WHY WE SHOULD WORK ON MATHEMATICS IN PRE-SCHOOL AND PRIMARY SCHOOL

AN ANALYSIS OF PRESERVICE AND IN-SERVICE SCHOOL TEACHERS' VIEWS OF MATHEMATICS

European team ANFoMAM

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On the cover: Activities from the "Geometry, writing, expression" workshop held at the Department of Education Sciences, Roma Tre University, in November and December 2018
(still image by Fulvia Subania)

General index

Introduction

Within the framework of the ANFoMAM project, seven questionnaires (Qi) have been designed to explore the beliefs and attitudes towards mathematics of pre-service and in-service teachers in Spain, France and Italy: six of them related to the six areas of mathematics explored in the workshops designed in the project and a further one devoted to general aspects of mathematics and its teaching:

Q0.- General questionnaire about mathematics
Q1.- The understanding of arithmetic algorithms
Q2.- Solving and representing arithmetic problems
Q3.-Relationships between arithmetic and geometry
Q4.-Mental computation and use of the calculator
Q5.- History of mathematics and its teaching
Q6.- Geometric constructions and geometric problem-solving

Each questionnaire has been given to several groups of pre-service teachers studying in the universities that are partners in the project, Université de Bordeaux in France, Università Roma Tre in Italy, Universidad de Zaragoza and Universidad Pública de Navarra, in Spain, and also to several groups of in-service teachers during their training courses in Tokalon Association, in Italy.

The collected data will be analysed in order to study:

- The way in which the participants lived each area of mathematics, and mathematics in general, during their school and university studies
- Their beliefs about the nature of each area of mathematics, and mathematics in general, and whether or not it should be worked on in some way at school
- Their attitudes towards the teaching of each area of mathematics, and mathematics in general.

Besides the analysis of these issues, there are other possible uses of the questionnaires. For instance, any professor that wants to implement one of the workshops designed in the project (intellectual product O2) may be interested in giving the participants the corresponding questionnaire, or the general one, in order to:

- Help the pre-service or in-service teachers to:
    - Remember and revive in some way their previous experiences with that area of maths
    - Be aware of their own beliefs about that area of mathematics and its teaching
    - Reflect on their own attitude toward teaching maths as future or current teachers at school

- Obtain an overview of the experiences, beliefs and attitudes of the participants about the issues the workshop deals with. In this regard, the analysis of the answers collected during the project, provided in this document, will give the professor an idea of the student profiles they will find in the workshop.

- Be aware, at the end of the workshop implementation, of any shifts that may have occurred in the beliefs and attitudes of the participants.

For this last aim, it is not necessary to give the participants the questionnaire again at the end of the workshop. The professor could select some questions or, if he or she prefers, the following alternative questions could be formulated to the participants in order to assess both the quality of the workshops and the results:

   - What has this workshop provided for you?
   - Have your beliefs changed in some way either about the nature of this area of maths or about its teaching? Describe these changes
   - Do you have any suggestions that might improve the workshop?
   - Assign a value (from 1, disappointing, to 5, excellent) to your experience at the workshop

Needless to say, any of the designed questionnaires can be used independently of the corresponding workshop to get to know a group of students at any given moment.

We initially planned to include an analysis of the participants' knowledge of mathematical and didactical contents, but the reaction of some participants to this has dissuaded us from doing so in all the questionnaires. For instance, in the questionnaire about solving arithmetical problems, we thought that the inclusion of some particular problems might make some of the participants feel intimidated. Such feelings would not help them reflect on their lived experiences and beliefs. As it is explained in the description of the workshops, a key factor is the creation of a relaxed atmosphere, which can provide a pleasant experience for the participants. Clearly, beginning the workshop with questions, which might be seen as a test of their mathematical knowledge, could make them feel uncomfortable and as such create a strained atmosphere.

Theoretical framework

In our project, when we talk about **beliefs**, we follow the definition of Vause (2011). The author (p. 22) defines beliefs as a reservoir of values and ideas on which teachers rely to act in situations and justify their actions. Beliefs may be *personal* (highly dependent on the history of the subject; integrated over time, through different educational experiences) or *shared* (linked to ideas shared within an institution). These beliefs are then distinguished from knowledge, which, according to Vause (p. 26), is a set of contents and skills related to a field that can be empirically validated. Despite this distinction, syncretisms exist between knowledge and beliefs in a teacher's practices, which leads Vause (pp. 27-28) to speak of *working knowledge*, which is a mixture of beliefs, knowledge from practice and more theoretical knowledge.

