP	bi-direc	–	no	yes	headway deviations from the given service pattern and required trains	MINLP	CPLEX
Wang et al. (2017a)	TS, RCP[1]	bi-direc	dynamic	yes	yes	operation cost,trains required, headway variations	MINLP	SQP, CPLEX
This paper	TS,RCP	bi-direc	dynamic	yes	yes	headway variations, load factor variations, number of depot operations	MINLP	iterative approach, CPLEX

* Symbols description for Table 1: line planning (LP); train scheduling (TS); Rolling stock circulation planning (RCP); Speed profile (SP); bi-direction single line (bi-direc); Uni-direction single line(Uni-direc); mixed integer linear programming (MILP); mixed integer nonlinear programming (MINLP); Cplex solver (CPLEX);Gams software (GAMS); genetic algorithm (GA); adaptive large neighborhood search (ALNS); sequential quadratic programming (SQP).

[1] In Wang et al. (2017), the passenger demand oriented train schedule is optimized first. Based on the optimal train schedule, the rolling stock circulation plan is then calculated by adjusting the departure and arrival times of train services.
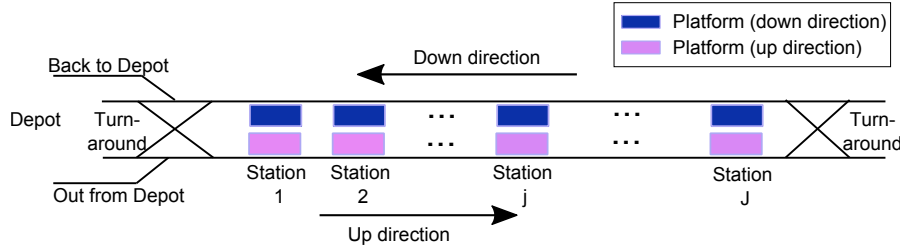
Figure 2: The layout of an urban rail transit line

approach is based on equivalent reformulations from the original multi-objective MINLP problem. The latter approach can return better solutions and can serve as benchmark for the other two approaches.

- The performance of the proposed integrated model and solution approaches is evaluated by using the practical data obtained from the Beijing Yizhuang line. The solutions obtained by the new approaches are compared with those generated by the state-of-the-art approach (i.e., the two-stage approach in Wang et al. (2017a)) and the approach used by traffic planners in practice, in terms of headway variations, maximum load factor violations, and required depot operations. We also perform a multi-objective study and investigate the scalability of the fastest solution approach.

3. Problem statement and formulation assumptions

3.1. Problem statements

We consider a double-track urban rail transit line with the layout given in Figure 2, where the depot is connected with station 1. The total number of stations is J and the turnaround operations occur at station 1 and station J. We define the operating direction from station 1 to J as the up direction and the operating direction from station J to 1 as the down direction. The operations of the rolling stocks in the up and down directions are connected via turnaround operations.

When considering daily train scheduling problem for urban rail transit lines, the headway in the regular train schedule changes several times; because the passenger demand in the peak hours is significantly higher than that in the off-peak hours. For example, the number of passengers traveling between two stations in Beijing Yizhuang line is shown in Figure 3, where the passenger demand has different patterns for weekdays and weekends. There are several settings of the headways in regular train schedules to satisfy the passenger demand, as illustrated by the black line of Figure 4. In irregular train schedules, the headways between train services (as it is shown by the red dashed-line in Figure 4) can be adjusted to satisfy the passenger demand better than when using the regular train schedules. Here, we consider irregular train schedules in this paper.

Passenger demand oriented train scheduling usually optimizes the departure and arrival times of train services without consideration of the rolling stock circulation plan. In particular, some studies only consider one operating direction of the urban rail transit line. Furthermore, the turnaround operation of trains at station J and the passenger demand on the other direction are not considered. However, if we solve the train scheduling problem without consideration of the rolling stock circulation constraints, then there could be no feasible rolling stock circulation plan
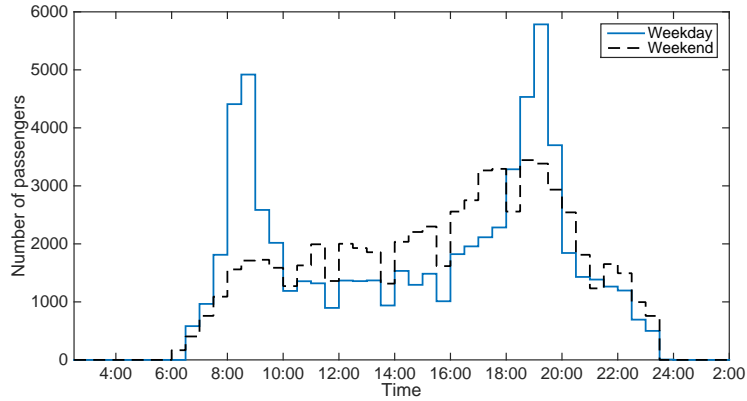
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Figure 3: The number of passengers traveling from Songjiazhuang to Xiaocun in Beijing Yizhuang line
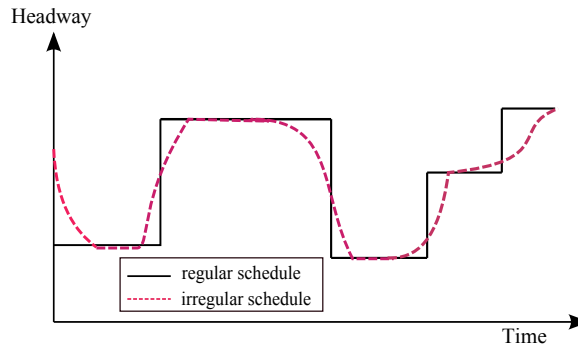

Figure 4: The headway between trains in regular and irregular train schedules

for the train schedules, e.g., the available rolling stocks are not enough. For this reason, the train schedules need to be adjusted in the rolling circulation planning phase, where the departure and arrival times of the train services should be changed and some of the train services even need to be canceled. Hence, we propose to integrate train scheduling and rolling stock circulation planning with consideration of passenger demand and to obtain train schedules and rolling stock circulation plans simultaneously.

A small example is presented in Figure 5, showing the train schedule for 18 train services and the circulation plan for 6 rolling stocks. In Figure 5, this urban rail transit line has 4 stations and a depot is connected to station 1. The rolling stocks performing train services 1-6 come out from the depot directly. After completing a train service, a rolling stock can turnaround at station 1 or go back to the depot. We take rolling stock 1 as an example. After performing train service 1, rolling stock 1 turns around at station 1 and operates service 7. Later on, rolling stock 1 performs service 13 and goes back to the depot. We observe that the turnaround operation of rolling stocks at station 1 and the operations related with the depot are not included in the definition of train services. Even for this small example, we can observe that the train schedule and the rolling stock circulation plan are strongly interrelated and the integrated optimization approaches generate benefits both for the rail operators and the passengers.
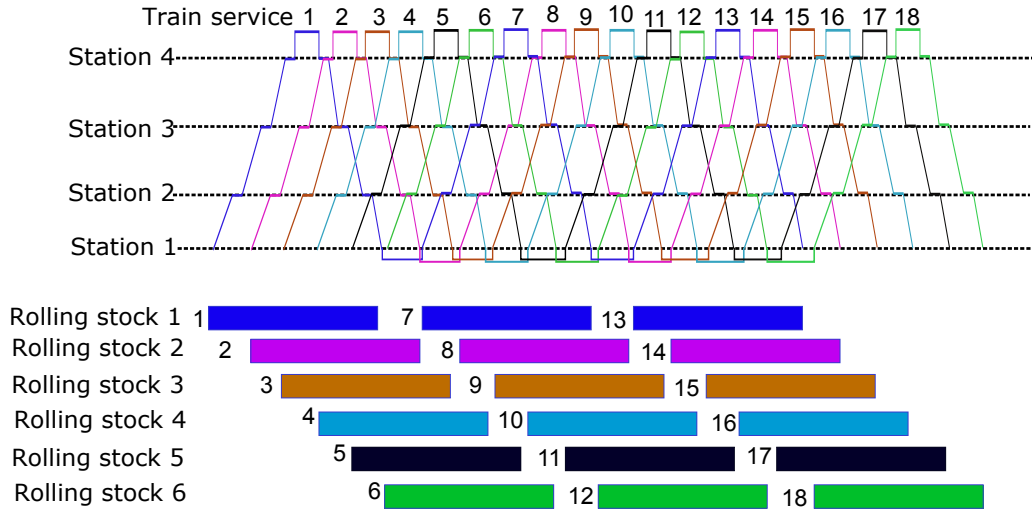
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Figure 5: Small example of a train schedule and the corresponding rolling stock circulation plan

3.2. Assumptions

The assumptions for the integrated train scheduling and rolling stock circulation planning problem have already been introduced in Section 1. Here, we give more detailed descriptions regarding those assumptions:

(1) The overtaking or meeting of trains does not occur in normal operations of urban rail transit lines, where trains usually run in a first-in first-out order from station 1 to J or from station J to 1 (see Figure 2).

(2) The station layout for urban rail transit lines is simple and normally consists of two platforms. Each platform is used for an operating direction, as shown in Figure 2. Each platform can only accommodate one train at a time.

(3) There are different topology structures for the urban rail transit lines, where a line may have a single depot or multiple depots (Yue et al., 2017). Here, we only consider the topology with one depot that is connected with station 1. However, the model formulations and solution approaches proposed in this paper can be extended to other topology structures.

(4) The stop patterns for urban rail transit lines involve full-length services (i.e., services that start from station 1, go to station J, turnaround at station J, go back to station 1, and stop at all the stations), short-turning services (i.e., services that turnaround at the intermediate stations other than station J), and stop-skipping services (i.e., services that skip some stations). However, most of the urban rail transit lines are only operated with full-length train services. Here, we only consider the full-length train services.

(5) The rolling stocks used in urban rail transit lines are electrical multiple units (EMUs), which consist of motor cars and trailer cars. No separated locomotives are needed for the operations of EMUs. In theory, the composition of the EMUs can change according to the passenger demand to reduce the operational costs and to enhance the passenger

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satisfaction. The rolling stocks consist of more cars during peak hours and less cars during off-peak hours. However, in order to simplify the practical operations, the composition of rolling stocks does not change in the daily operations of urban rail transit lines, such as the Beijing Yizhuang line. Therefore, the decomposition and composition of rolling stocks are not included in the rolling stock circulation planning.

(6) In urban rail transit lines, the available rolling stocks can operate from the early morning to the late evening, i.e., the whole operating period of a day. For this reason, we do not consider the maximum number of train services (or the maximum working times) that a rolling stock can operate. However, the capacity of the EMUs, i.e., the maximum number of on-board passengers, is indeed considered in the integrated optimization problem. The operation of rolling stocks inside the depot is out of the scope of this paper, since this type of operation is normally scheduled when the train schedule and the rolling stock circulation plan are fixed (Flamini and Pacciarelli, 2008).

(7) Urban rail transit lines focus on the number of passengers traveling between two consecutive stations (Yue et al.,2017), e.g., the number of passengers traveling between Songjiazhuang and Xiaocun for a weekday and a weekend day is shown in Figure 3. We thus optimize the train schedule and rolling stock circulation plan based on a sectional passenger demand without consideration of the origin and destination of passengers.

(8) Since the number of passengers that are on board of the trains is a large integer value, the approximation error, caused by treating this number as a real-valued variable, is small.

4. Mathematical formulation

The mathematical model is presented in this section to simultaneously optimize the train schedule and rolling stock circulation plan. First, the notations and decision variables are introduced. Then, the constraints and objective functions of the passenger demand oriented train scheduling and rolling stock circulation planning are formulated.

4.1. Notations and decision variables

Table 2 lists all the parameters and subscripts used in the model formulation. Moreover, Table 3 gives the decision variables of the integrated passenger oriented train scheduling and rolling stock circulation planning model. Since the integrated model is built on the basis of train services, the departure and arrival times at stations, the running times and dwell times, and the load factors of train services are important elements for the train scheduling model. Moreover, the behaviors of rolling stocks are important for the circulation plan. Specifically, the considered behaviors include (1) whether the rolling stock directly comes from the depot or not, (2) whether the rolling stock enters the depot or not immediately after a train service is executed, and (3) whether the rolling stock serves other train services afterwards or not.

4.2. Systematic constraints

In this subsection, systematic constraints will be formulated to make the train schedule satisfying the passenger demand and guaranteeing the feasibility of the rolling stock circulation plan simultaneously. The constraints are next described in detail.

Table 2: Parameters and subscripts for the model formulation

Notations	Definition
$\mathbf{J}$	set of stations
$\mathbf{I}$	set of train services
i, i'	index of train services, $i, i' \in \mathbf{I}$
j, j'	index of stations, $j, j' \in \mathbf{J}$
J	total number of stations
I	total number of train services
up	the up operating direction
dn	the down operating direction
$r_{j,\min}^{\mathrm{up}}$	minimal running time from station j to $j+1$, $j \in \mathbf{J}/\{J\}$
$r_{j,\max}^{\mathrm{up}}$	maximal running time from station j to $j+1$, $j \in \mathbf{J}/\{J\}$
$r_{j,\min}^{\mathrm{dn}}$	minimal running time from station j to $j-1$, $j \in \mathbf{J}/\{1\}$
$r_{j,\max}^{\mathrm{dn}}$	maximal running time from station j to $j-1$, $j \in \mathbf{J}/\{1\}$
$r_{J,\min}^{\mathrm{turn}}$	minimal turnaround time at station J
$r_{J,\max}^{\mathrm{turn}}$	maximal turnaround time at station J
$r_{1,\min}^{\mathrm{turn}}$	minimal turnaround time at station 1
$r_{1,\max}^{\mathrm{turn}}$	maximal turnaround time at station 1
$\tau_{j,\min}$	minimal dwell time at station j
$\tau_{j,\max}$	maximal dwell time at station j
$h_{\min}$	minimal headway between two consecutive train services
$h_{\max}$	maximal headway between two consecutive train services
t_{start}	start time of the operating period, $t_{\mathrm{start}} = t_0$
t_{end}	end time of the operating period, $t_{\mathrm{end}} = t_K$
$\kappa_{j,k}^{\mathrm{up}}$	number of passengers traveling between stations j and $j+1$ in time interval $[t_{k-1}, t_k)$ in the up direction, $j \in \mathbf{J}/\{J\}$, $k \in \{1, 2, \ldots, K\}$
$\kappa_{j,k}^{\mathrm{dn}}$	number of passengers traveling between stations j and $j-1$ in time interval $[t_{k-1}, t_k)$ in the down direction, $j \in \mathbf{J}/\{1\}$, $k \in \{1, 2, \ldots, K\}$
$\lambda_{j,k}^{\mathrm{up}}$	passenger traveling rates between stations j and $j+1$ in time interval $[t_{k-1}, t_k)$ in the up direction, $j \in \mathbf{J}/\{J\}$, $k \in \{1, 2, \ldots, K\}$
$\lambda_{j,k}^{\mathrm{dn}}$	passenger traveling rates between stations j and $j-1$ in time interval $[t_{k-1}, t_k)$ in the down direction, $j \in \mathbf{J}/\{1\}$, $k \in \{1, 2, \ldots, K\}$
$\tilde{\lambda}_j^{\mathrm{up}}$	passenger traveling rates between stations j and $j+1$ in the up direction, $j \in \mathbf{J}/\{J\}$
$\tilde{\lambda}_j^{\mathrm{dn}}$	passenger traveling rates between stations j and $j-1$ in the down direction, $j \in \mathbf{J}/\{1\}$
t_{enter}	entry time from station 1 to the depot
t_{exit}	exit time from the depot to station 1
$\sigma_{\max}$	maximum allowable load factor
C_{train}	capacity of trains in terms of the maximum number of on-board passengers
N_{rs}	number of available rolling stocks
M	a sufficiently large number
ε	a small positive number
$\gamma_1, \gamma_2, \gamma_3$	positive weights in the objective function

Table 3: Variables for the model formulation

Notations	Definition
$d_{i,j}^{\text{dn}}$	departure time of train service i at station j in the down direction
$d_{i,j}^{\text{up}}$	departure time of train service i at station j in the up direction
$a_{i,j}^{\text{dn}}$	arrival time of train service i at station j in the down direction
$a_{i,j}^{\text{up}}$	arrival time of train service i at station j in the up direction
$r_{i,j}^{\text{up}}$	running time of train service i from station j to $j+1$
$r_{i,j}^{\text{dn}}$	running time of train service i from station j to $j-1$
$\tau_{i,j}^{\text{dn}}$	dwell time at station j in the down direction for train service i
$\tau_{i,j}^{\text{up}}$	dwell time at station j in the up direction for train service i
$\sigma_{i,j}^{up}$	load factor of train service i between stations j and $j+1$ in the up direction
$\sigma_{i,j}^{dn}$	load factor for train service i between stations j and $j-1$ in the down direction
ξ_i	0-1 binary variable, if the rolling stock for train service i does not come from the depot, $\xi_i = 1$; otherwise, $\xi_i = 0$
δ_i	0-1 binary variable, if the rolling stock for train service i does not enter the depot, $\delta_i = 1$; otherwise, $\delta_i = 0$
$\beta_{i,i'}$	0-1 binary variable, if train services i is connected with service i' with $i < i'$, $\beta_{i,i'} = 1$; otherwise, $\beta_{i,i'} = 0$
$y_{i,i'}$	0-1 binary variable, if the rolling stock for train service i that enters the depot can be used for service i' with $i < i'$, $y_{i,i'} = 1$; otherwise, $y_{i,i'} = 0$

4.2.1. Departure and arrival constraints of train services

The integrated train scheduling and circulation planning problem determines the departure and arrival times of train services with consideration of the passenger demand. The departure and arrival times in the up direction should satisfy

$$
\begin{aligned}
d_{i,j}^{\text{up}} &= a_{i,j}^{\text{up}} + \tau_{i,j}^{\text{up}}, \\
a_{i,j+1}^{\text{up}} &= d_{i,j}^{\text{up}} + r_{i,j}^{\text{up}},
\end{aligned}
\tag{1}
$$

for $i \in \mathbf{I}$ and $j \in \mathbf{J}/\{J\}$, where $r_{i,j}^{\text{up}}$ is the running time from station j to $j+1$ for train service i. Furthermore, the departure and arrival times in the down direction can be expressed as

$$
\begin{aligned}
d_{i,j}^{\text{dn}} &= a_{i,j}^{\text{dn}} + \tau_{i,j}^{\text{dn}}, \\
a_{i,j-1}^{\text{dn}} &= d_{i,j}^{\text{dn}} + r_{i,j}^{\text{dn}},
\end{aligned}
\tag{2}
$$

for $i \in \mathbf{I}$ and $j \in \mathbf{J}/\{1\}$, where $r_{i,j}^{\text{dn}}$ is the running time from station j to $j-1$ for train service i. Moreover, the running times and dwell times should satisfy the following constraints

$$
\begin{aligned}
r_{j,\min}^{\text{up}} \leq r_{i,j}^{\text{up}} \leq r_{j,\max}^{\text{up}}, &\quad r_{j,\min}^{\text{dn}} \leq r_{i,j}^{\text{dn}} \leq r_{j,\max}^{\text{dn}}, \\
\tau_{j,\min} \leq \tau_{i,j}^{\text{up}} \leq \tau_{j,\max}, &\quad \tau_{j,\min} \leq \tau_{i,j}^{\text{dn}} \leq \tau_{j,\max},
\end{aligned}
\tag{3}
$$

for $i \in \mathbf{I}$ and $j \in \mathbf{J}$.