We also agree with the definition of **attitudes**, in the sense of Zan and Di Martino (2014):

> When students describe their own relationship to mathematics, nearly all of them refer to one or more of these three dimensions: emotions, vision of mathematics and perceived competence[1]. These dimensions and their mutual relationships therefore characterize students' relationship with mathematics, suggesting a three-dimensional model for **attitude** (TMA) (Fig. 1):
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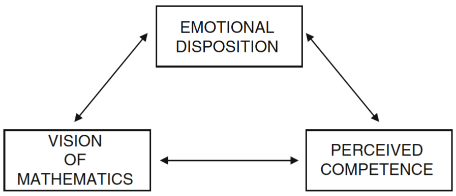
> 
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>
> **Students' Attitude in Mathematics Education,**
> **Fig. 1** The TMA model for attitude (Di Martino and Zan 2010)
>
> The multidimensionality highlighted in the model suggests the inadequacy of the positive/negative dichotomy for attitude, which referred only to the emotional dimension. In particular the model suggests considering an attitude as negative when at least one of the three dimensions is negative. In this way, it is possible to outline different profiles of negative attitude towards mathematics. Moreover, in the study a number of profiles characterized by failure and unease emerge. A recurrent element is a low perceived competence, perhaps reinforced by repeated school experience perceived as failures, often accompanied by an instrumental vision of mathematics.

In our questionnaires, many items are expressed as beliefs (in the sense of Vause, 2011). For the analysis of the results collected, we use the three dimensions defined by Green (1971) to characterize beliefs: *quasi-logical structure*, *psychological centrality* (the degree of conviction) and *cluster structure*.

Each individual organizes their beliefs with their own logic. This can be described as a quasi-logical structure. Unique for each person, this dimension reflects the thought and perspective of the person in question. Beliefs may contradict each other even if they are held by the same subject, whereas the knowledge system does not normally contain contradictions.

[1] Perception that we have of ourselves knowing that we are able to do, feel, express, be or become something.

The dimension of the psychological centrality of beliefs implies that there are beliefs that are more important to an individual than others. One could say that the most important ones are psychologically more central and that the others are peripheral in the individual's belief system. Thus, beliefs have their own psychological strength, that is, they are distinguished by the degree of conviction with which they are held by the individual. The degree of conviction may vary from one belief to another. The most central beliefs are the strongest. They are generally considered 100% safe, while peripheral ones can be replaced more easily.

The last dimension, the cluster structure, is based on the fact that beliefs are grouped together. As Green (p. 41) states: "Nobody holds a belief in total independence of all other beliefs. Beliefs always occur in sets or groups". This cluster structure even allows individuals to have conflicting beliefs within their own belief system. The grouping property can help explain some of the inconsistencies found in an individual's belief system.

Report of questionnaire Q0: General questionnaire about mathematics

Table of contents

1. Abstract

We present here the process of design of a general questionnaire (appendix P0) about beliefs and attitudes concerning to the nature of mathematics and its teaching in Primary education. It has been given both to in-service teachers and to pre-service teachers from the partner institutions of the project ANFoMAM. The analysis of the collected data provides several profiles of future and current teachers based on their experience with mathematics during their childhood. As expected, the answers are related to the participants' provenance as well as their personal or professional situation. Some modifications of the questionnaire are proposed for the future concerning the self-perception of the participants regarding their mathematical competence (appendix Q0[2]).

2. Questionnaire design: background and goals

The general questionnaire was designed by researchers from all the institutions that are partners in the project during our first international meeting in Rome (September, 2018). We intended to design a questionnaire to collect information about the beliefs and attitudes towards mathematics of the pre-service and in-service teachers who are studying in our institutions. The starting point were the ideas we shared in the Symposium Numeracy and Beyond in the 5th International Congress of Educational Sciences and Development (Santander, May 2017), where we interchanged different experiences related to work carried out to improve the training of both pre-service and in-service primary school teachers in mathematics in France, Italy, Norway and Spain (Celi & De Simone, 2018; Campión Arrastia et al, 2017; Lekaus et al., 2015).

As university teachers in charge of primary teachers' courses, we had verified the poor experiences with mathematics that our students typically reported from their school years (Gil Clemente, Millán Gasca, 2016). We also shared the belief that feelings such as frustration and unease were often associated with a rigid view of elementary mathematics as rote learning of procedures and computation, which results in participants lacking of confidence (Celi et al., 2020) in their capacity to teach mathematics with enthusiasm. A priori, we expected the general questionnaire to distinguish between two distinct profiles; the abovementioned one and a contrasting profile of a teacher who views mathematics as an engaging and dynamic subject, strongly related to human experience. Our expectation was that participants with

[2] Questionnaire Q0 : https://docs.google.com/forms/d/1J15LUMwkk45JHBpA6iUUCVw5ZPnRhc4e-yfPWahirnk/copy

the latter profile would seek to teach maths at school not only with a utilitarian target, but also with the educative purpose of contributing to children's growth.

However, obtaining an overview of the relationship of our students with mathematics was not the only purpose of questionnaire. We also intended that, by completing this questionnaire, the participants would become aware of three related aspects: their 'lived experiences' (Van Manen, 2016), their beliefs about the nature of mathematics and their beliefs about the objectives of teaching maths to primary school children. Without doubt, a greater consciousness of these factors will help them adopt the best approach to living new experiences in the mathematics workshops that will be designed in the project. For this reason, we did not want to include in this questionnaire arithmetical problems or any other question that might make them feel embarrassed about their lack of mathematical knowledge. We prefer to include questions that led them to write down words related to their mathematics experience (numbers 1 and 3) or questions (number 2) in which they have to assign a lower or higher weight to some aspects related to mathematical activity. In addition, by means of some multiple-choice questions (numbers 4 and 5), they can express who or what helped or hindered their learning of mathematics during their childhood. The following question (number 6) provide them with a list of general beliefs about both the nature of mathematics and the aims of learning it from an early age at school, to which the participants have to express their degree of agreement. The last two questions (numbers 7 and 8) ask them about their most recent mathematical studies and current professional status.

3. Data collecting

The questionnaire has been given to the following groups of participants:

- Two groups of students of the second year of Primary Education Degrees of the Universidad Pública de Navarra (Upna) in Spain
- Two groups of students of the second year of Primary Education Degrees of the Universidad de Zaragoza (Unizar) in Spain
- Two groups of students of the fourth year of Primary Education Degrees in Université de Bordeaux (UB), in France
- One group of students of the Università Roma Tre (URT) in Italy
- A group of in-service teachers participating in updating courses in Tokalon Association (TOK) in Rome.

The distribution of in-service and pre-service teachers is collected in the following table.

Table 1: Distribution of participants

	Number of pre-service teachers	Number of in-service teachers	Total of Participants	in-service/ pre-service
Upna	83	1	84	0 approx.
Unizar	87	4	91	0 approx.
UB	37	24	61	2/3 approx.
URT-TOK	44	86	130	2 approx.
	251	115	366	

The following graph shows the differences of ratio between in-service (maestro) and pre-service teachers (estudiantes) when we take into account their origin.

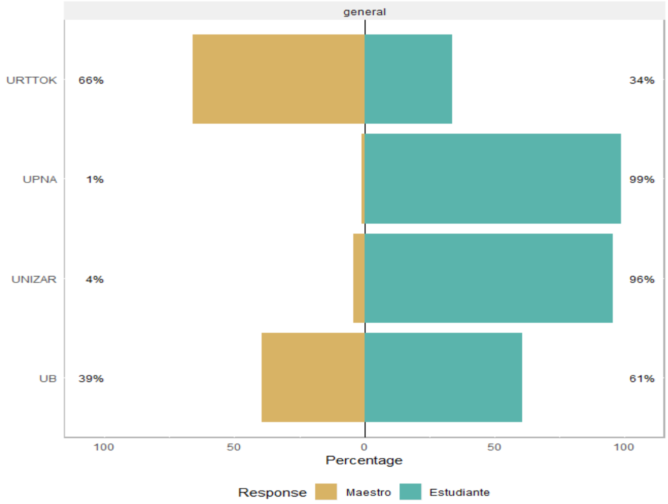


Figure 1

4. Data processing and analysis

The main part of the questionnaire consists of six questions:

1) Words associated to mathematics (open answer)
2) Weights assigned to 14 aspects of mathematics in the Likert scale (from 1, the lowest weight, to 4, the highest)
3) The three most difficult themes of mathematics (open answer)
4) About people or things that have been a help to learn mathematics
5) About people or things that have been a hindrance to learning mathematics
6) Degree of agreement with 12 statements about mathematics

We analyse first the answers to questions numbers 2 and 6:

Question 2: As can be appreciated from Figure 3, participants' experience of maths is mainly associated with reasoning, attention and perseverance, followed by experimentation and construction, while fantasy and speed occupy the last positions in the ranking. If the analysis takes into account the participants' institutions, the main differences between them can be seen in the responses to the words creativity and dialogue, which have a very high weight only for the Italian group (see Figure 4). All the other universities give creativity a very low weight, but especially in the case of dialogue, the response from Upna contrasts dramatically with Rome.

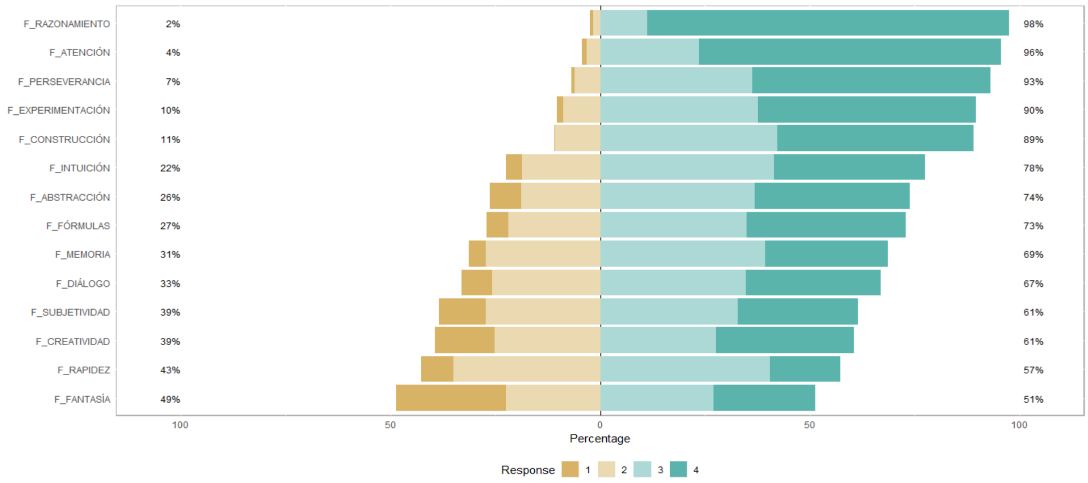


Figure 2

Key:

F_razonamiento: reasoning F_atención: attention F_perseverancia: perseverance F_experimentación: experimentation F_construcción: construction	F_intuición: intuition F_abstracción: abstraction F_fórmulas: formulae F_memoria: memory F_diálogo: dialogue	F_subjetividad: subjectivity F_creatividad: creativity F_rapidez: speed F_fantasía: fantasy