Remark. *For the real-valued variables in Table 3, if we choose the departure and arrival times (i.e., $d_{i,j}^{\text{dn}}$, $d_{i,j}^{\text{up}}$, $a_{i,j}^{\text{dn}}$, and $a_{i,j}^{\text{up}}$) at stations as independent variables, then the running times (i.e., $r_{i,j}^{\text{up}}$ and $r_{i,j}^{\text{dn}}$), dwell times (i.e., $\tau_{i,j}^{\text{up}}$ and $\tau_{i,j}^{\text{dn}}$) will become dependent variables.*
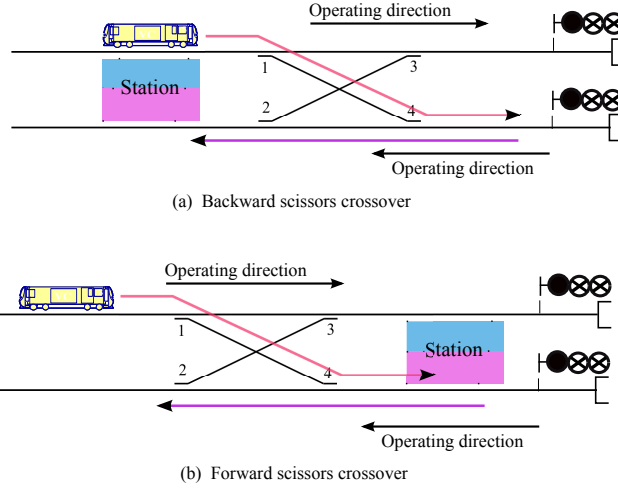
13

(a) Backward scissors crossover

(b) Forward scissors crossover

Figure 6: The two types of layouts for turnaround stations

4.2.2. Headway constraints

In order to guarantee the safety of train operations, minimum headways between train services are calculated based on the infrastructure data and the train characteristics. In addition, a maximum headway between two consecutive trains is introduced for urban rail transit systems to ensure a certain level of services. Since the indices of train services increase with the departure time at station 1 in the up direction and there is no overtaking anywhere on the urban rail transit line, according to Assumption (1), the order of train services holds the same for all the stations. Hence, the headway constraints between train services i and $i+1$ can be formulated as

$$
\begin{aligned}
h_{\min} \leq d_{i+1,j}^{\mathrm{up}} - d_{i,j}^{\mathrm{up}} \leq h_{\max}, \\
h_{\min} \leq d_{i+1,j}^{\mathrm{dn}} - d_{i,j}^{\mathrm{dn}} \leq h_{\max},
\end{aligned}
\tag{4}
$$

for $i \in \mathbf{I}/\{I\}$ and $j \in \mathbf{J}$.

4.2.3. Turnaround constraints

The turnaround operations allow trains to change from the up/down direction to the down/up direction. Since we only consider full-length train services, the turnaround operations can only occur at stations 1 and J. The turnaround operations are usually the bottleneck of urban rail transit systems and mainly determine the maximum capacity of the line. To ensure the safe operation of the trains, the turnaround constraints must be satisfied, which depend on the layout of the turnaround station and on the route settings for the turnaround operations. For urban rail transit systems, the layouts of turnaround stations are different from each other, especially for the stations that are connected with depots. In general, there are two types of layouts, as given in Figure 6. Figure 6(a) illustrates the case that the double crossovers are behind the station, so each train stops at the station and all passengers alight from the train. Then, the train starts the turnaround operations as shown in Figure 6(a). In this case, the train uses switch (1,4) to arrive at the stop track, changes its operating direction, and runs to the next station in the other operating direction. Figure 6(b) illustrates the case that the double crossovers are before the
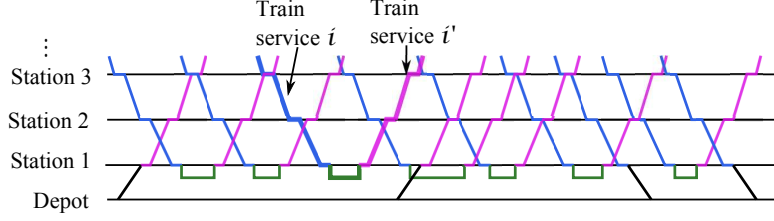
14

Figure 7: The turnaround operation at station 1 and the connection between train services

station, so the train will pass the crossover (1,4) before arriving at the station, to let passengers get off. Moreover, the train changes the operating direction while dwelling at the station. After the boarding process of passengers, the train can depart from the station. However, Figure 6 only illustrates one possibility for the turnaround operation, while there are other turnaround possibilities, such as using switch (2,3) if the headways between trains are relatively small.

In this paper, we only consider the case of the backward double crossover and only one crossover is used for turnaround operations. For all the services, trains need to turnaround at station J, where the operating direction changes from up to down. For service $i \in \mathbf{I}$, its turnaround operation at station J should satisfy

$$r_{J,\min}^{\text{turn}} \leq a_{i,J}^{\text{dn}} - d_{i,J}^{\text{up}} \leq r_{J,\max}^{\text{turn}}, \tag{5}$$

where $d_{i,J}^{\text{up}}$ is the departure time of service i at station J in the up direction, $a_{i,J}^{\text{dn}}$ is the arrival time of service i at station J in the down direction, $r_{J,\min}^{\text{turn}}$ and $r_{J,\max}^{\text{turn}}$ are the minimal and maximal turnaround times at station J. Furthermore, the departure and arrival times of two consecutive train services should satisfy the following constraints for the turnaround operation at station J

$$d_{i+1,J}^{\text{up}} - a_{i,J}^{\text{dn}} \geq 0, \forall i \in \mathbf{I}/\{I\}, \tag{6}$$

which means that service $i+1$ can start the turnaround operation only when service i is arrived at station J in the down direction.

Furthermore, the rolling stocks could go back to the depot or turnaround at station 1 to serve other services, as shown in Figure 7. If services i and i' are connected with each other, i.e., they are served by the same rolling stock and connected by the turnaround operation at station 1, then the arrival and departure times of train services i and i' should satisfy the turnaround constraints. However, if services i and i' are not connected, i.e., $\beta_{i,i'} = 0$, then there is no constraint regarding these two train services. By introducing a sufficiently large positive number M, the turnaround constraints for the train services at station 1 can be formulated as

$$a_{i',1}^{\text{up}} - d_{i,1}^{\text{dn}} \geq r_{1,\min}^{\text{turn}} - M(1 - \beta_{i,i'}),$$
$$a_{i',1}^{\text{up}} - d_{i,1}^{\text{dn}} \leq r_{1,\max}^{\text{turn}} + M(1 - \beta_{i,i'}), \tag{7}$$

where $r_{1,\min}^{\text{turn}}$ and $r_{1,\max}^{\text{turn}}$ are the minimal and maximal turnaround times at station 1. When $\beta_{i,i'} = 0$, i.e., train service i does not need to connect with train service i', the turnaround constraints (7) become $a_{i',1}^{\text{up}} - d_{i,1}^{\text{dn}} \geq r_{1,\min}^{\text{turn}} - M$ and $a_{i',1}^{\text{up}} - d_{i,1}^{\text{dn}} \leq r_{1,\max}^{\text{turn}} + M$. These values will be satisfied automatically due to the big M value. When $\beta_{i,i'} = 1$, i.e., train service i is connected with train service i', the turnaround constraints (7) become $a_{i',1}^{\text{up}} - d_{i,1}^{\text{dn}} \geq r_{1,\min}^{\text{turn}}$ and $a_{i',1}^{\text{up}} - d_{i,1}^{\text{dn}} \leq r_{1,\max}^{\text{turn}}$, which mean that the turnaround operations at station 1 must satisfy the minimal and maximal turnaround time constraints.

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4.2.4. Passenger demand constraints

Passenger demand is a significant factor for train scheduling, since the number of train services and the headways between train services highly depend on this factor. The passenger demand estimation heavily relies on manual data collection, such as passenger surveys to estimate the origin-destination (OD) matrix (Zhao et al., 2007). However, with the introduction of automatic fare collection systems and automatic passenger counting systems, more accurate passenger demand information can be obtained by the urban rail operators.

Due to the complexity of train scheduling models and to historical reasons, there are two ways to describe the passenger demand in the literature: OD-independent passenger demand and OD-dependent passenger demand. For the OD-independent passenger demand, the origin and destination of each passenger are not considered, while the number of passengers and the passenger arrival rate at stations are used (Elberlein et al., 2001; Wang et al., 2015a; Niu and Zhou, 2013; Barrena et al., 2014). For the OD-dependent passenger demand, the origin and destination of each passenger are taken into account during train scheduling (Niu et al., 2015; Wang et al., 2014, 2015b; Corman et al., 2017).

For the simplicity of the train scheduling model and for the easy usage of the practical passenger demand data, we adopt the OD-independent passenger demand. As mentioned in Yue et al. (2017), the number of passengers that travel between two consecutive stations is an important factor for urban rail transit systems. In practice, the passenger volume between two adjacent stations (also called sectional passenger demand because the segment between two consecutive stations is usually defined as a section) is counted for a certain time period, such as 30 minutes, 15 minutes or 5 minutes. In this paper, we also adopt the sectional passenger demand, as in Yue et al. (2017) and Sun et al. (2014). Figure 3 gives the sectional passenger demand between two consecutive stations, i.e., Songjiazhuang and Xiaocun in Beijing Yizhuang line, where the number of passengers traveling between these two stations is given every 30 minutes. Moreover, the passenger traveling rates on sections are introduced to describe the passenger demand (Wang et al., 2016). The passenger traveling rates for the up direction can be written as

$$\tilde{\lambda}_j^{\text{up}}(t) = \begin{cases} \lambda_{j,1}^{\text{up}}, & \text{if } t \in [t_0, t_1) \\ \lambda_{j,2}^{\text{up}}, & \text{if } t \in [t_1, t_2) \\ \dots & \dots \\ \lambda_{j,k}^{\text{up}}, & \text{if } t \in [t_{k-1}, t_k) \\ \dots & \dots \\ \lambda_{j,K}^{\text{up}}, & \text{if } t \in [t_{K-1}, t_K) \end{cases}, \forall j \in \mathbf{J}/\{J\}. \tag{8}$$

with

$$\lambda_{j,k}^{\text{up}} = \kappa_{j,k}^{\text{up}}/(t_k - t_{k-1}), \forall k \in \{1, 2, \dots, K\}, \tag{9}$$

where $\kappa_{j,k}^{\text{up}}$ is the number of passengers traveling between stations j and $j+1$ in time interval $[t_{k-1}, t_k)$. Note that the operating time period $[t_{\text{start}}, t_{\text{end}}]$ is split into K time slots with the splitting time instants $t_1, t_2, \dots,$ and t_{K-1}, while we have $t_0 = t_{\text{start}}$ and $t_K = t_{\text{end}}$. Similarly, the passenger traveling rates $\tilde{\lambda}_j^{\text{dn}}(\cdot)$ for the down direction can be formulated.

The passenger satisfaction for the urban rail transit systems is affected by the crowdedness of train services, which is normally represented by the load factor. This factor is defined as the ratio between the number of on-board passengers and the capacity of trains (including the number of seats and the allowable standing passengers). According to the operating practice, the load factors of trains are larger than 1, and even reach 1.2 in some extreme cases, for most of the

Beijing urban rail transit lines. These values are caused, e.g., by the huge passenger demand, the limited number of available trains, the minimal headway between trains, and the non-optimal train schedules. In order to guarantee the passenger satisfaction, we include the following load factor constraints in our formulation

$$\sigma_{i,j}^{\text{up}} \leq \sigma_{\max}, \sigma_{i,j}^{\text{dn}} \leq \sigma_{\max}, \forall i \in \mathbf{I}, j \in \mathbf{J}, \tag{10}$$

where $\sigma_{\max}$ is the maximal allowable load factor determined by the rail operator. Here, we only give the equations for the calculation of the load factor $\sigma_{i,j}^{\text{up}}$. The load factor $\sigma_{i,j}^{\text{dn}}$ can be computed in a similar way. As we mentioned before, the load factor is the ratio between the number of onboard passengers and the capacity of the circulating trains, i.e.,

$$\sigma_{i,j}^{\text{up}} = \frac{p_{i,j}^{\text{up}}}{C_{\text{train}}}, \forall i \in \mathbf{I}, j \in \mathbf{J}, \tag{11}$$

where C_{train} is the train capacity and $p_{i,j}^{\text{up}}$ is the number of passengers on board train service i between stations j and $j+1$. The latter can be calculated as follows:

$$p_{i,j}^{\text{up}} = \int_{d_{i-1,j}^{\text{up}}}^{d_{i,j}^{\text{up}}} \tilde{\lambda}_j^{\text{up}}(t)dt, \forall i \in \mathbf{I}/\{1\}, j \in \mathbf{J}. \tag{12}$$

We note that for the first train service, i.e., $i = 1$, the number of onboard passengers $p_{1,j}^{\text{up}}$ should be computed by

$$p_{1,j}^{\text{up}} = \int_{t_{\text{start}}}^{d_{1,j}^{\text{up}}} \tilde{\lambda}_j^{\text{up}}(t)dt, \forall j \in \mathbf{J}. \tag{13}$$

4.2.5. Rolling stock circulation constraints

The rolling stock circulation plan is characterized by binary variables ξ_i, δ_i, and $\beta_{i,i'}$ for $i, i' \in \mathbf{I}$. For the small example of train schedule and rolling stock circulation plan of Figure **??**, we have $\xi_i = 0$ for $i \in \{1, 2, \ldots, 6\}$ and $\xi_i = 1$ for $i \in \{7, 8, \ldots, 18\}$, which means that the rolling stocks for train services 1, 2, $\ldots$, and 6 come out from the depot. In addition, we have $\delta_i = 1$ for $i \in \{1, 2, \ldots, 12\}$ and $\delta_i = 0$ for $i \in \{13, 14, \ldots, 18\}$, which means that the rolling stocks for train services 13, 14, $\ldots$, and 18 go back to the depot after completing these train services. Furthermore, we have $\beta_{1,7} = 1$ and $\beta_{7,13} = 1$ for rolling stock 1, which means that train services 1, 7, and 13 are operated by the same rolling stock, i.e., rolling stock 1. Similarly, the circulation plan of the other rolling stocks can be obtained based on the values of $\beta_{i,i'}$ with $i, i' \in \{1, 2, \ldots, 18\}$.

In Figure 7, the rolling stock assigned to perform a train service can come from the depot directly or can just be the same of a previous service. Furthermore, when a rolling stock ends its current service, it can go back to the depot or can perform another service. Train services i and i' in Figure 7 are assigned to the same rolling stock, i.e., service i is connected with service i', so we have $\beta_{i,i'} = 1$. Based on the definitions of the binary variables ξ_i and $\beta_{i,i'}$ in Table 3, we have

$$\xi_{i'} = \sum_{i \in \mathbf{I}} \beta_{i,i'}, \tag{14}$$

which basically means that if there is no service connected with service i', i.e., $\sum_{i \in \mathbf{I}} \beta_{i,i'} = 0$, then the rolling stock serving i' must come from the depot. On the other hand, if the rolling stock

serving i' does not come from depot directly, then there must exist only one service connected with train service i', i.e., $\sum_{i \in \mathbf{I}} \beta_{i,i'} = 1$, since $\xi_{i'}$ is a binary variable and its maximum value is equal to 1. We thus also have the following constraint

$$\sum_{i \in \mathbf{I}} \beta_{i,i'} \leq 1. \tag{15}$$

Furthermore, for binary variables δ_i and $\beta_{i,i'}$, we have

$$\delta_i = \sum_{i' \in \mathbf{I}} \beta_{i,i'}, \tag{16}$$

which means that if service i is not connected with any service, i.e., $\sum_{i' \in \mathbf{I}} \beta_{i,i'} = 0$, then the rolling stock serving i enter the depot. Otherwise, the rolling stock serving i does not enter the depot and it must perform another service. Each rolling stock can perform at most one service at a time, i.e., service i can only be connected with another service. We thus have

$$\sum_{i' \in \mathbf{I}} \beta_{i,i'} \leq 1. \tag{17}$$

In addition, for any two services i and i' with $i < i'$, when $\beta_{i,i'} = 1$, i.e., i is connected with i', then the rolling stock serving i must not go back to the depot, i.e., $\delta_i = 1$. But this rolling stock must turn around at station 1 and perform service i'. Hence, the rolling stock serving i' does not directly come from the depot, i.e., $\xi_{i'} = 1$. The rolling stock circulation plan should satisfy the following constraints:

$$\begin{aligned} \delta_i + \xi_{i'} &\leq 2 + M(1 - \beta_{i,i'}), \\ \delta_i + \xi_{i'} &\geq 2 - M(1 - \beta_{i,i'}). \end{aligned} \tag{18}$$

When $\beta_{i,i'} = 0$, the constraints given in (18) will be satisfied automatically due to the big M.

In practical operations of an urban rail transit line, the number of rolling stocks available for the daily operation is limited and is denoted by N_{rs}. In this paper, we only consider one depot. All the rolling stocks are in the depot at the start of the daily operations and need to go back to the depot at the end of the daily operations. When $\xi_i = 0$, i.e., the rolling stock for train service i comes out from the depot, we need to check whether there is still a rolling stock inside the depot to perform service i. The checking condition is basically the following: the difference between the total number of exiting operations and the total number of entering operations should be less than or equal to the number of available rolling stocks. This condition can be formulated as

$$\sum_{m=1}^{i} (1 - \xi_m) - \sum_{m=1}^{i-1} (1 - \delta_m) y_{m,i} \leq N_{\mathrm{rs}}, \tag{19}$$

where $\sum_{m=1}^{i} (1 - \xi_m)$ and $\sum_{m=1}^{i-1} (1 - \delta_m) y_{m,i}$ are the total numbers of exiting and entering operations regarding the depot until the departure of train service i at station 1 in the up direction. In particular, binary-valued variable $y_{m,i}$ is introduced to denote whether the rolling stocks entering the depot after service m could be used to perform service i. This constraint is defined as $[y_{m,i} = 1] \Leftrightarrow [d_{m,1}^{\mathrm{dn}} - d_{i,1}^{\mathrm{up}} + t_{\mathrm{enter}} + t_{\mathrm{exit}} \leq 0]$. The previous definition is equivalent to the following linear constraint

$$d_{m,1}^{\mathrm{dn}} - d_{i,1}^{\mathrm{up}} + t_{\mathrm{enter}} + t_{\mathrm{exit}} \leq M(1 - y_{m,i}). \tag{20}$$

4.3. Objective function

The urban rail operators aim to minimize the operating cost and to provide a certain service level to passengers at the same time. Since the passenger demand in urban rail transit lines varies significantly along the daily time, it is important for urban rail operators to schedule trains according to the passenger demand fluctuation. A certain service level can be achieved by the load factor constraints (10) introduced in Section 4.2.4, where the maximum allowable load factor σ_{max} can take a value predefined by the urban rail operators based on their experiences. In order to distribute the passengers to the train services, the difference between the actual maximal load factor of each train service and the mean value of the actual maximal load factors of all train services is minimized in the objective function, i.e.,

$$f_{load} = \sum_{i \in \mathbf{I}} \left(\left| \max_{j \in \mathbf{J}} \sigma_{i,j}^{up} - \sigma_{mean}^{up} \right| + \left| \max_{j \in \mathbf{J}} \sigma_{i,j}^{dn} - \sigma_{mean}^{dn} \right| \right), \tag{21}$$

where

$$\sigma_{mean}^{up} = \frac{\sum_{i \in \mathbf{I}} \max_{j \in \mathbf{J}} \sigma_{i,j}^{up}}{I} \text{ and } \sigma_{mean}^{dn} = \frac{\sum_{i \in \mathbf{I}} \max_{j \in \mathbf{J}} \sigma_{i,j}^{dn}}{I}. \tag{22}$$

Apart from the crowdedness of train services, the passenger satisfaction is also influenced by the traveling time, which involves the waiting time at stations and the onboard time. Because we consider the OD-independent passenger demand in this paper, the onboard time and waiting time of passengers cannot be calculated accurately. Moreover, the passengers are usually more sensitive to the waiting time than to the onboard time (Niu and Zhou, 2013). We thus only consider the waiting time of passengers, which can be minimized by maximizing the regularity of train services. Based on the accumulated or sectional passenger demand, it is impossible to calculate the waiting time accurately. In Yue et al. (2017), the waiting time of passengers is approximated by the number of queuing passengers in the various sections (i.e., the segments between any two adjacent stations). In Sun et al. (2014), the waiting time of passengers is formulated as the product of the number of passengers traveling on the sections and the average waiting time of passenger groups in each time interval. However, it would be hard to distinguish the passengers that are already on-board with the passengers that are waiting at platforms, since the origin and destination of passengers are not directly considered in the passenger demand profile. As stated in Niu and Zhou (2013), an even schedule with a constant headway between consecutive vehicles can reduce total waiting time, when the passenger arrival pattern at stations follows some particular probability distributions, such as uniform and Poisson distributions. However, Niu and Zhou (2013) considered the train scheduling problem for a heavily congested urban rail transit line, where an even headway timetable may lead to longer passenger waiting times during oversaturated periods, since some passengers may not be able to board the next arrival train. In the current paper, we assume that all passengers get on the first coming train and their satisfaction (or dissatisfaction) is indicated by the on-board crowdedness, i.e., the load factor. Furthermore, for a short time period the passenger arrival pattern at stations can be assumed to follow uniform or Poisson distributions. We thus minimize the headway variation between consecutive train services in order to limit the waiting time of passengers. The headway variation is defined as the absolute value of the difference between the current headway and the mean of the neighboring headways. The headway variation $f_{headway}^{up}$ in the up direction can be formulated as

$$f_{headway}^{up} = \sum_{i \in \mathbf{I}/\{1\}, j \in \mathbf{J}} \left| (d_{i,j}^{up} - d_{i-1,j}^{up}) - \frac{\sum_{m=\max(i-i_1,2)}^{\min(i+i_2,I)} (d_{m,j}^{up} - d_{m-1,j}^{up})}{\min(i+i_2,I) - \max(i-i_1,2)} \right|, \tag{23}$$

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where i_1 and i_2 are the numbers of neighboring train services before and after train service i. In particular, (23) can be simplified and rewritten as

$$f_{\text{headway}}^{\text{up}} = \sum_{i \in \mathbf{I}/\{1\}, j \in \mathbf{J}} \left| (d_{i,j}^{\text{up}} - d_{i-1,j}^{\text{up}}) - \frac{d_{\min(i+i_2,I),j}^{\text{up}} - d_{\max(i-i_1,2)-1,j}^{\text{up}}}{\min(i+i_2,I) - \max(i-i_1,2)} \right|. \tag{24}$$

The headway variation $f_{\text{headway}}^{\text{dn}}$ in the down direction can be calculated in a similar way. We then have

$$f_{\text{headway}} = f_{\text{headway}}^{\text{up}} + f_{\text{headway}}^{\text{dn}}. \tag{25}$$

The operating cost of urban rail operators is mostly affected by the number of required rolling stocks and the number of services in the train schedule. In the integrated train scheduling and rolling stock circulation planning formulation, the total number of train services I is considered as a parameter and not as a variable. However, in practice, the possible value of I is defined in an interval. We denote as $[I_{\min}, I_{\max}]$. An optimal total number of train services I can be decided by solving the integrated optimization problem multiple times and by taking different values of I in the interval $[I_{\min}, I_{\max}]$. The number of rolling stocks required and the operational complexity of the integrated optimization problem can be reduced by minimizing the number of entering/exiting depot operations, i.e.,

$$f_{\text{depot}} = \sum_{i \in \mathbf{I}} \delta_i. \tag{26}$$

In this paper, we consider the load factor variation, the headway variation, and the number of entering/exiting depot operations as the objective function components and formulate the overall problem as a multi-objective optimization problem. The linear weighted method is used to handle the three objective function components and the objective function is formulated as follows:

$$f = \gamma_1 \frac{f_{\text{load}}}{f_{\text{load,nom}}} + \gamma_2 \frac{f_{\text{headway}}}{f_{\text{headway,nom}}} + \gamma_3 \frac{f_{\text{depot}}}{f_{\text{depot,nom}}}, \tag{27}$$

where $f_{\text{load,nom}}$, $f_{\text{headway,nom}}$, and $f_{\text{depot,nom}}$ are the nominal values (or normalization factors) for each of the objective function components, which are calculated by solving the optimization problem related to each single objective function component; γ_1, γ_2, and γ_3 are positive weights to denote the importance of these three objectives. Specifically, if the three weights are set equal to 1, we obtain a Pareto-optimal solution. Moreover, the values of γ_1, γ_2, and γ_3 can be decided according to the urban rail operators' preferences and the practical operating situations.

4.4. Mathematical model

The integrated train scheduling and rolling stock circulation planning optimization problem can be formulated as follows:

$$\begin{cases} \min f = \gamma_1 \frac{f_{\text{load}}}{f_{\text{load,nom}}} + \gamma_2 \frac{f_{\text{headway}}}{f_{\text{headway,nom}}} + \gamma_3 \frac{f_{\text{depot}}}{f_{\text{depot,nom}}}, \\ \text{s.t. constraints (1)-(7), (10), (14)-(20).} \end{cases} \tag{28}$$

The integrated optimization problem given in (28) is a mixed integer nonlinear programming (MINLP) problem. The nonlinearities of the proposed model come from the load factor constraints (10), where the calculation of the on-board passengers is the integration of a piecewise constant function between the departure times of consecutive train services. Moreover, the

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headway variations f_{headway} and the load factor variations f_{load} in the objective function involve absolute functions, which are also nonlinear. The integer variables are introduced by the rolling stock circulation constraints to describe the entering or exiting of trains at the depot.

Remark. *The formulation given in* (28) *can be considered as a fundamental model for train scheduling and rolling stock circulation planning problems for an urban rail transit line. It can also be extended in many ways to satisfy a variety of particular requirements, such as OD-dependent passenger demand, no entering/exiting operations of rolling stocks in the depot for a certain period, and a maximum number of train services that a rolling stock can serve.*

5. Solution approach 1 - iterative nonlinear programming approach

In this section, a new iterative nonlinear programming (INP) approach is proposed to solve the MINLP problem. For each iteration, the estimated values of $\hat{\xi}_i$, $\hat{\delta}_i$, and $\hat{\beta}_{i,i'}$ are used for ξ_i, δ_i, and $\beta_{i,i'}$, respectively. This approach eliminates the binary variables in the original MINLP problem and the resulting optimization problem is a nonlinear programming problem, which can be solved, e.g., by using multi-start sequential quadratic programming and active-set algorithms. Based on a local optimum of the nonlinear programming problem, the new estimated values for ξ_i, δ_i, and $\beta_{i,i'}$ are calculated. By solving the nonlinear programming problem in an iterative way, we get a local optimum of the original MINLP problem. The detailed procedure of the INP method is given by Algorithm 1.

Algorithm 1 The procedure of the INP method

1: **Input** : feasible initial departure and arrival times for the up and down directions, i.e., $d_{i,j}^{\text{up}}(0)$, $d_{i,j}^{\text{dn}}(0)$, $a_{i,j}^{\text{up}}(0)$, $a_{i,j}^{\text{dn}}(0)$ for $i = 1,\ldots,I$ and $j = 1,\ldots,J$, $p_{\max}$, convergence tolerance ς, maximum number of iterations $p_{\max}$;

2: iteration index $p \leftarrow 0$;

3: calculate initial estimates $\hat{\xi}_i(p)$, $\hat{\delta}_i(p)$, and $\hat{\beta}_{i,i'}(p)$ by solving the MILP problem with the objective to minimize the number of depot operations f_{depot} and with constraints (7) and (14)-(18) based on $d_{i,j}^{\text{up}}(p)$, $d_{i,j}^{\text{dn}}(p)$, $a_{i,j}^{\text{up}}(p)$, and $a_{i,j}^{\text{dn}}(p)$;

4: calculate the initial objective function value $f(p)$ by using (27);

5: **Repeat**

6:      $p = p + 1$;

7:      substitute the estimated values $\hat{\xi}_i(p-1)$, $\hat{\delta}_i(p-1)$, and $\hat{\beta}_{i,i'}(p-1)$ into the original MINLP problem and get a new nonlinear programming problem;

8:      obtain the best sub-optimal departure and arrival times $d_{i,j}^{\text{up},*}(p)$, $d_{i,j}^{\text{dn},*}(p)$, $a_{i,j}^{\text{up},*}(p)$, and $a_{i,j}^{\text{dn},*}(p)$ by solving the nonlinear problem with multiple initial points;

9:      compute estimates $\hat{\xi}_i(p)$, $\hat{\delta}_i(p)$, and $\hat{\beta}_{i,i'}(p)$ by solving the MILP problem with the objective to minimize the number of depot operations f_{depot} and with constraints (7) and (14)-(18) based on $d_{i,j}^{\text{up},*}(p)$, $d_{i,j}^{\text{dn},*}(p)$, $a_{i,j}^{\text{up},*}(p)$, and $a_{i,j}^{\text{dn},*}(p)$;

10:      calculate the objective value $f(p)$ by using (27);

11: **Until** $p = p_{\max}$ or $|f(p) - f(p-1)| \leq \varsigma$

12: **Return** $d_{i,j}^{\text{up},*}(p)$, $d_{i,j}^{\text{dn},*}(p)$, $a_{i,j}^{\text{up},*}(p)$, $a_{i,j}^{\text{dn},*}(p)$, $\hat{\xi}_i(p)$, $\hat{\delta}_i(p)$, $\hat{\beta}_{i,i'}(p)$, $f(p)$

Since the values of the binary variables, i.e., ξ_i, δ_i, and $\beta_{i,i'}$, are fixed for the nonlinear programming problem in each iteration, the value of f_{depot} in the objective function keeps constant. Hence, the number of the entering/exiting depot operations f_{depot} can be eliminated from the objective function. However, in order to guide the nonlinear programming problem at each iteration and to minimize the number of depot operations, a new penalty term is added in the objective function for the nonlinear programming problem. The newly added penalty term is fixed according to the value of $\beta_{i,i'}$ with $i' > i$. The penalty is defined as follows

$$f_{\text{penalty}} = \begin{cases} 0 & \text{for } \beta_{i,i'} = 1, \\ r_{1,\min}^{\text{turn}} - \left(a_{i',1}^{\text{up}} - d_{i,1}^{\text{dn}}\right) & \text{for } \beta_{i,i'} = 0 \text{ and } a_{i',1}^{\text{up}} - d_{i,1}^{\text{dn}} < r_{1,\min}^{\text{turn}}, \\ \left(a_{i',1}^{\text{up}} - d_{i,1}^{\text{dn}}\right) - r_{1,\min}^{\text{turn}} & \text{for } \beta_{i,i'} = 0 \text{ and } a_{i',1}^{\text{up}} - d_{i,1}^{\text{dn}} > r_{1,\max}^{\text{turn}}. \end{cases} \tag{29}$$

When $\beta_{i,i'}$ is equal to 1, i.e., train services i and i' are connected, during the whole optimization process $\beta_{i,i'}$ is equal to 1 due to the constraints in (7); the penalty is then set to zero. However, when $\beta_{i,i'}$ is equal to 0, a penalty term is added and the difference between $a_{i',1}^{\text{up}}$ and $d_{i,1}^{\text{dn}}$ varies in the interval $[r_{1,\min}^{\text{turn}}, r_{1,\max}^{\text{turn}}]$; in this way, $\beta_{i,i'}$ has a higher probability to be equal to 1. Furthermore, the calculation of the estimates $\hat{\xi}_i$, $\hat{\delta}_i$, and $\hat{\beta}_{i,i'}$ is done by solving a mixed integer linear programming (MILP) problem, where the departure and arrival times of the services obtained by the nonlinear programming problem are not allowed to change. The MILP problem can be formulated as follows

$$\begin{cases} \min f = \sum_{i \in \mathbf{I}} \delta_i, \\ \text{s.t. constraints (7) and (14)-(18),} \end{cases} \tag{30}$$

where the objective function is to maximize the number of connected train services and all the constraints related with ξ_i, δ_i, and $\beta_{i,i'}$ are included in the MILP formulation.

We note that the INP approach deals with a sequence of nonlinear approximations of the original MINLP problem. We thus employ multiple starting points for the INP approach.

6. Solution approach 2 - MILP approach

In this section, the original mixed integer nonlinear programming (MINLP) problem will be transformed into mixed integer linear programming (MILP) problems by transforming and/or approximating the nonlinear terms into mixed logical dynamic models. The resulting MILP problem can be solved by several existing commercial and free solvers, such as CPLEX, Xpress-MP,GLPK (see, e.g., Linderoth and Ralphs (2005); Atamturk and Savelsbergh (2005)).