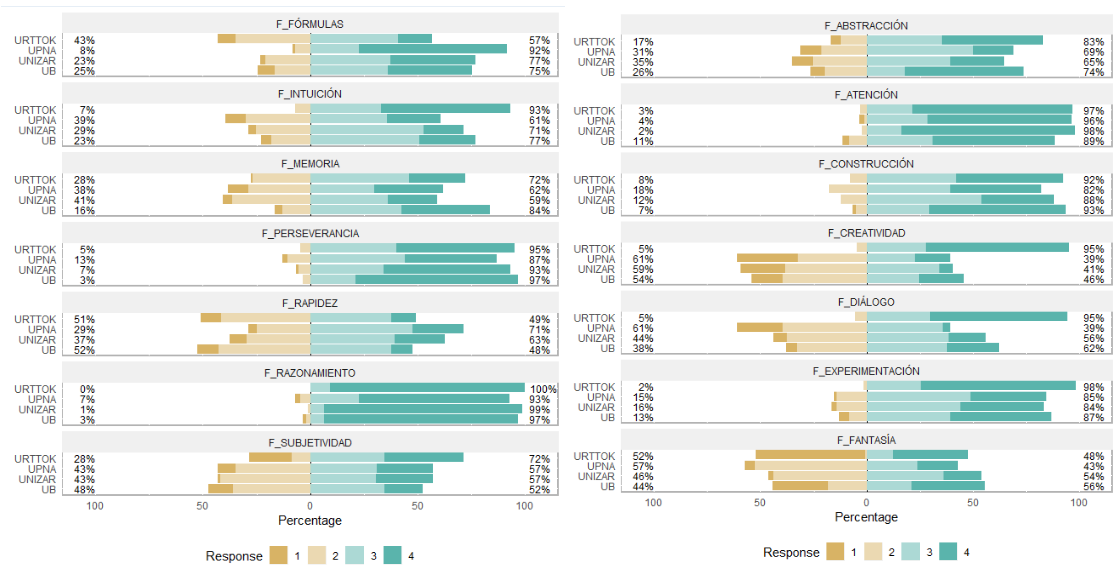


Figure 3

Question 6: The main agreement is given on the beliefs that one's ability at mathematics can be improved over time and that it structures your mind although this belief is less strong in Upna. There is also a general consensus that mathematics is not always the same thing. If the analysis takes into account the participants' institutions, the greatest discrepancies appear again with the Italian group, who consider that mathematics is fun, not beautiful and not difficult, while the rest of participants think the opposite.

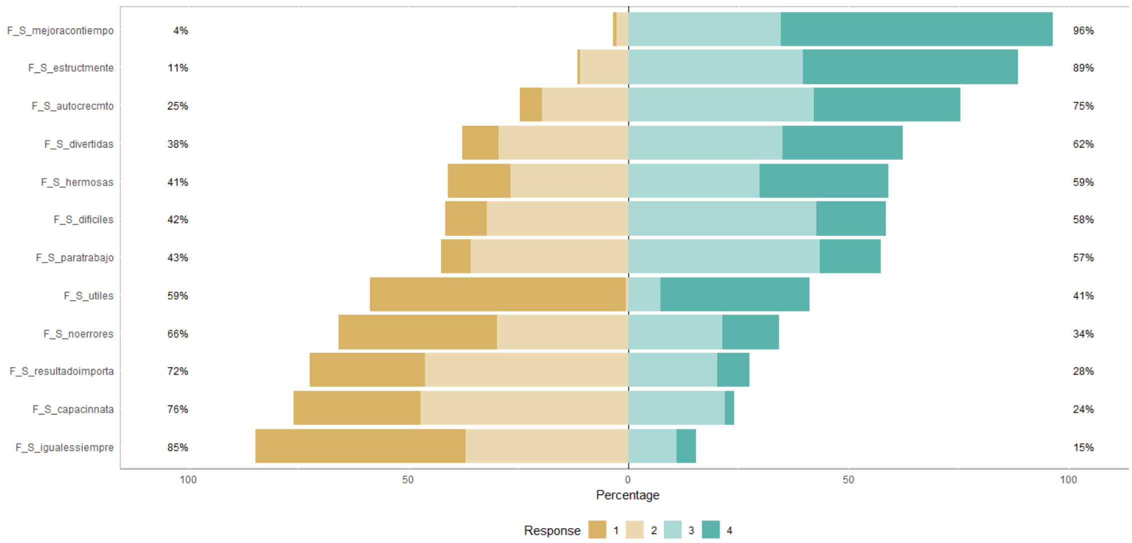


Figure 4

Key:

F_S_mejoracontiempo: ability can be improved	F_S_paratrabajo: essential to find a job
F_S_estructmente: structures one's mind	F_S_utiles: useful in day-to-day life
F_S_autocrecmto: contributes to personal growth	F_S_noerrores: mistakes must be avoided
F_S_divertidas: maths is a fun	F_S_resultadoimporta: result is what matters
F_S_hermosas: maths is beautiful	F_S_capacinnata: necessary to be gifted
F_S_dificiles: maths is difficult	F_S_igualessiempre: always the same thing

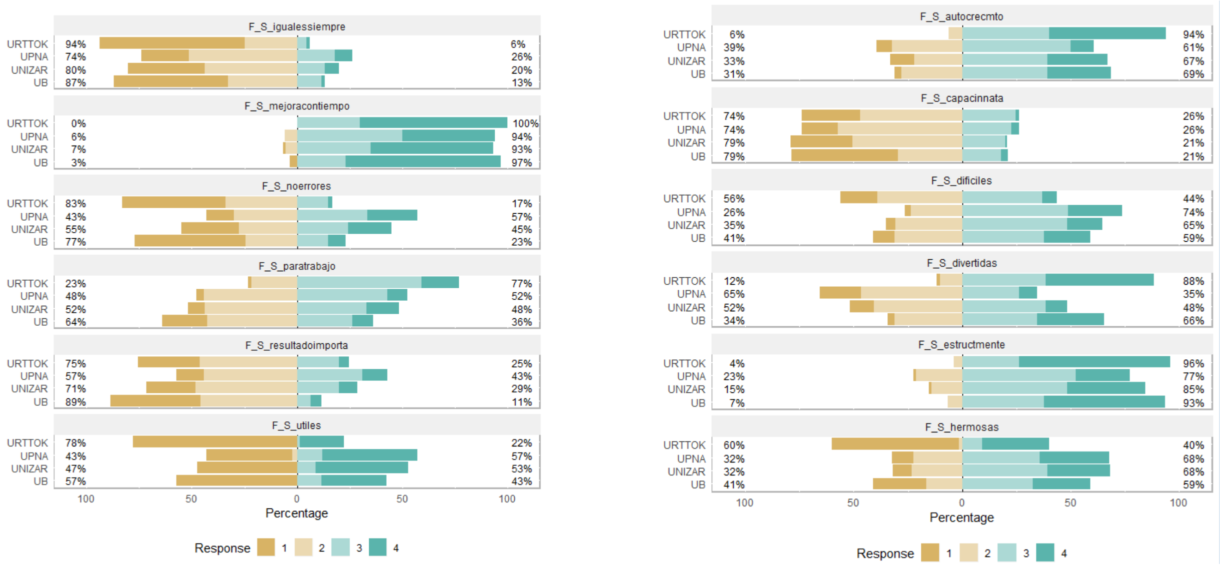


Figure 5

Análisis de los grupos de variables

Following Green's theory of clusters, we look for groups of variables which appear together in the data by using a technique of multivariate data analysis, called the principal component analysis.

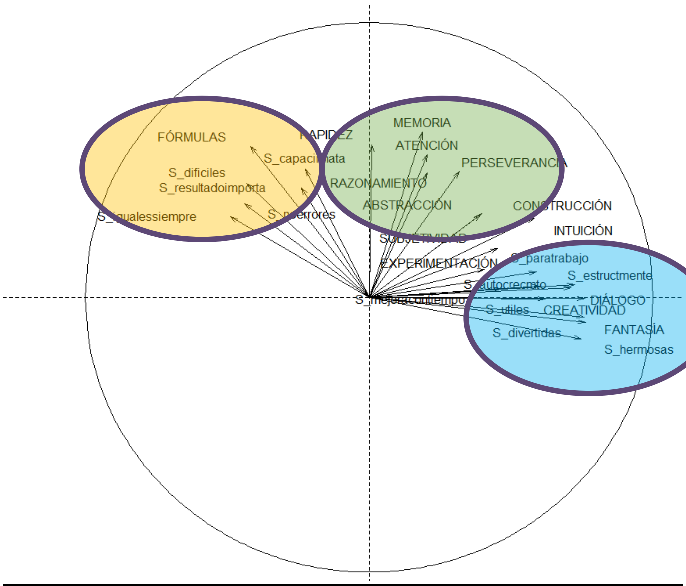


Figure 6

We can distinguish three groups of variables:

Group 1) A first group consisting of variables related to a technical and static view of mathematics: Assertions such as *maths is always the same thing, the result is what matters and words such as formulae* denote that maths is understood as something static, something 'given' which is not susceptible to modification. Maths is seen as *difficult* and as *a subject for which it is necessary to be gifted*. This means that mathematics is not considered as something which is natural and linked to human experience. Instead, it is seen as a technical competence, a mere skill (Peters, 1966), which is not easy for some students, who have to practice in order to gain in *speed* and accuracy (*avoiding making mistakes*).