As for the objective function, the absolute functions (involved in the load factor variations and the headway fluctuations) are handled by introducing new real-valued variables (i.e., ω_i^{up}, ω_i^{dn}, $q_{i,j}^{\text{up}}$, and $q_{i,j}^{\text{dn}}$). The new variables should satisfy the following constraints

$$\begin{cases} \omega_i^{\text{up}} \geq \max_{j \in \mathbf{J}} \sigma_{i,j}^{\text{up}} - \sigma_{\text{mean}}^{\text{up}}, \\ \omega_i^{\text{up}} \geq -\max_{j \in \mathbf{J}} \sigma_{i,j}^{\text{up}} + \sigma_{\text{mean}}^{\text{up}}, \\ \omega_i^{\text{dn}} \geq \max_{j \in \mathbf{J}} \sigma_{i,j}^{\text{dn}} - \sigma_{\text{mean}}^{\text{dn}}, \\ \omega_i^{\text{dn}} \geq -\max_{j \in \mathbf{J}} \sigma_{i,j}^{\text{dn}} + \sigma_{\text{mean}}^{\text{dn}}, \end{cases} \tag{31}$$

and

$$
\begin{cases}
q_{i,j}^{\text{up}} \geq (d_{i,j}^{\text{up}} - d_{i-1,j}^{\text{up}}) - \frac{d_{\min(i+i_2,I),j}^{\text{up}} - d_{\max(i-i_1,2)-1,j}^{\text{up}}}{\min(i+i_2,I) - \max(i-i_1,2)}, \\
q_{i,j}^{\text{up}} \geq -(d_{i,j}^{\text{up}} - d_{i-1,j}^{\text{up}}) + \frac{d_{\min(i+i_2,I),j}^{\text{up}} - d_{\max(i-i_1,2)-1,j}^{\text{up}}}{\min(i+i_2,I) - \max(i-i_1,2)}, \\
q_{i,j}^{\text{dn}} \geq (d_{i,j}^{\text{dn}} - d_{i-1,j}^{\text{dn}}) - \frac{d_{\min(i+i_2,I),j}^{\text{dn}} - d_{\max(i-i_1,2)-1,j}^{\text{dn}}}{\min(i+i_2,I) - \max(i-i_1,2)}, \\
q_{i,j}^{\text{dn}} \geq -(d_{i,j}^{\text{dn}} - d_{i-1,j}^{\text{dn}}) + \frac{d_{\min(i+i_2,I),j}^{\text{dn}} - d_{\max(i-i_1,2)-1,j}^{\text{dn}}}{\min(i+i_2,I) - \max(i-i_1,2)}.
\end{cases}
\tag{32}
$$

The objective function can then be approximated by

$$
f = \gamma_1 \frac{(\omega_i^{\text{up}} + \omega_i^{\text{dn}})}{f_{\text{load,nom}}} + \gamma_2 \frac{(q_{i,j}^{\text{up}} + q_{i,j}^{\text{dn}})}{f_{\text{headway,nom}}} + \gamma_3 \frac{f_{\text{depot}}}{f_{\text{depot,nom}}}.
\tag{33}
$$

It is easy to verify that when we minimize the objective function (33) subject to (31) and (32), the optimal value of ω_i^{up} and ω_i^{dn} will be equal to $\left| \max_{j \in \mathbf{J}} \sigma_{i,j}^{\text{up}} - \sigma_{\text{mean}}^{\text{up}} \right|$ and $\left| \max_{j \in \mathbf{J}} \sigma_{i,j}^{\text{dn}} - \sigma_{\text{mean}}^{\text{dn}} \right|$. This also holds for $q_{i,j}^{\text{up}}$ and $q_{i,j}^{\text{dn}}$. So the original objective function (27) will also be optimized.

As for the nonlinear load factor constraints (10), these will be transformed into an exact mixed logical dynamic model by introducing new binary-valued variables and real-valued variables in Section 6.1. In addition, we will also propose an approximated mixed logical dynamic model for handling constraints (10), to reduce the computational effort, in Section 6.2.

6.1. Accurate MILP model

Regarding the nonlinear load factor constraints, the calculation of the onboard passengers is an integral of the passenger traveling rates between two consecutive departures times, as given in expression (12). Binary variable $x_{i,j,k}^{\text{up}}$ with $i \in \mathbf{I}$, $j \in \mathbf{J}$, and $k \in \{1,2,\ldots,K\}$ is introduced to indicate whether the departure time $d_{i,j}^{\text{up}}$ of train service i at station j is in time interval $[t_{k-1},t_k]$ or not, i.e.,

$$
x_{i,j,k}^{\text{up}} = \begin{cases} 1 & \text{if the departure time } d_{i,j}^{\text{up}} \text{ is in } [t_{k-1},t_k], \\ 0 & \text{if the departure time } d_{i,j}^{\text{up}} \text{ is not in } [t_{k-1},t_k]. \end{cases}
\tag{34}
$$

Based on the definition of $x_{i,j,k}^{\text{up}}$, we have

$$
d_{i,j}^{\text{up}} \leq \sum_{k=1}^{K} x_{i,j,k}^{\text{up}} t_k, \forall i,j,
$$

$$
d_{i,j}^{\text{up}} \geq \sum_{k=1}^{K} x_{i,j,k}^{\text{up}} t_{k-1}, \forall i,j,
\tag{35}
$$

$$
\sum_{k=1}^{K} x_{i,j,k}^{\text{up}} = 1, \forall i,j.
$$

The calculation of the onboard passengers includes three scenarios as illustrated in Figure 8, where the departure times of two consecutive trains can be in the same time interval (Figure 8(a)), in two consecutive time intervals (Figure 8(b)), or in two non-consecutive time intervals (Figure 8(c)). For the scenario given in Figure 8(a), i.e., the passenger arrival rate is a constant
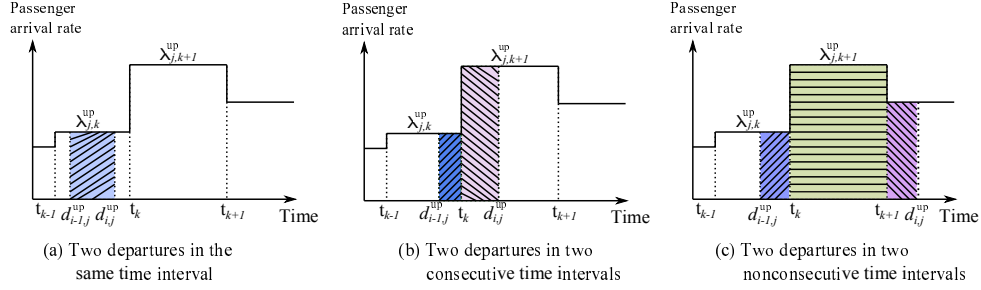
23

(a) Two departures in the same time interval

(b) Two departures in two consecutive time intervals

(c) Two departures in two nonconsecutive time intervals

Figure 8: Three scenarios for the calculation of onboard passengers

between the departures of services i and $i-1$. When train service i departs from station j, the number of on-board passengers can be calculated by

$$p_{i,j}^{\mathrm{up}} = \sum_{k=1}^{K} \lambda_{j,k}^{\mathrm{up}} \left(x_{i,j,k}^{\mathrm{up}} d_{i,j}^{\mathrm{up}} - x_{i-1,j,k}^{\mathrm{up}} d_{i-1,j}^{\mathrm{up}} \right). \tag{36}$$

For the scenario of Figure 8(b), i.e., when the passenger arrival rate changes once between the departure times $d_{i-1,j}^{\mathrm{up}}$ and $d_{i,j}^{\mathrm{up}}$, the number of onboard passengers can be calculated by

$$p_{i,j}^{\mathrm{up}} = \sum_{k=1}^{K} \lambda_{j,k}^{\mathrm{up}} \left(t_k - d_{i-1,j}^{\mathrm{up}} \right) x_{i-1,j,k}^{\mathrm{up}} + \sum_{k=1}^{K} \lambda_{j,k}^{\mathrm{up}} \left(d_{i,j}^{\mathrm{up}} - t_{k-1} \right) x_{i,j,k}^{\mathrm{up}}. \tag{37}$$

Moreover, when the two departures are in two non-consecutive time intervals (Figure 8(c)), the number of onboard passengers is computed as

$$p_{i,j}^{\mathrm{up}} = \sum_{k=1}^{K} \lambda_{j,k}^{\mathrm{up}} \left(t_k - d_{i-1,j}^{\mathrm{up}} \right) x_{i-1,j,k}^{\mathrm{up}} + \sum_{k=1}^{K} \lambda_{j,k}^{\mathrm{up}} \left(d_{i,j}^{\mathrm{up}} - t_{k-1} \right) x_{i,j,k}^{\mathrm{up}} +$$
$$\sum_{k=1}^{K} \lambda_{j,k}^{\mathrm{up}} \left(\sum_{\ell=1}^{k-1} x_{i-1,j,\ell}^{\mathrm{up}} - \sum_{\ell_1}^{k} x_{i,j,\ell}^{\mathrm{up}} \right) (t_k - t_{k-1}). \tag{38}$$

Note that (36) and (37) are two specific cases of (38). We will thus use (38) to calculate the number of onboard passengers. The load factor constraints given in (10) can be rewritten as

$$\sum_{k=1}^{K} \lambda_{j,k}^{\mathrm{up}} \left(t_k - d_{i-1,j}^{\mathrm{up}} \right) x_{i-1,j,k}^{\mathrm{up}} + \sum_{k=1}^{K} \lambda_{j,k}^{\mathrm{up}} \left(d_{i,j}^{\mathrm{up}} - t_{k-1} \right) x_{i,j,k}^{\mathrm{up}} +$$
$$\sum_{k=1}^{K} \lambda_{j,k}^{\mathrm{up}} \left(\sum_{\ell=1}^{k-1} x_{i-1,j,\ell}^{\mathrm{up}} - \sum_{\ell_1}^{k} x_{i,j,\ell}^{\mathrm{up}} \right) (t_k - t_{k-1}) \le \sigma_{\max} C_{\mathrm{train}}, \tag{39}$$

where $\lambda_{j,k}^{\mathrm{up}}$, t_k, and t_{k-1} are constants, and the product $d_{i,j}^{\mathrm{up}} x_{i,j,k}^{\mathrm{up}}$ can be replaced by new real-valued variables $z_{i,j,k}^{\mathrm{up}}$ with the following constraints

$$\sum_{k=1}^{K} z_{i,j,k}^{\mathrm{up}} = d_{i,j}^{\mathrm{up}},$$
$$z_{i,j,k}^{\mathrm{up}} \le x_{i,j,k}^{\mathrm{up}} M, \tag{40}$$
$$z_{i,j,k}^{\mathrm{up}} \ge 0.$$

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6.2. Approximated MILP model

Since the passenger traveling rates in the up and down directions are piecewise constant functions (see (8)), the calculation of the onboard passengers is given in (38). For the up direction, we approximate the calculation of the onboard passengers given in (12) and (38) as follows

$$p_{i,j}^{\mathrm{up}} \approx \tilde{\lambda}_j^{\mathrm{up}}(d_{i,j}^{\mathrm{up}}) \cdot (d_{i,j}^{\mathrm{up}} - d_{i-1,j}^{\mathrm{up}}), \tag{41}$$

where $\tilde{\lambda}_j^{\mathrm{up}}(\cdot)$ is considered as a constant during time interval $[d_{i-1,j}^{\mathrm{up}}, d_{i,j}^{\mathrm{up}}]$ and the value of the passenger arrival rate is taken as $\tilde{\lambda}_j^{\mathrm{up}}(d_{i,j}^{\mathrm{up}})$, i.e., the passenger arrival rate when service i departs from station j.

Remark. *Here we approximate the piecewise constant function in time interval $[d_{i-1,j}^{\mathrm{up}}, d_{i,j}^{\mathrm{up}}]$ by a constant, which is equal to the passenger traveling rate at $d_{i,j}^{\mathrm{up}}$, i.e., $\tilde{\lambda}_j^{\mathrm{up}}(d_{i,j}^{\mathrm{up}})$. However, the constant could also be equal to $\tilde{\lambda}_j^{\mathrm{up}}(d_{i-1,j}^{\mathrm{up}})$, $\tilde{\lambda}_j^{\mathrm{up}}\left(\frac{d_{i,j}^{\mathrm{up}}+d_{i-1,j}^{\mathrm{up}}}{2}\right)$, or the passenger traveling rate at any point between $d_{i-1,j}^{\mathrm{up}}$ and $d_{i,j}^{\mathrm{up}}$. Note that due to the approximation errors, the maximum load factor constraints could be violated when computing the train schedule and rolling stock circulation plan by using the original nonlinear model.*

As we introduced in Section 4.2.4, the whole operating period $[t_{\mathrm{start}}, t_{\mathrm{end}}]$ is split into K sub-intervals with splitting points $t_1, t_2, t_3, \ldots, t_{K-1}$. We propose the binary variable $y_{i,j,k}^{\mathrm{up}}$ with $i \in \mathbf{I}$, $j \in \mathbf{J}$, and $k \in \{0, 1, 2, \ldots, K\}$ to indicate whether the departure time $d_{i,j}^{\mathrm{up}}$ of train service i at station j is smaller than t_k or not. The definition of binary variable $y_{i,j,k}^{\mathrm{up}}$ is given as follows

$$[d_{i,j}^{\mathrm{up}} \le t_k] \Leftrightarrow [y_{i,j,k}^{\mathrm{up}} = 1], \tag{42}$$

which means that if $d_{i,j}^{\mathrm{up}}$ is less than t_k, then $y_{i,j,k}^{\mathrm{up}}$ is equal to 1; otherwise, $y_{i,j,k}^{\mathrm{up}}$ is equal to 0. We note that $y_{i,j,0}^{\mathrm{up}} = 0$ and $y_{i,j,K+1}^{\mathrm{up}} = 1$ for all $i \in \mathbf{I}$ and $j \in \mathbf{J}$. Based on the transformation properties given in (Williams, 1999), the definition of $y_{i,j,k}^{\mathrm{up}}$ given in (42) can be reformulated by the following linear inequalities

$$\begin{cases} (t_K - t_k)y_{i,j,k}^{\mathrm{up}} \le t_K - d_{i,j}^{\mathrm{up}}, \\ (t_0 - t_k - \varepsilon)y_{i,j,k}^{\mathrm{up}} \le d_{i,j}^{\mathrm{up}} - t_k - \varepsilon, \end{cases} \tag{43}$$

where t_0 and t_K are the minimal and maximal values of $d_{i,j}^{\mathrm{up}}$, and ε is a small positive number, typically the machine precision. The small number is introduced to transform a strict equality into a non-strict inequality to fit the MILP framework. Here we use the passenger traveling rates at $d_{i,j}^{\mathrm{up}}$, and we have

$$x_{i,j,k}^{\mathrm{up}} = y_{i,j,k}^{\mathrm{up}} - y_{i,j,k-1}^{\mathrm{up}}, \forall i \in \mathbf{I}, j \in \mathbf{J}, k \in \{1, 2, \ldots, K\}. \tag{44}$$

According to the definition of $y_{i,j,k}^{\mathrm{up}}$, the passenger traveling rate $\tilde{\lambda}_j^{\mathrm{up}}(d_{i,j}^{\mathrm{up}})$ can be calculated by

$$\begin{aligned} \tilde{\lambda}_j^{\mathrm{up}}(d_{i,j}^{\mathrm{up}}) = & y_{i,j,1}^{\mathrm{up}}\lambda_{j,1}^{\mathrm{up}} + (1 - y_{i,j,1}^{\mathrm{up}})y_{i,j,2}^{\mathrm{up}}\lambda_{j,2}^{\mathrm{up}} + \cdots + (1 - y_{i,j,k-1}^{\mathrm{up}})y_{i,j,k}^{\mathrm{up}}\lambda_{j,k}^{\mathrm{up}} + \ldots \\ & + (1 - y_{i,j,K-1}^{\mathrm{up}})y_{i,j,K}^{\mathrm{up}}\lambda_{j,K}^{\mathrm{up}}. \end{aligned} \tag{45}$$

It can be observed that (45) is still nonlinear because of the products of binary variables, i.e., $y_{i,j,k-1}^{\text{up}} y_{i,j,k}^{\text{up}}$ for $k = 2, 3, \ldots, K$. These products can be replaced by auxiliary logical variables $\tilde{y}_{i,j,k-1}^{\text{up}} = y_{i,j,k-1}^{\text{up}} y_{i,j,k}^{\text{up}}$, i.e., $[\tilde{y}_{i,j,k-1}^{\text{up}} = 1] \leftrightarrow [y_{i,j,k-1}^{\text{up}} = 1] \wedge [y_{i,j,k}^{\text{up}} = 1]$, with $\wedge$ denoting "and". This replacement is equivalent to the following linear constraints (Williams, 1999)

$$\begin{cases} -y_{i,j,k-1}^{\text{up}} + \tilde{y}_{i,j,k-1}^{\text{up}} \leq 0, \\ -y_{i,j,k}^{\text{up}} + \tilde{y}_{i,j,k-1}^{\text{up}} \leq 0, \\ y_{i,j,k-1}^{\text{up}} + y_{i,j,k}^{\text{up}} - \tilde{y}_{i,j,k-1}^{\text{up}} \leq 1. \end{cases} \tag{46}$$

By replacing the product of binary variables with the new auxiliary logical variables, the function $\tilde{\lambda}_j^{\text{up}}(d_{i,j}^{\text{up}})$ can be rewritten as the following linear form

$$\tilde{\lambda}_j^{\text{up}}(d_{i,j}^{\text{up}}) = y_{i,j,1}^{\text{up}} \lambda_{j,1}^{\text{up}} + (y_{i,j,2}^{\text{up}} - \tilde{y}_{i,j,1}^{\text{up}}) \lambda_{j,2}^{\text{up}} + \cdots + (y_{i,j,k}^{\text{up}} - \tilde{y}_{i,j,k-1}^{\text{up}}) \lambda_{j,k}^{\text{up}} + \cdots + (y_{i,j,K}^{\text{up}} - \tilde{y}_{i,j,K-1}^{\text{up}}) \lambda_{j,K}^{\text{up}}. \tag{47}$$

When substituting (47) in (41) to calculate the number of onboard passengers, we will encounter the products of binary-valued variables (i.e., $y_{i,j,k}^{\text{up}}$ and $\tilde{y}_{i,j,k}^{\text{up}}$) and real-valued variables (i.e., $d_{i,j}^{\text{up}}$). By introducing new auxiliary variables $z_{i,j,k}^{\text{up},1} = y_{i,j,k}^{\text{up}} d_{i,j}^{\text{up}}$, $z_{i,j,k}^{\text{up},2} = \tilde{y}_{i,j,k}^{\text{up}} d_{i,j}^{\text{up}}$, $z_{i,j,k}^{\text{up},3} = y_{i,j,k}^{\text{up}} d_{i-1,j}^{\text{up}}$, and $z_{i,j,k}^{\text{up},4} = \tilde{y}_{i,j,k}^{\text{up}} d_{i-1,j}^{\text{up}}$, (41) can be rewritten as

$$\begin{aligned} p_{i,j}^{\text{up}} \approx &z_{i,j,1}^{\text{up},1} \lambda_{j,1}^{\text{up}} + (z_{i,j,2}^{\text{up},1} - z_{i,j,1}^{\text{up},2}) \lambda_{j,2}^{\text{up}} + \cdots + (z_{i,j,k}^{\text{up},1} - z_{i,j,k-1}^{\text{up},2}) \lambda_{j,k}^{\text{up}} + \cdots + (z_{i,j,K}^{\text{up},1} - z_{i,j,K-1}^{\text{up},2}) \lambda_{j,K}^{\text{up}} \\ &- z_{i,j,1}^{\text{up},3} \lambda_{j,1}^{\text{up}} - (z_{i,j,2}^{\text{up},3} - z_{i,j,1}^{\text{up},4}) \lambda_{j,2}^{\text{up}} - \cdots - (z_{i,j,k}^{\text{up},3} - z_{i,j,k-1}^{\text{up},4}) \lambda_{j,k}^{\text{up}} - \cdots - (z_{i,j,K}^{\text{up},3} - z_{i,j,K-1}^{\text{up},4}) \lambda_{j,K}^{\text{up}}. \end{aligned} \tag{48}$$

The definition of these auxiliary variables satisfies, by taking $z_{i,j,k}^{\text{up},1}$ as an example, $[y_{i,j,k}^{\text{up}} = 0] \Leftarrow [z_{i,j,k}^{\text{up},1} = 0]$ and $[y_{i,j,k}^{\text{up}} = 1] \Leftarrow [z_{i,j,k}^{\text{up},1} = d_{i,j}^{\text{up}}]$. It follows that $z_{i,j,k}^{\text{up},1} = y_{i,j,k}^{\text{up}} d_{i,j}^{\text{up}}$ is equivalent to the following linear constraints

$$\begin{cases} z_{i,j,k}^{\text{up},1} \leq t_K y_{i,j,k}^{\text{up}}, \\ z_{i,j,k}^{\text{up},1} \geq t_0 y_{i,j,k}^{\text{up}}, \\ z_{i,j,k}^{\text{up},1} \leq d_{i,j}^{\text{up}} - t_0 (1 - y_{i,j,k}^{\text{up}}), \\ z_{i,j,k}^{\text{up},1} \geq d_{i,j}^{\text{up}} - t_K (1 - y_{i,j,k}^{\text{up}}), \end{cases} \tag{49}$$

where t_0 and t_K are the minimal and maximal possible values of $d_{i,j}^{\text{up}}$. Similarly, the auxiliary variables $z_{i,j,k}^{\text{up},2}$, $z_{i,j,k}^{\text{up},3}$ and $z_{i,j,k}^{\text{up},4}$ can be transformed into linear constraints. The approximated load factor constraints in the up and down directions can be rewritten as linear constraints for the integrated train scheduling and circulation planning problem studied in this paper.

As given in (8), the passenger traveling rate functions $\tilde{\lambda}_j^{\text{up}}(\cdot)$ and $\tilde{\lambda}_j^{\text{dn}}(\cdot)$ are piecewise constant functions with K sub-functions, where the length of each time interval can be 30 minutes, 15 minutes, or even less. In the linearization process of the load factor constraints, the number of auxiliary variables and constraints grows linearly with the number of sub-functions. Therefore, we can approximate the original passenger traveling rate functions by a piecewise constant function with much less sub-functions as follows:

$$\tilde{\lambda}_j^{\text{up,approx}}(t) = \begin{cases} \hat{\lambda}_{j,1}^{\text{up}}, & \text{if } t \in [t_0, t_1') \\ \hat{\lambda}_{j,2}^{\text{up}}, & \text{if } t \in [t_1, t_2') \\ \cdots & \cdots \\ \hat{\lambda}_{j,K'}^{\text{up}}, & \text{if } t \in [t_{K'-1}', t_{K'}') \end{cases} , \forall j \in \mathbf{J}/\{J\}. \tag{50}$$
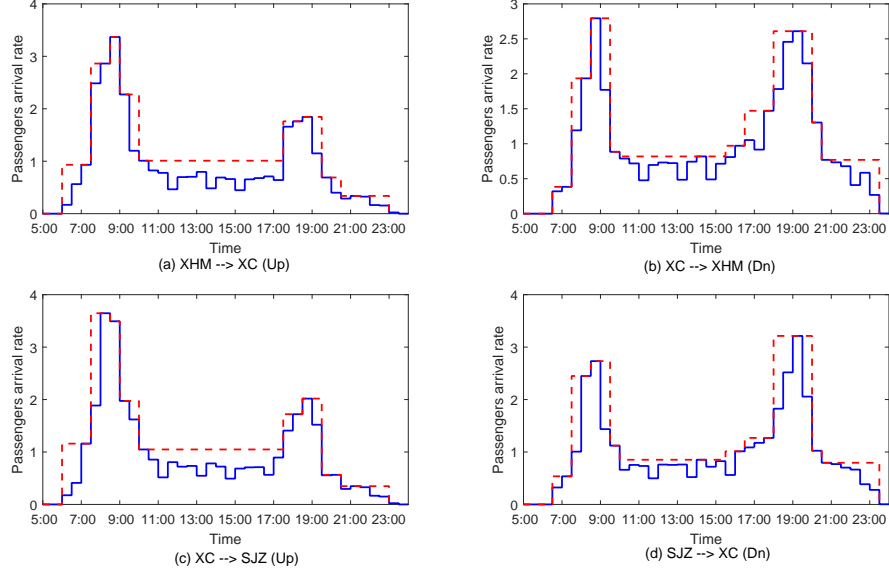
Figure 9: An example of approximated passenger traveling rates between XHM and SJZ in the up and down directions

with $t_{K'} = t_K$ and $K' < K$. For example, the passenger traveling rate functions corresponding to the passenger demand of Figure 5 with 34 non-zero sub-functions can be approximated by a piecewise constant function with 11 sub-functions, as illustrated in Figure 9. It is worth to note that these approximations do not cause the violation of the load factor constraints, because the approximated piecewise constant functions are always larger or equal to the original passenger traveling rate functions.

7. Case study

We now present numerical experiments to illustrate the effectiveness and efficiency of our integrated model and solution approaches. The sequential quadratic programming algorithm, implemented in the *fmincon* function of the Matlab optimization toolbox, is employed to solve the nonlinear programming problems, while CPLEX, through the interface function of the Matlab OPTI toolbox, is used to solve the MILP problems. All the experiments are performed on a 3.6 GHz Intel Core i7-3520M CPU running with a 64-bit windows operating system and 16G RAM. In addition, the solutions of the proposed INP, MILP-acc, MILP-app approaches are compared with those of the two-stage approach, i.e., the non-integrated approach, proposed by Wang et al. (2017a). Specifically, in the two-stage approach the train schedule is first calculated through the sequential quadratic programming algorithm and the rolling stock circulation plan is then obtained via the adjustments of the optimized train schedule by solving an MILP problem (see Wang et al. (2017a) for details). Furthermore, the train schedule and rolling stock circulation plan currently used in the Beijing Yizhuang line are also evaluated and compared with the train schedules and rolling stock circulation plans computed in this paper.
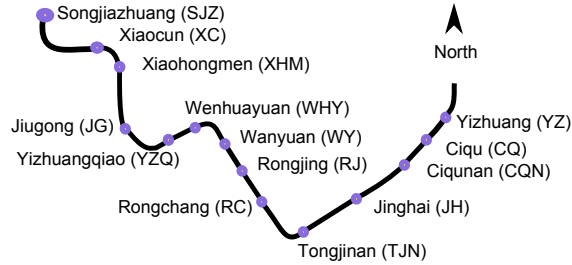
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Figure 10: The layout of the Beijing Yizhuang line

7.1. Set-up

The proposed integrated model and solution approaches are applied to the Beijing Yizhuang line, which was put into operation on December 30, 2010. The current daily ridership of this line is around 200,000. This line has 14 stations and is 23.23 km long. There is one depot that connects with Yizhuang station. The layout of the Beijing Yizhuang line is illustrated in Figure 10, where the direction from Yizhuang to Songjiazhuang is defined as the up direction and the direction from Songjiazhuang to Yizhuang is defined as the down direction. The running times between stations and the dwell times at stations are determined by train characteristics, grade profile, resistances, and passenger demand. In the train schedule used in the current practice, the running times and dwell times have the same values for all the train services and those values are also used for the proposed integrated optimization model. For the turnaround operations at Songjiazhuang and Yizhuang stations, the turnaround time must be larger than 120 s and smaller than 720 s based on the practical operating requirements.

The daily operating period of the Beijing Yizhuang line is from 5:20 till 23:30. In particular, the first train service departs from Yizhuang station at 5:20 and the last train service departs from Yizhuang station at 22:05. In order to provide a consistent service to passengers, the departure times of the first and last train services must not be changed, which can be easily included in the integrated optimization model by specifying $d_{1,1}^{\mathrm{up}}$ and $d_{N,1}^{\mathrm{up}}$ equal to 5:20 and 22:05, respectively. The passenger demand of the Beijing Yizhuang line in a weekday is illustrated in Figure 11, which shows the variations of passenger demand over the time of the day. In particular, the quantities of passengers traveling between two consecutive stations are denoted by different colors. Note that the passenger demand on the section between Ciqu and Yizhuang stations is equal to zero, since passengers are not allowed to enter and exit Yizhuang station in the current operation of the Beijing Yizhuang line. In Figure 11, the sections adjacent to Songjiazhang station have a higher passenger demand when compared with the sections adjacent to Yizhuang station. This is because Songjiazhuang station is the only transfer station and is nearby the city center. In Figure 11 (a), the morning peak hours are between 7:00 and 9:30 and the maximum volume of passengers in the line is reached between 8:00 and 8:30 in the up direction. In addition, the evening peak hours are between 17:00 and 19:00 in the up direction. Note that the passenger volumes of the evening peak hours are smaller than those of the morning peak hours. Similar observations are obtained from Figure 11 (b) for the down direction, but the evening hours are, in this case, between 16:30 and 20:00.

The capacity of trains in the Beijing Yizhuang line is 1440 passengers, including the number of seated passengers and the allowable standing passengers. With the increase of the passenger demand, the maximum load factor of some train services can be larger than 0.9, and even close to
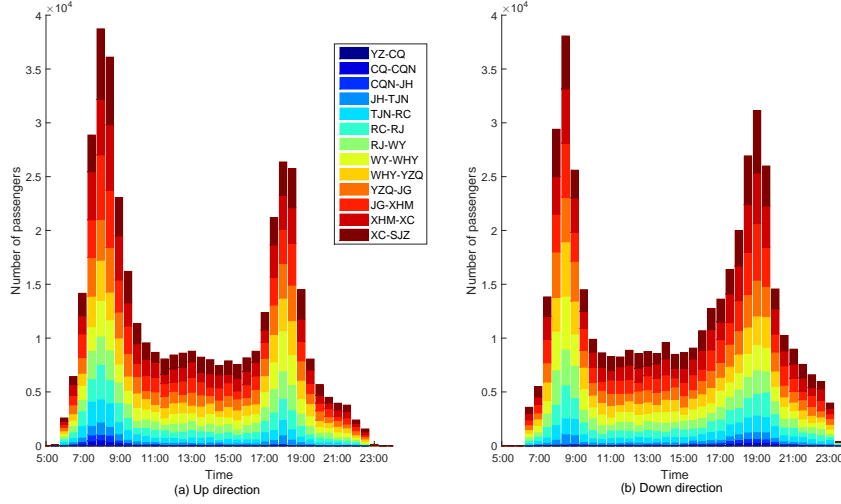
Figure 11: The passenger demand of the Beijing Yizhuang line

1 during the morning peak hours, for the current train schedule (which is generated by the traffic planner with the help of graphic tools) applied in the Beijing Yizhuang line. Since the number of available rolling stocks for a daily operation is limited to 13 in the current practice, this is a practical challenge for the train scheduling and rolling stock circulation planning problems of the Beijing Yizhuang line. In this case study, the maximum load factor σ_{max} of each train service is set as 0.9 to guarantee the passenger satisfaction. In addition, the minimal and maximal headways between train services are taken as 240 s and 660 s, respectively. The weights γ_1, γ_2, and γ_3 introduced in (27) are set equal to 1. Moreover, the nominal values for the objective function components $f_{\text{load,nom}}$, $f_{\text{headway,nom}}$, and $f_{\text{depot,nom}}$, are 31.66, 192.49, and 13, respectively. These values are obtained by solving the three single-objective optimization problems with 122 services. We observe that the number of services in the practical train schedule is also 122; we thus use the same number of services to compute the nominal values. Furthermore, the maximum computation time for the solution approaches in this case study is set as 5 hours.

7.2. Computational results

We evaluate the overall performance of the proposed integrated optimization model and solution approaches from the point of view of effectiveness and efficiency, such as their computation times and their objective function values. A detailed quantitative analysis of the proposed methods is given afterwards, including the following aspects:

- a comparison between the three integrated approaches and the non-integrated approach, i.e., the two-stage approach proposed in Wang et al. (2017a);

- a comparison between the best solutions obtained by non-integrated and integrated approaches and the practical solution generated by the traffic planners to show the practical value of the proposed solution approaches;

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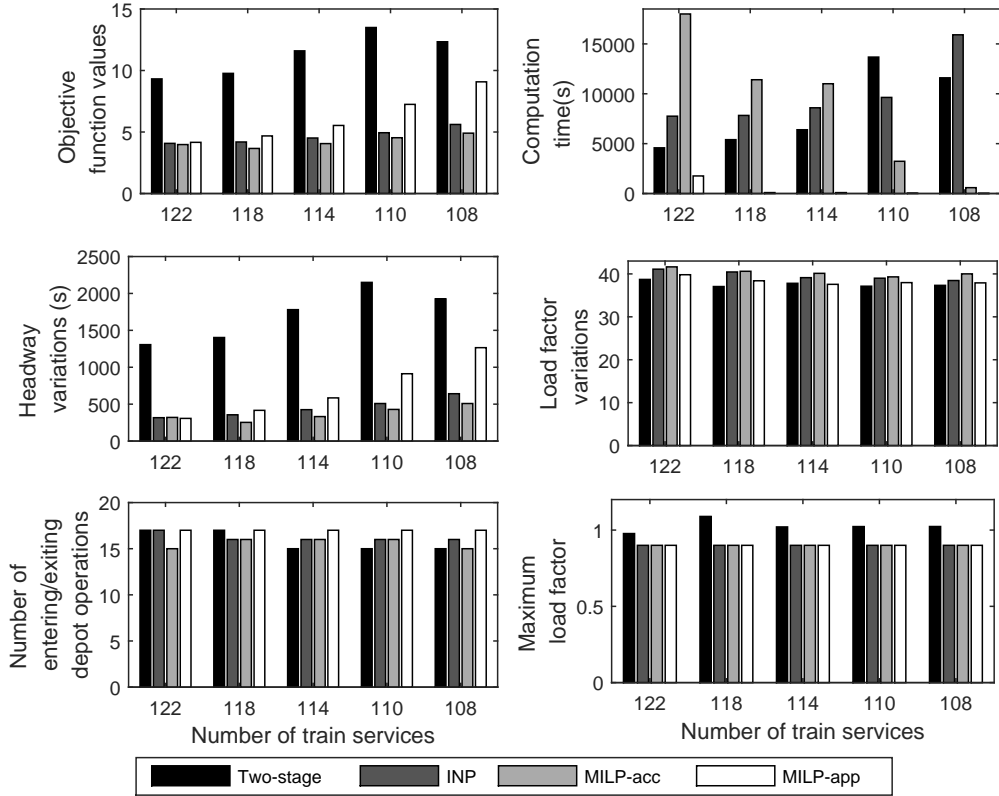
Figure 12: Results of the four solution approaches (i.e., two-stage, INP, MILP-acc, and MILP-app) for 122, 118, 114, 110, and 108 train services

- performance analysis for different passenger demand data to show the effects of demand variations and the scalability of the fastest approach, i.e., the MILP-app approach;

- performance analysis for different weights in the multi-objective optimization problem to show Pareto-optimal solutions obtained by the approach providing the best near-optimal solutions, i.e., the MILP-acc approach.