The attitude towards teaching maths that may be expected from those participants who have given a high weight to these variables is:

- Providing pupils with closed tasks and mechanical procedures that allow them to gain in speed and accuracy
- Giving students repetitive activities in order to prevent them from making mistakes
- Checking and evaluating the results of the tasks, rather than the processes followed by the pupils
- A fixed mindset (Dweck, 2006) with respect to their pupils, which leads to them regarding each student as having a fixed capacity for maths.

Group 2) A second group consisting of variables related to a dynamic and less technical view of mathematics. Words such as *dialogue* and *creativity* denote a vision of maths as something which pupils can argue about, something that can be 'made' instead of being 'given' and that can help people communicate with each other. The reference to *fantasy* connects maths with the children's world, while assertions such as *maths is a fun* and *maths is beautiful* show that maths provides enjoyment at the same time as *contributing to personal growth*. The participants with this view of mathematics also think that maths is *useful in day-to-day life* and *essential to find a job*, which reinforces the view of maths as something related to personal life (Orón Semper & Blasco, 2019) and not merely technical. The idea that maths *is an ability that can be improved with* *time* allows us to think that they consider maths as something connected with human nature, not reserved for a minority of people with a special gift.

The attitude towards teaching maths that may be expected from those participants who have given a high weight to these variables is:

- Providing pupils with open tasks which encourage creativity
- Giving students activities which are linked to their day-to-day life
- Looking for tasks that make human sense for children, leaving room for fantasy and imagination
- Promoting the sharing and the interchanging of strategies in the classroom by means of dialogue
- A growth mindset (Dweck, 2006) with respect to their pupils i.e. the conviction that their pupils' performance with mathematical tasks can be improved

Group 3) A third group consisting of variables associated with words such as *memory, attention* and *perseverance*, all of them referring to aspects of mathematical learning that can be improved by means of work and effort. Together with this emphasis on work, there also appear words such as *abstraction* and *reasoning*. The attitude towards teaching maths of those participants who have given a high weight to these variables is difficult to predict. The fact that these variables are not correlated to the variables belonging to the other two groups means that the participants who have highly valued variables of Group 3 could have either a dynamic and personal vision of mathematics or a static and technical one.

i) If the variables of this third group appear together with those of the first group, it is expected that they correspond to participants who have a technical and static view of maths, attributing the *difficulty* of the subject to its *abstract* nature and the need for *reasoning* to perform the tasks. Although these participants think that one *needs to be gifted* for doing maths, pupils may overcome the difficulties by doing training tasks, i.e. mechanical and repetitive procedures.

ii) If this third group of variables appear together with those of the second group, it is expected that they correspond to participants with a dynamic view of maths, who think that aspects such as *abstraction* and *reasoning* may be promoted by providing the pupils with a great variety of concrete tasks. If the pupils work, with *attention* and *perseverance*, on a series of different concrete representations of the same mathematical concept, the understanding of the abstract concept will follow.

Analysis of the profile of the participants of different institutions

If we represent each participant in our study by means of the value that he or she has assigned to the 26 variables which appear in Figures 2 and 4, i.e. as a point in a 26-dimensional space, the statistics technique of the Discriminant Analysis represents the projection of those points onto a plane determined by two axes, LD1 and LD2, each of which is obtained as a linear combination of the 26 variables considered in our study. Among all the possible projections, the analysis looks for one which puts the individuals of different institutions as far apart as possible, while placing the individuals from the same institution as close as possible. Figure 7 shows the number of participants from each institution that belong to the expected group following this projection.

Specifically, the axis LD1, described in Figure 9 as a specific linear combination of all the variables, separates the Spanish participants (in the first and fourth quadrants) from the Italian ones (mainly in the second quadrant), as can be seen in Figure 7. Moreover, the axis LD2, also described in Figure 9, separates the French participants (in the third and fourth quadrants) from the majority of the Italian participants.

It can be seen in Figure 7 that Spanish participants are divided between the positive (upper) semi-plane and the negative (lower) one.