7.2.1. Performance comparison between the integrated and non-integrated approaches

For the passenger demand given in Figure 11, the number of train services used in the practical train schedule generated by the traffic planners is 122 and the number of rolling stocks required is 13. When solving the train scheduling problem by using the proposed approaches, the number of train services can be reduced up to 108. In what follows, the train schedules and rolling stock circulation plans with 122, 118, 114, 110, and 108 services are obtained by the proposed solution approaches. For this case study, the allowed adjustments of the departure/arrival times are set equal to 80 s for the generation of a feasible rolling stock circulation plan in the

two-stage approach. After the adjustments, the resulting train schedules could violate some constraints, e.g., the maximum load factor constraints. In addition, for the two-stage approach and the INP approach, multiple initial solutions need to be employed to achieve a better performance, since the resulting optimization problem is nonlinear and non-convex. In the computational results presented in Table 4, the number of initial solutions is taken as 10.

The results for different numbers of train services obtained by the solution approaches are reported in Figure 12, where we observe that the performance measures of the integrated approaches (especially the INP approach and the MILP-acc approach) are, on average, much better when compared with those of the non-integrated approach, i.e., the two-stage approach. In particular, the objective function values of the MILP-app approach are slightly higher than the values of the other two integrated approaches. This deterioration of the performance is due to the approximation adopted for the passenger arrival rates, as illustrated in Figure 9, where the approximations cause sharp changes of the passenger demand. These sharp changes then result in sudden (or non-continuous) changes in the headways. Therefore, the headway variations of the MILP-app approach are much larger than the ones of the other two integrated solution approaches, especially when the number of train services is reduced for the same passenger demand. Moreover, the objective function value increases with the decrease of the number of train services, which is also mainly caused by the increase of the headway variations.

Load factor fluctuations for the different solution approaches are close to each other. In particular, the load variations obtained by the two-stage approach (i.e., the non-integrated approach) are smaller when compared with the other approaches (i.e., the integrated approaches). This result indicates that the integrated problem is indeed a multi-objective optimization problem, where a better performance for one objective is achieved at the cost of a worse performance for other objectives. The load factor variation decreases, in general, with the reduction of the number of train services. In addition, the average load factor for all the train services increases with the decrease of the number of train services. The latter result is because the total number of passengers traveling in the line is fixed. The solutions obtained by the two-stage approach violate the maximum load factor constraints, as shown in Figure 12, where the maximum load factor for this case study is chosen as 0.9 and the maximum load factors obtained by the two stage approach are larger than 0.9, and even reach 1 in some cases. However, the solutions obtained by all the integrated approaches proposed in this paper satisfy the load factor constraints. The smallest number of depot operations is 15, which is obtained by the MILP-acc approach for the case with 122 train services. The number of depot operations for most cases is 17 and decreases with the reduction of train services, except for the MILP-app approach, where the approximation of the passenger demand could be the reason for the non-decrease. It is worth to note that the number of rolling stocks required for all the cases is 13, which is mostly related to the passenger demand in the morning and evening peak hours.

The computation time of the MILP-app approach is the smallest since this approach introduces approximations for the passenger demand. On average, the MILP-acc approach outperforms the other approaches in terms of the objective function values. However, for the case with 122 train services, the optimality gap of the MILP-acc approach is 20.32% when the maximum computation time of 5 hours is reached. In addition, we can observe that with the reduction of the number of train services, the computation time of the MILP-acc approach and the MILP-app approach decreases, which may be caused by the reduction of the feasible solution space. The computation time of the two-stage approach and of the INP approach increases with the decrease of the number of train services. Differently, the computation time of the two-stage approach is higher than that of the INP approach for the case with 110 services. Since we solve the nonlin-

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Table 4: Performance comparison of the practical train schedule and the trains schedules with 122 services obtained by the two-stage and MILP-acc approaches

Solution approach	Computa-tion time [s]	Objective function value	Headway variation [s]	Load factor variation	Number of exits (Depot)	Maximal load factor	Average load factor
Practice	–	16.94	2790	35.35	20	0.988	0.458
Two-stage	4582	9.32	1307	38.72	17	0.977	0.457
MILP-acc	18000	3.98	320	41.49	15	0.9	0.457
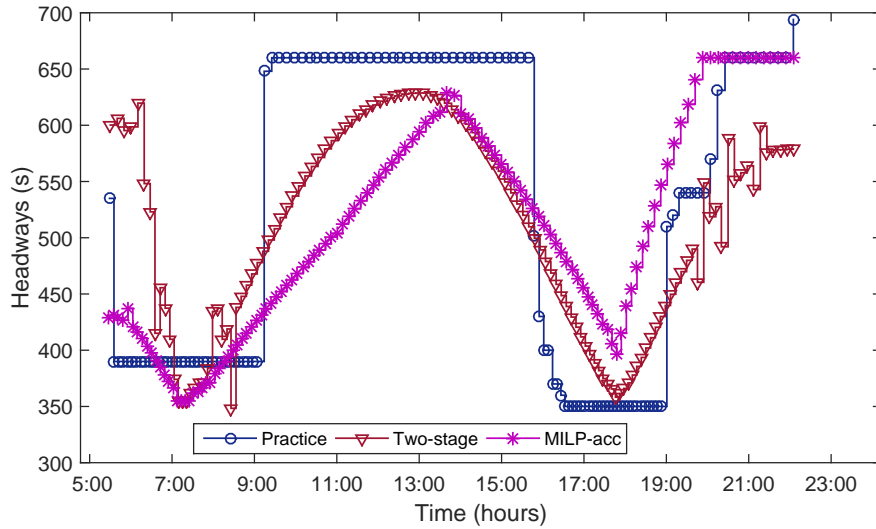
Figure 13: The headways for the train schedules with 122 train services for the Beijing Yizhuang line

ear non-convex optimization problems in the two-stage approach and in the INP approach, their computation time to get a local minimum is not deterministic.

7.2.2. Performance comparison of solutions obtained by the approaches and the traffic planners

The train schedule and rolling stock circulation plan used in the current practice of the Beijing Yizhuang line are designed manually by the traffic planners based on their experiences and the whole design process may take more than one week. The practical train schedule is regular, where the headways for the peak and off-peak hours are constants. As it has been mentioned before, the number of train services in the practical train schedule is 122. We thus compare the practical solution with the best solutions with 122 services obtained by the non-integrated approach (i.e., the two-stage approach) and the most accurate integrated approach (i.e., the MILP-acc approach).

The performance comparison of the obtained train schedules and rolling stock circulation plans is given in Table 4. As it can be observed from Table 4, the MILP-acc approach results in the best-known (near-optimal) solution. When compared to the practical solution, the headway variation is reduced from 2790 s to 320 s and the number of entering/exiting depot operations is reduced from 20 to 15. The headways for the train schedule generated by traffic planners are

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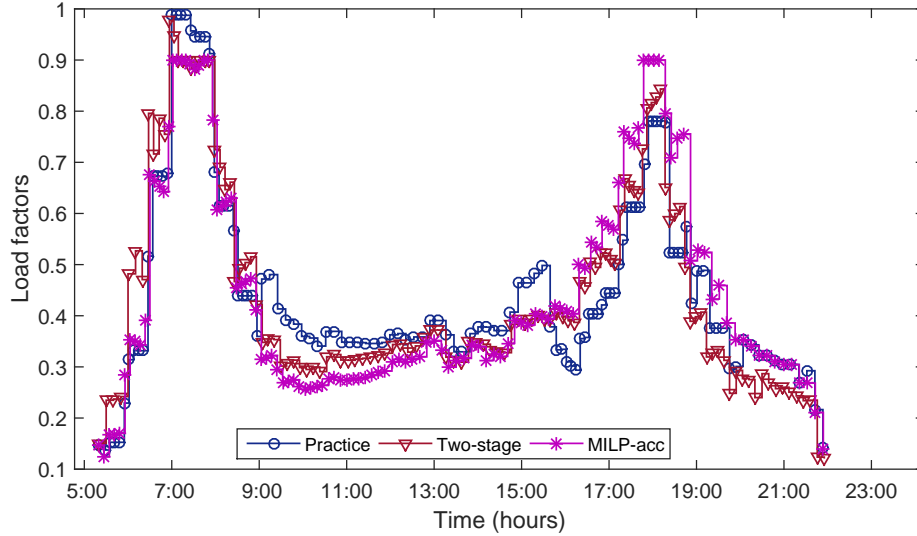
Figure 14: The load factors for the train schedules with 122 train services for the Beijing Yizhuang line

shown in Figure 13, where the headways in the off-peak hours are 660 s and the headways in the morning and evening peak hours are 390 s and 350 s, respectively. Due to the sudden fluctuation of the headways between the peak and off-peak hours, the headway variation defined in (25) is quite large (is equal to 2790 s) for the practical train schedule. Moreover, since the passenger demand varies much slower and smoother when compared with the sudden change of headways, the load factors of several trains during the headway transition period are higher than 0.9, and even reach 0.988 for the practical train schedule, as shown in Figure 14. In addition, the number of entering/exiting depot operations is 20, while the number of required rolling stocks is 13 in the practical rolling stock circulation plan ((Figure A.18 of Appendix A).

When comparing the headways related to the various approaches, the smoothness of the headways obtained by the MILP-acc approach is the best, as shown in Figure 13. It is worth to note that the smoothness of the headways obtained by the two-stage approach is also good for most of the train services, while the headways change sharply for a few services. This is because in the two-stage approach the train schedule is calculated first and is then adjusted when computing the rolling stock circulation plan. The second stage adjustments can cause some constraint violations. The sudden changes of the headways, reported in Figure 13, thus result in the violation of maximal load factor constraints. Specifically, the maximal load factor is 0.977, which is a bit smaller than the one of the practical solution. However, this value is larger than the predefined maximal load factor (0.9). The solution obtained by the MILP-acc approach is much better than those of the traffic planners and of the non-integrated approach. With the MILP-acc approach, there is no violation of the maximum load factor constraints. Moreover, the headway variation is hugely reduced and the number of entering/exiting depot operations is also reduced with the cost of a limited increase in the load factor variation.

The rolling stock circulation plans with 122 services obtained by the two-stage approach and the MILP-acc approach are given in Figures A.19-A.20 of Appendix A. Furthermore, the train

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schedule obtained by the MILP-acc approach is also given in Figures A.21-A.23 of Appendix A. Moreover, the departure times in the up direction (i.e., the start time of train services) and the arrival times in the down direction (i.e., the end time of train services) at Yizhuang station are provided in Table B.8 of Appendix B. The number of on-board passengers for these 122 train services in the up direction is given in Table B.9.

7.2.3. Performance analysis for different passenger demand data

We compare the train schedules and the rolling stock circulation plans under different passenger demand data by using the fastest integrated approach, i.e., the MILP-app approach. In order to evaluate the effects of the small variations of the passenger demand to the train schedules and the rolling stock circulation plans, we have fixed the number of train services, which is taken as 114. The passenger demand data given in Figure 11 is considered as the nominal passenger demand, i.e., the demand factor is equal to 1. In the following experiments, we vary the passenger demand data by using different demand factors, which are taken as 0.9, 0.95, 1.05, and 1.1.

Table 5 gives the performance comparison of the solutions with 114 train services under different passenger demand factors. It can be observed from Table 5 that with the increase of the passenger demand, the value of the objective function also grows. Similar trends hold for all the objective function components, i.e., the headway variations, the load factor variations, and the number of exits from the depot. In particular, the number of exits from the depot is 15 and the number of rolling stocks required is 12 when the passenger demand factors are equal to 0.9 and 0.95. Furthermore, when the passenger demand increases, more rolling stocks need to be put into operation, e.g., the numbers of rolling stocks required are 14 and 15 when the demand factors are equal to 1.05 and 1.1. Moreover, the average load factor for the train services increases from 0.438 to 0.536 when the passenger demand factor varies from 0.9 to 1.1. This is because the number of services is fixed to 114 for all the demand scenarios.

The headways and load factors for each demand factor in the obtained train schedules are given in Figures 15 and 16, respectively. As it can be observed from Figure 15, the headway between trains in the morning and evening peak hours decreases with the increase of the passenger demand. Since the departure times of the first and last train services are fixed for all scenarios, the optimization process distributes all the train services in the operating period as equally as possible, when all the constraints are satisfied. Therefore, the headways between train services in the off-peak hours are generally smaller, when the passenger demand is decreased. In Figure 16, the load factors of most of the train services become larger with the increase of the passenger demand. However, since we set the maximum load factor as 0.9, the load factor of some train services in peak hours is equal to 0.9, which is achieved by varying the headways between train services.

In order to evaluate the scalability of the fastest integrated approach, i.e., the MILP-app approach, we enlarge the passenger demand to increase the scale of the integrated optimization problem. Due to the transport capacity limit of the Beijing Yizhuang line, the maximum passenger demand factor is set as 2.5. The passenger demand factors for the scalability evaluation are thus chosen as 1, 1.25, 1.5, 1.75, 2, 2.25, and 2.5. The model sizes of the integrated optimization problems corresponding to these passenger demand factors are given in Table 6, in terms of the number of constraints, binary variables, real-valued variables, and non-zeros. The computation time and optimality gap for the integrated optimization problems under different passenger demand data are shown in Figure 17. In addition, the number of entering/exiting depot operations and the number of rolling stock required are also given in Figure 17. The computation time increases with the number of train services in Figure 17. In particular, when the number

Table 5: Performance comparison of the trains schedules and circulation plans with 114 services under different passenger demand data obtained by the approximated MILP approach

Passenger demand factor	Computation time [s]	Objective function value	Headway variation [s]	Load factor variation	Number of exits (Depot)	Maximal load factor	Average load factor	Number of trains required
0.9	4020	3.90	305	36.90	15	0.9	0.438	12
0.95	145	4.62	440	37.43	15	0.9	0.463	12
1	93	5.53	585	37.57	17	0.9	0.487	13
1.05	748	6.80	792	38.92	19	0.9	0.512	14
1.1	66	8.40	1086	38.66	20	0.9	0.536	15
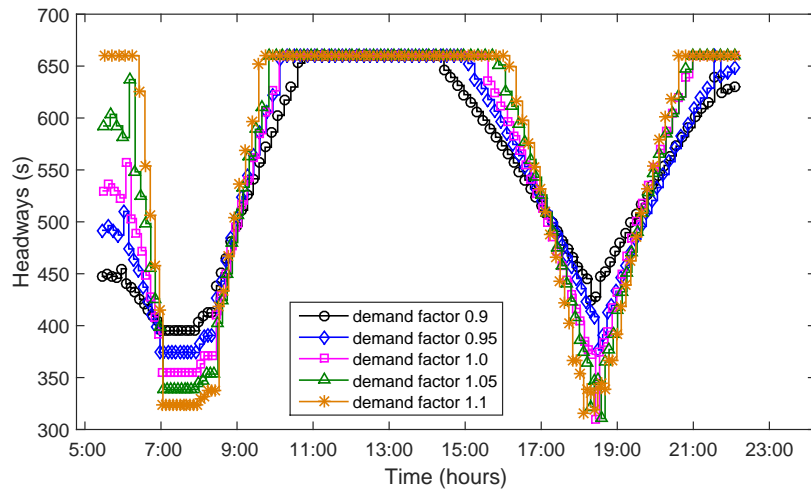
Figure 15: Headways of the train schedules with different passenger demand data obtained by the approximated MILP approach
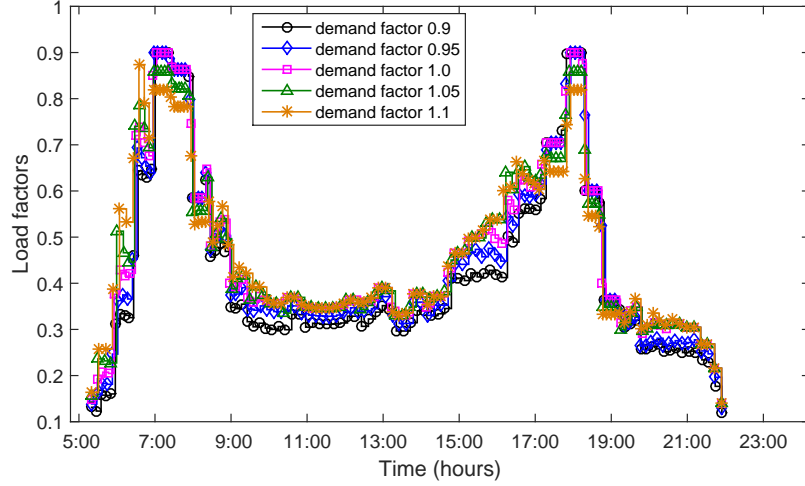
Figure 16: Load factors of the train schedules with different passenger demand data obtained by the approximated MILP approach

Table 6: The model sizes of the integrated optimization problems for different passenger demand data when using the MILP-app approach

Passenger demand factor	Number of train services	Original MILP		Reduced MILP[1]			
		Number of constraints	Number of variables	Number of constraints	Number of binaries	Number of real variables	Number of non-zeros
1	110	84,005	27,124	11,000	2,845	1,568	36,482
1.25	125	100,480	33,009	19,250	4,964	2,541	63,673
1.5	138	115,793	38,598	28,319	7,238	3,626	93,309
1.75	150	130,666	44,045	36,585	9,437	4,566	120,654
2	200	199,472	68,926	60,445	17,773	6,440	205,666
2.25	220	230,313	80,097	68,469	21,203	6,776	236,049
2.5	250	280,165	98,194	80,522	26,710	7,114	282,689

of train services is 150, an optimal solution cannot be found within 5 hours of computation and the optimality gap is 0.67%. Since the maximum computation time is always set to 5 hours, the optimality gap increases significantly with the number of train services. In order to satisfy the increasing passenger demand, the number of entering/exiting depot operations and the number of required rolling stocks also increase. For the case with 250 services, the number of entering/exiting depot operations is 59, while the number of rolling stocks is 35. A feasible solution for the latter case is obtained within 10 minutes of computation; however, it may take a much longer time to find the best-known solution. The optimality gap is 44.47% for the given time limit of computation.
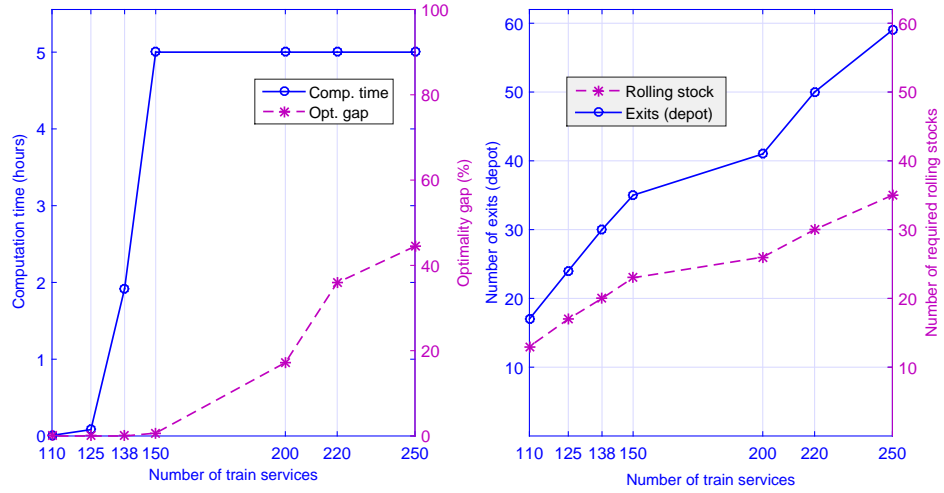
Figure 17: The computation time, the optimality gap, the number of entering/exiting depot operations, and the number of required rolling stocks for different passenger demand data and for an increasing number of train services

Table 7: Performance comparison of train schedules and circulation plans with different weights in the objective function obtained by the MILP-acc approach

γ_1	γ_2	γ_3	Computa- tion time [s]	Load factor variation	Headway variation [s]	Number of exits (Depot)
1	1	1	587	40.01	508.8	15
0.1	1	1	307	39.99	508.7	15
0.5	1	1	694	39.90	511.9	15
5	1	1	1111	38.11	540.5	15
10	1	1	919	37.20	620.8	15
1	0.1	1	5382	38.59	589.5	14
1	0.5	1	1310	39.99	509.6	15
1	5	1	267	39.72	500.7	16
1	10	1	135	39.78	500.6	16
1	1	0.1	202	39.39	502.0	16
1	1	0.5	347	39.40	501.8	16
1	1	5	1873	39.92	548.6	14
1	1	10	3489	40.71	637.9	13
1	0	0	322	**33.08**	782.0	17
0	1	0	212	39.78	**500.6**	16
0	0	1	1306	40.00	645.0	**13**

7.2.4. Performance analysis for different weights in the multi-objective optimization problem

In order to evaluate the influence of different weights in the multi-objective optimization problem, we use the MILP-acc approach because this approach gives the best-known solutions. Moreover, the number of train services is taken as 108. For all the scenarios, the MILP-acc approach obtains the optimal solutions (i.e., the optimality gap is zero[2]). In each row of Table 7 but the last three rows, we keep two weights constants (i.e., 1) and change the other weight (i.e., 0.1, 0.5, 5, 10). In the last three rows of Table 7, we also report the Pareto-optimal solutions computed by using the ε-constraint method, where an optimization problem with a bi-objective function is solved to optimize two performance indicators with the same weight, while the other one performance indicator is indirectly optimized via the insertion of an additional bound constraint, forcing the value of the latter indicator to the best possible (bold) value in the corresponding row of Table 7.

Detailed computational results for all these weight settings and the non-dominated solutions obtained by the ε-constraint method are given in Table 7, where the computation time, the headway variation, the load factor variation, the number of exiting depot operations for each setting are given. The best values for the headway variation, the load factor variation, and the number of exiting depot operations are 500.6, 33.08, and 13, respectively. For the results of Table 7, the computation time required by the CPLEX solver to find the optimal solution is influenced by the values of the three weights. In addition, the load factor variation decreases with the increase of γ_1. Similar trends also hold for the headway variation and the number of entering/exiting depot operations when γ_2 and γ_3 increase, respectively.

7.3. Discussion

Based on the computational results provided in the previous subsections, we can conclude that the solutions obtained by the integrated approaches (i.e., the INP approach, the MILP-acc approach, and the MILP-app approach) are better than those obtained by the non-integrated approaches (i.e., the two-stage approach and the practical approach used by traffic planners). The MILP-acc approach yields the best-known (near-optimal) solutions, while the performance of the INP approach is also competitive. Moreover, the computation time of the MILP-app approach is the smallest among all the solution approaches. The latter approach can be applied to achieve the best overall trade-off when both the computation time and the objective function value are considered.

Under the same passenger demand, the objective function value increases when the number of services decreases. This is mainly because less available services result in larger headway variations. When the number of train services is fixed and the passenger demand varies, the objective function value deteriorates with the increase of the passenger demand. When looking at the specific objective function components, the headway variations, the load variations, and the number of entering/exiting depot operations are all increased when the passenger demand increases. This is because the headways during the peak hours become smaller to satisfy the load factor constraints. Moreover, smaller headways result in more depot entering/exiting operations and additional rolling stocks.

To investigate the scalability of the MILP-app approach, we enlarge the passenger demand factor up to 2.5 and the number of services reaches up to 250. Both these values are about two

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[1]The reduced MILP problem is obtained by the presolving process of CPLEX.
[2]If the optimality gap is smaller than 10^{-6}, we consider it as zero.

times the values used in practice. With this more complex setting, a feasible solution is found easily, however, the optimality gap is still 44.47% after a computation time equal to 5 hours. The weights γ_1, γ_2, and γ_3 in the objective function are used to denote the relative importance of the considered performance indicators. The weight setting indeed affects the optimal solutions and the computation time. In general, the value of each objective function component decreases with the increase of the corresponding weight, while the other components may deteriorate their performance. We conclude that the studied problem is inherently a multi-objective optimization problem.

8. Conclusions and future research

In this paper, we have tackled the integration of the passenger demand oriented train scheduling and the rolling stock circulation planning by using a multi-objective mixed integer nonlinear programming model. Three solution approaches have been developed: an iterative nonlinear programming (INP) approach and two mixed integer linear programming (MILP) methods (i.e., an accurate MILP method and an approximated MILP method). Our aim is to compute train schedules (i.e., departure and arrival times of all train services) and rolling stock circulation plans (including entering/exiting depot operations of rolling stocks and connections between train services) simultaneously. The performance of the proposed optimization approaches has been compared with the state-of-the-art approach (i.e., the two-stage approach) of Wang et al. (2017a) and the practical approach used by the traffic planners, from the viewpoints of solution quality and computational efficiency. The comparison is based on the real-world data of Beijing Yizhuang line. According to the experimental results, the accurate MILP approach yields the best-known solutions. When taking the computation time into account, the approximated MILP approach is the fastest algorithm. Furthermore, the performance of the INP approach is competitive with the MILP-acc approach in terms of solution quality.

For our future research, one interesting extension is to employ short-turning train services to speed up the circulation of rolling stocks and to transport more passengers efficiently; because the passenger demand is not equally distributed to all the stations in the urban rail transit line. Also, multiple depots could be considered in urban rail transit lines and the track storage can be used for the daily operations, especially in case of large passenger demand. As other promising research directions, we could investigate the positioning of rolling stocks for urban rail transit lines with multiple depots. Furthermore, we could explore the benefits of rolling stock composition changes, e.g., in terms of transport capacity and energy-efficiency. Moreover, we assume that the rolling stocks are available for the whole operating period; however, in practice, the rolling stocks would have maintenance works. In future work, we thus would consider the integration of the maintenance scheduling, including daily check and double-weekly check as, e.g., in Giacco et al. (2014), with the train scheduling and rolling stock circulation planning for urban rail transit systems. In addition, a different setting of binary variables for rolling stock circulation planning could also be an interesting topic to speed-up the solution process. Furthermore, the responsive behavior of passengers to service frequencies and the sensitive changes of passenger arrival rates to departure times are not formulated in this paper. Further work could address the issue of directly incorporating a demand model in order to capture various possible effects of varying the train schedules. Finally, the service frequencies, stop patterns, etc. could also be optimized based on an origin-destination dependent passenger demand.

Acknowledgment

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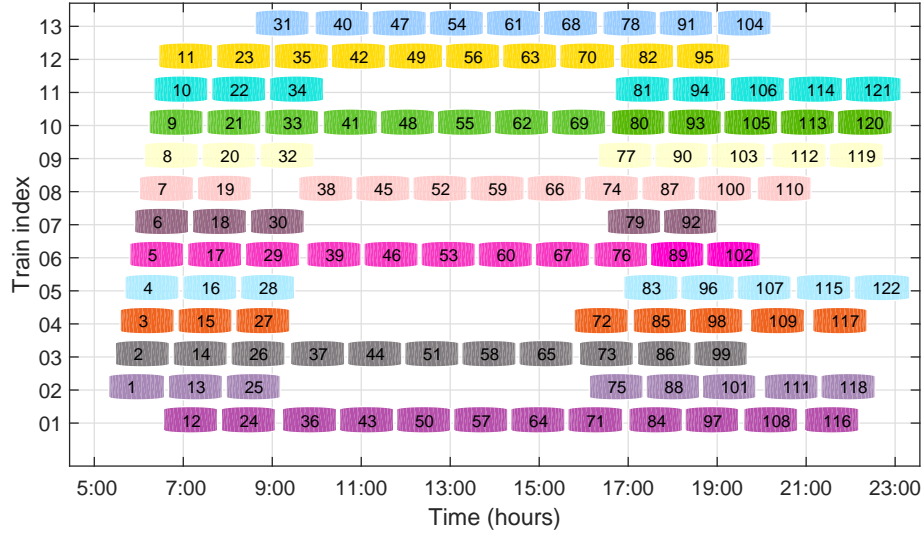
Figure A.18: Rolling stock circulation plan with 122 train services for the current practice of the Beijing Yizhuang line

Appendix A. Train schedules and rolling stock circulation plans with 122 train services

We report here the rolling stock circulation plans with 122 train services for the Beijing Yizhuang line (regarding the experiments in Section 7.2.2) obtained by: the traffic planner (Figure A.18), the two-stage approach (Figure A.19), and the MILP-acc approach (Figure A.20), respectively. Furthermore, the detailed train schedule with 122 servies obtained by the MILP-acc approach is given in Figures A.21, A.21, and A.23, illustrating the morning, afternoon, and evening schedules, respectively.

In Figures A.18-A.20, the number of required rolling stocks is 13 for all the three rolling stock schedules. In particular, only 12 rolling stocks are put into operation during the morning-peak hours in the solution obtained by the two-stage approach, which results in the violations of the maximum load factor constraints. For the MILP-acc approach, the entering/exiting depot operations are 15, which is less than those of the practical solution and of the solution obtained by the two-stage approach.

Appendix B. Detailed results for 122 train services via the MILP-acc approach

The departure times in the up direction (i.e., the start time of train services) and the arrival times in the down direction (i.e., the end time of train services) at Yizhuang station are provided in Table B.8, which is linked to the train schedules given in Figures A.21-A.23 of Appendix A. In addition, the number of on-board passengers for the 122 train services in the up direction is given in Table B.9.

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Figure A.19: Rolling stock circulation plan with 122 train services via the two-stage approach for the Beijing Yizhuang line

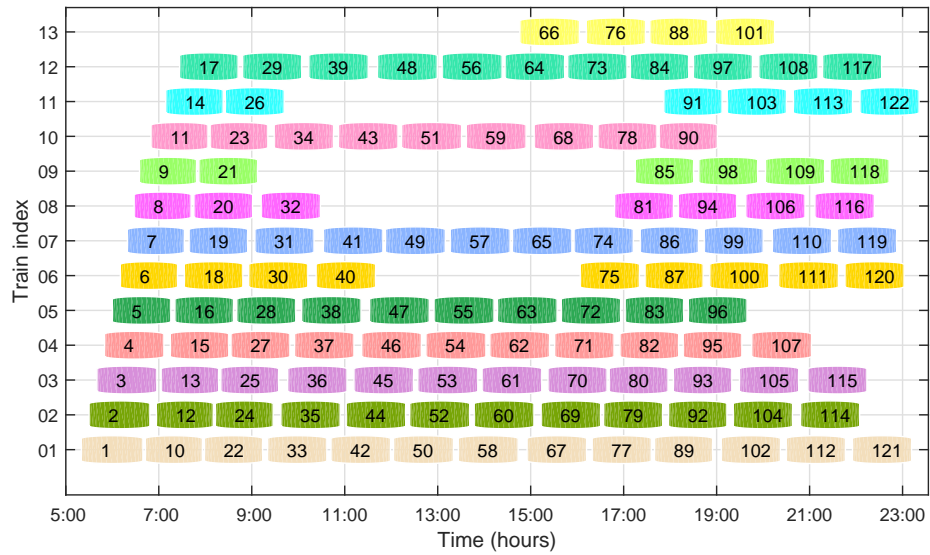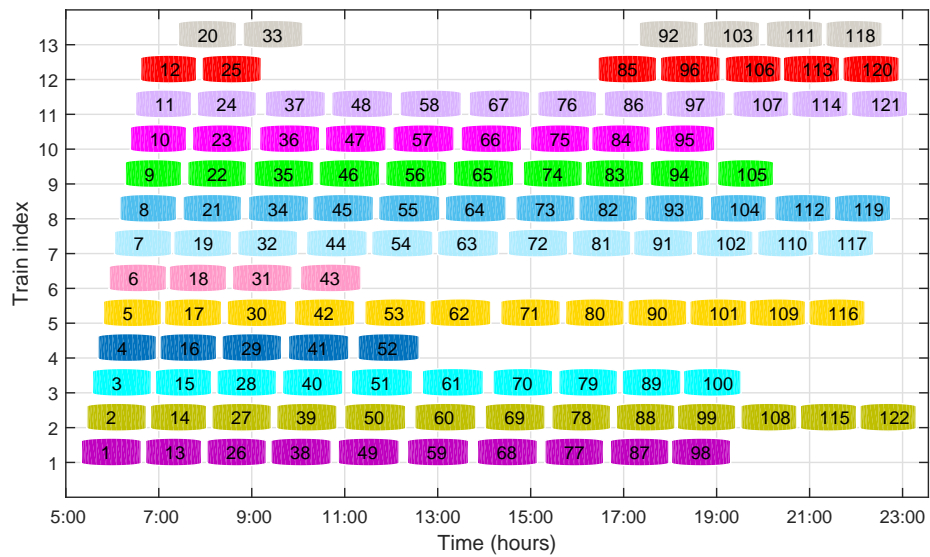

Figure A.20: Rolling stock circulation plan with 122 train services via the MILP-acc approach for the Beijing Yizhuang line

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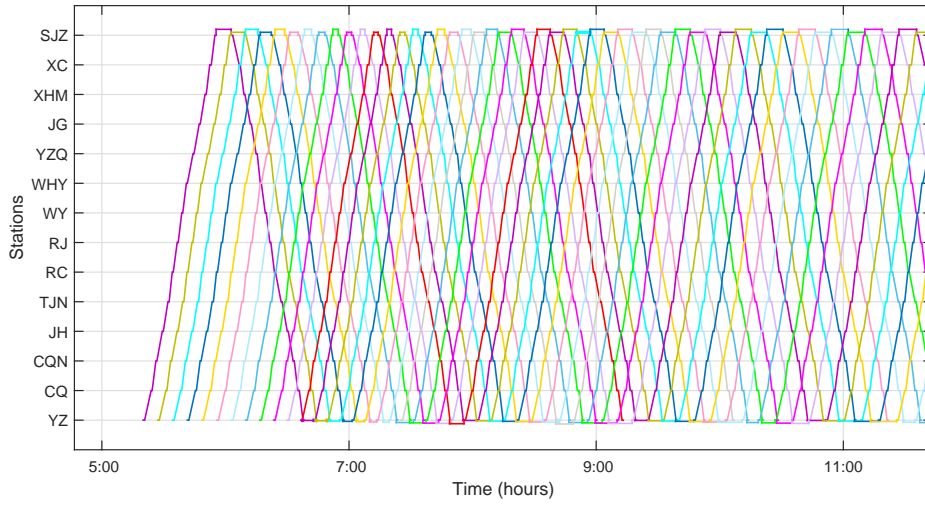
Figure A.21: Train schedule with 122 train services via the MILP-acc approach for the morning hours of the Beijing Yizhuang line
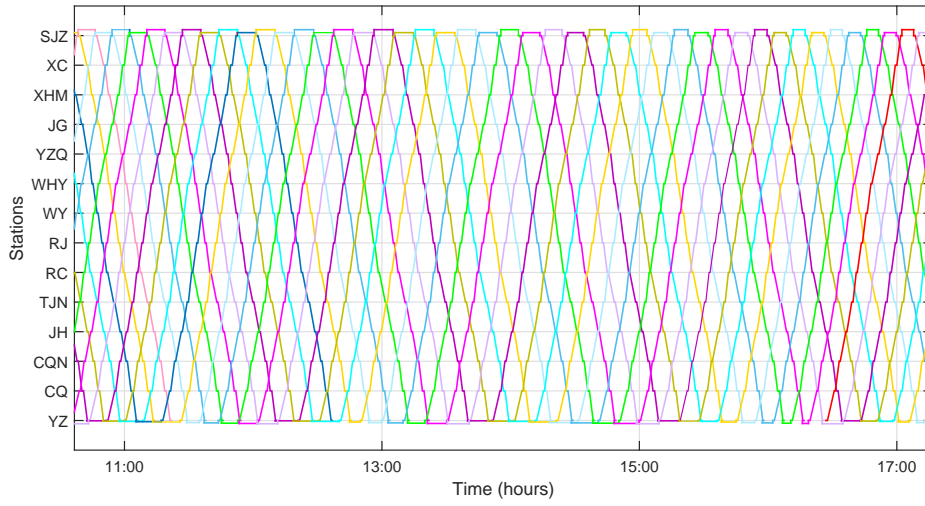
Figure A.22: Train schedule with 122 train services via the MILP-acc approach for the afternoon hours of the Beijing Yizhuang line

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Table B.8: Departure and arrival times of the train services at the Yizhuang (YZ) station obtained by the MILP-acc approach for the Beijing Yizhuang line

Service number	Departure time (up)	Arrival time (dn)	Service number	Departure time (up)	Arrival time (dn)	Service number	Departure time (up)	Arrival time (dn)
1	5:20:00	6:37:26	42	9:55:05	11:13:09	83	16:10:32	17:26:07
2	5:27:09	6:43:56	43	10:02:53	11:21:02	84	16:18:47	17:34:23
3	5:34:20	6:50:26	44	10:10:46	11:29:01	85	16:26:54	17:42:32
4	5:41:29	6:56:56	45	10:18:45	11:37:04	86	16:34:54	17:50:33
5	5:48:36	7:03:26	46	10:26:49	11:45:13	87	16:42:45	17:58:29
6	5:55:53	7:09:57	47	10:34:58	11:53:27	88	16:50:28	18:06:17
7	6:02:53	7:16:26	48	10:43:13	12:01:48	89	16:58:04	18:13:58
8	6:09:50	7:22:56	49	10:51:33	12:10:14	90	17:05:32	18:21:33
9	6:16:41	7:29:25	50	10:59:57	12:18:46	91	17:12:52	18:29:01
10	6:23:24	7:35:53	51	11:08:29	12:27:23	92	17:20:04	18:36:22
11	6:30:01	7:42:22	52	11:17:09	12:36:07	93	17:27:07	18:43:37
12	6:36:32	7:48:49	53	11:25:54	12:44:56	94	17:34:06	18:50:44
13	6:42:57	7:55:17	54	11:34:47	12:53:51	95	17:40:51	18:57:45
14	6:49:15	8:01:44	55	11:43:47	13:02:51	96	17:47:28	19:04:40
15	6:55:27	8:08:11	56	11:52:53	13:11:58	97	17:54:23	19:11:23
16	7:01:34	8:14:38	57	12:02:06	13:21:10	98	18:01:43	19:18:07
17	7:07:29	8:21:04	58	12:11:27	13:30:28	99	18:09:17	19:24:50
18	7:13:25	8:27:29	59	12:20:54	13:39:52	100	18:17:11	19:31:33
19	7:19:20	8:33:55	60	12:30:27	12:49:21	101	18:25:23	19:39:06
20	7:25:16	8:40:20	61	12:40:08	13:58:47	102	18:33:53	19:47:01
21	7:31:17	8:46:44	62	12:49:56	14:08:38	103	18:42:42	19:55:21
22	7:37:20	8:53:09	63	12:59:50	14:18:24	104	18:51:49	20:04:11
23	7:43:27	8:59:34	64	13:09:52	14:18:24	105	19:01:14	20:13:31
24	7:49:39	9:05:54	65	13:20:00	14:38:16	106	19:10:58	20:23:15
25	7:55:50	9:12:21	66	13:30:12	14:48:16	107	19:21:01	20:33:18
26	8:02:09	9:18:50	67	13:40:41	14:58:40	108	19:31:19	20:43:40
27	8:08:33	9:25:16	68	13:51:07	15:08:39	109	19:42:00	20:54:40
28	8:15:02	9:31:52	69	14:01:17	15:18:36	110	19:53:00	21:05:40
29	8:21:37	9:38:33	70	14:11:27	15:28:28	111	20:04:00	21:16:40
30	8:28:17	9:45:18	71	14:21:20	15:38:11	112	20:15:00	21:27:40
31	8:35:02	9:52:08	72	14:31:09	15:47:49	113	20:26:00	21:38:40
32	8:41:52	9:59:04	73	14:40:51	15:57:19	114	20:37:00	21:49:40
33	8:48:48	10:06:05	74	14:50:24	16:06:42	115	20:48:00	22:00:40
34	8:55:49	10:13:12	75	14:59:50	16:15:59	116	20:59:00	22:11:40
35	9:02:55	10:20:23	76	15:09:08	16:25:09	117	21:10:00	22:22:40
36	9:10:06	10:27:40	77	15:18:18	16:34:12	118	21:21:00	22:33:40
37	9:17:23	10:35:02	78	15:27:20	16:43:08	119	21:32:00	22:44:40
38	9:24:45	10:42:29	79	15:36:14	16:51:57	120	21:43:00	22:55:40
39	9:32:12	10:50:01	80	15:45:01	17:00:40	121	21:54:00	23:06:40
40	9:39:44	10:57:39	81	15:53:39	17:09:16	122	22:05:00	23:19:25
41	9:47:22	11:05:21	82	16:02:10	17:17:45	–	–	–

Table B.9: Number of on-board passengers for the train services in the up direction of the Beijing Yizhuang line

Service number	YZ - CQ	CQ - CQN	CQN - JH	JH - TJN	TJN - RC	RC - RJ	RJ - WY	WY - WHY	WHY - YZQ	YZQ - JG	JG- XHM	XHM XC	XC SJZ
1	0	8	9	28	19	33	55	78	86	144	152	202	210
2	0	7	10	17	62	57	60	63	56	72	97	179	170
3	0	7	10	17	62	57	60	65	87	173	208	242	176
4	0	7	10	17	65	85	116	117	146	195	207	240	175
5	0	8	13	37	95	132	141	120	149	200	212	246	180
6	0	21	18	43	91	127	136	115	143	193	213	305	409
7	0	21	18	43	90	126	135	114	156	278	509	388	482
8	0	21	18	42	89	147	186	260	255	332	503	383	477
9	0	21	18	60	215	308	241	293	250	326	494	376	468
10	0	31	52	85	238	303	237	288	246	321	487	388	565
11	0	37	52	84	234	298	233	284	242	378	647	972	738
12	0	36	52	82	230	293	229	331	466	768	750	956	726
13	0	35	51	81	282	371	524	462	596	754	736	939	713
14	0	35	68	157	467	435	540	455	586	742	725	924	701
15	0	87	113	176	461	429	533	449	578	732	715	951	1107
16	0	90	110	170	447	415	516	435	560	720	1093	1018	1296
17	0	90	110	170	447	415	516	517	692	745	1185	1018	1296
18	0	90	110	170	441	544	521	656	736	745	1185	1018	1296
19	0	90	107	216	423	633	521	656	736	745	1185	1018	1296
20	0	108	100	232	430	643	529	667	748	758	1204	1078	1284
21	0	111	101	233	433	647	533	671	753	783	1106	1225	1270
22	0	113	102	235	437	653	538	638	588	844	1085	1237	1282
23	0	114	103	238	445	526	573	576	531	854	1097	1250	1296
24	0	114	109	180	451	443	573	576	531	854	1097	1250	1296
25	0	84	123	173	462	453	586	589	543	873	1122	1126	914
26	0	83	124	175	467	458	594	596	549	717	676	873	758
27	0	85	126	178	473	464	553	462	525	520	644	884	768
28	0	86	128	173	344	406	412	368	530	527	653	897	779
29	0	77	80	134	246	402	418	373	537	534	662	909	775
30	0	58	59	136	250	407	423	378	544	541	636	586	656
31	0	58	60	137	253	413	429	373	447	485	579	492	665
32	0	59	60	139	249	351	331	281	309	485	586	498	673
33	0	60	55	85	209	264	304	285	313	491	594	505	682
34	0	34	42	67	212	267	308	288	317	497	591	476	509
35	0	33	43	68	214	270	312	292	319	417	296	435	452
36	0	33	43	69	217	247	231	294	318	368	300	441	458
37	0	34	41	70	162	152	162	297	322	372	303	446	463
38	0	25	20	71	160	154	164	300	326	377	307	425	416
39	0	23	21	72	162	156	166	304	321	277	351	373	387
40	0	23	21	72	164	163	187	245	249	190	358	377	391
41	0	24	22	64	147	190	207	233	252	192	362	381	396
42	0	18	26	58	147	192	209	235	255	194	366	377	294
43	0	17	26	59	149	194	211	238	239	263	352	368	244
44	0	17	27	59	145	195	212	239	177	303	356	372	247
45	0	18	27	51	108	196	214	241	179	306	360	376	250
46	0	26	27	48	109	198	216	244	181	309	352	284	380
47	0	27	28	49	110	201	218	227	223	289	327	230	399
48	0	27	28	50	122	194	145	167	264	290	331	232	403
49	0	23	16	55	137	191	144	168	265	292	333	235	394
50	0	19	12	56	139	195	147	171	270	299	268	357	369
51	0	19	12	57	141	188	185	236	284	306	238	362	374
52	0	20	15	56	143	139	236	268	288	310	241	367	378
53	0	13	23	57	145	141	239	272	292	314	272	373	390
54	0	13	24	57	147	142	242	277	296	230	367	379	396
55	0	13	24	59	120	194	239	283	300	223	371	384	401
56	0	18	22	62	99	211	242	286	303	226	376	402	356
57	0	21	22	63	100	213	245	288	286	319	429	446	306
58	0	22	22	63	120	220	263	213	229	379	437	452	310
59	0	23	26	43	161	227	271	216	231	384	443	445	354
60	0	25	28	44	163	229	274	218	243	364	406	280	454

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Table B.9 Number of on-board passengers for the train services in the up direction of the Beijing Yizhuang line (continued from previous page)

| Service number	YZ - CQ	CQ - CQN	CQN - JH	JH - TJN	TJN - RC	RC - RJ	RJ - WY	WY - WHY	WHY - YZQ	YZQ - JG	JG- XHM	XHM XC	XC SJZ
61	0	25	28	44	162	220	199	277	308	339	398	284	460
62	0	25	21	58	156	206	160	287	311	343	402	287	457
63	0	23	16	59	158	209	163	291	316	300	303	415	434
64	0	23	16	60	162	191	227	307	327	237	264	419	439
65	0	22	25	67	169	162	263	310	329	238	266	421	406
66	0	19	31	70	173	167	270	319	314	293	367	415	310
67	0	19	31	69	147	186	251	290	189	343	393	413	308
68	0	22	29	59	88	200	238	282	184	334	383	384	342
69	0	26	29	59	87	198	236	278	217	341	390	270	414
70	0	26	28	57	107	202	253	202	323	343	387	267	409
71	0	21	21	42	142	205	254	199	319	338	381	280	408
72	0	16	18	41	140	202	251	196	311	331	261	381	411
73	0	16	18	40	132	168	188	271	291	324	236	375	406
74	0	17	14	48	114	132	164	270	287	320	233	370	400
75	0	19	12	48	112	130	162	267	283	280	326	378	396
76	0	19	12	47	111	143	195	264	283	217	363	373	391
77	0	18	16	46	109	181	223	261	279	214	358	368	385
78	0	12	22	46	107	179	220	257	275	223	358	375	304
79	0	12	21	45	106	176	223	247	265	396	361	374	297
80	0	12	21	45	108	194	248	236	259	390	355	368	293
81	0	13	13	46	108	193	245	232	255	384	350	353	378
82	0	13	13	45	106	190	241	229	282	420	409	322	450
83	0	13	13	44	120	223	311	372	410	443	406	317	443
84	0	13	15	76	231	284	345	379	404	436	400	312	436
85	0	14	19	84	227	279	339	373	397	429	477	616	652
86	0	14	19	83	224	275	334	421	526	682	681	782	664
87	0	13	19	82	227	354	544	633	661	699	669	769	653
88	0	16	22	106	236	403	556	623	650	687	658	756	650
89	0	22	24	105	232	396	546	612	639	675	701	777	770
90	0	22	24	103	228	389	537	670	688	783	775	775	757
91	0	21	23	101	262	433	608	889	727	771	760	760	743
92	0	21	24	181	340	459	603	871	713	756	745	745	728
93	0	24	26	185	337	454	597	862	705	748	771	759	836
94	0	23	25	180	326	440	578	835	691	738	872	748	818
95	0	23	24	176	319	420	537	620	696	726	852	731	800
96	0	24	24	145	229	408	543	730	761	761	893	766	838
97	0	17	15	120	236	431	575	675	772	805	945	733	765
98	0	15	15	124	244	446	595	698	784	652	520	522	691
99	0	16	16	129	254	430	470	447	450	473	472	544	720
100	0	16	16	97	153	269	316	468	492	491	566	748	
101	0	15	16	73	149	279	327	384	485	510	475	475	414
102	0	15	16	76	154	289	339	375	434	324	281	365	294
103	0	16	17	73	145	196	244	274	351	282	291	377	304
104	0	17	13	38	132	146	242	284	363	291	300	365	316
105	0	18	13	39	136	151	250	293	336	290	245	233	330
106	0	18	13	40	119	153	178	185	191	290	241	241	340
107	0	11	13	26	75	155	156	187	197	297	247	237	290
108	0	5	14	27	78	160	161	191	195	238	262	184	189
109	0	5	14	30	83	138	190	165	179	195	271	190	195
110	0	6	8	36	87	123	194	165	179	195	254	203	215
111	0	7	7	36	87	123	179	168	183	199	132	223	230
112	0	7	7	34	87	80	94	175	187	199	132	223	230
113	0	5	5	33	88	76	94	175	187	191	164	217	218
114	0	5	5	33	87	80	105	159	168	131	212	214	216
115	0	6	7	28	39	93	121	148	164	131	212	214	203
116	0	9	9	26	39	93	121	148	156	133	169	116	110
117	0	9	9	26	43	81	92	81	78	135	148	108	110
118	0	6	5	16	55	67	78	73	78	135	148	107	105
119	0	4	4	16	55	67	78	72	79	97	66	102	99
120	0	4	4	14	35	33	28	71	81	79	61	102	99
121	0	2	1	10	19	21	26	71	81	79	51	61	43
122	0	1	1	4	3	0	0	0	0	0	23	19	16
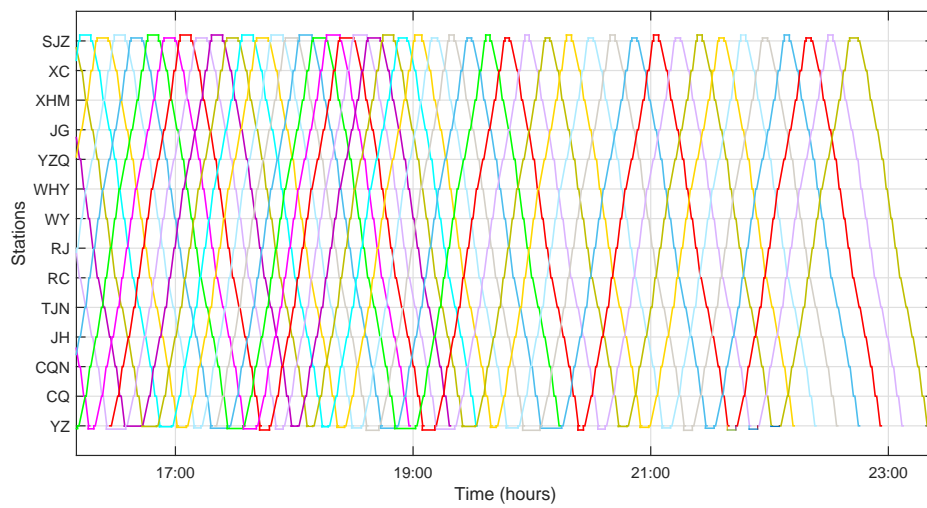
Figure A.23: Train schedule with 122 train services via the MILP-acc approach for the evening hours of the Beijing Yizhuang line