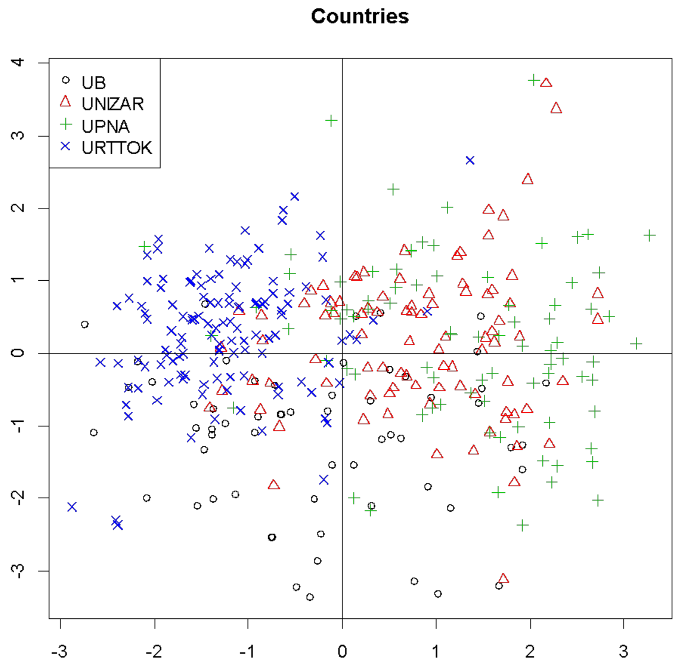
	Attention - creativity - beautiful - useful for finding a job intuition - speed - necessary be gifted - avoid making mistakes **Positive LD2**	
Negative LD1 Fantasy Dialogue Necessary be gifted A fun Memory Structures one's mind		**Positive LD1** Speed Attention Construction Always the same thing Formulae Avoid making mistakes
	Negative LD2 A fun- Memory - Perseverance - Fantasy - structure one's mind Construction - Abstraction	

Figure 7

grupo \ Grupo	UB	UNIZAR	UPNA	URTTOK
UB	32	7	0	4
UNIZAR	11	56	20	3
UPNA	2	16	55	1
URTTOK	16	12	9	122

Figure 8

	LD1	LD2		
EXP	0.18446838	0.1154512065	·	EXP: Experimentation
FAN	-0.66173517	-0.4861270180	·	FAN: Fantasy
CR	0.01887751	0.7015117094	·	CR: Creativity
RZ	-0.17584850	-0.1433206847		RZ: Reasoning
DG	-0.45664316	0.1945463710		DG: Dialogue
INT	-0.08010382	0.2672226298	·	INT: Intuition
MEM	-0.27182067	-0.5029879218	·	MEM: Memory
ABS	0.01170164	-0.2076723301	·	ABS: Abstraction
ATE	0.30061440	0.7115421464		ATE: Attention
RP	0.33813858	0.2181119463		RP: Speed
PER	-0.16349755	-0.4947876642		PER: Perseverance
CONS	0.21003362	-0.2481014539	·	CONS: Construction
SUJ	0.03286280	0.1750934522	·	SUJ: Subjectivity
FOR	0.18033813	-0.0007551256	·	FOR: Formulae
cap	-0.36467392	0.2266053515	·	cap: it is necessary to be gifted to perform well at maths
mej	0.09732893	-0.0189619859	·	mej: one's ability can be improved over time
uti	0.04126504	0.1093907104		uti: useful in day-to-day life
trb	0.08359742	0.4772304996		trb: essential to find a job
her	-0.09411722	0.4986212727	·	her: beautiful
res	0.02131315	0.1034917194	·	res: the result is what matters
dif	0.04262010	-0.1073154688	·	dif: difficult
div	-0.31081988	-0.5952555457		div: a fun
ig	0.27222073	0.0110757583		ig: always the same thing
aut	0.02148864	0.0134595290		aut: contributes to personal growth
noer	0.17627339	0.2180931591	·	noer: making mistakes must be avoided
estr	-0.23063977	-0.3697926939	·	estr: structures one's mind

Figure 9

Observing the variables that have the highest positive coefficients in the axis LD1, we can say that what distinguishes the Spanish participants is that they give a high weight to *attention, speed, construction* and *formulae*, at the same time as considering that mathematics *is always the same thing*. These features separate them from the Italian participants, who give a high weight to the variables that have the highest negative coefficients in the axis LD1, i.e. to *fantasy* and *dialogue*, as well as, to a lesser extent, *memory*, while they believe that *mathematics is a fun* and *structures one's mind*. At the same time, they think that, *in order to perform well at mathematics, it is necessary to be 'gifted'*.

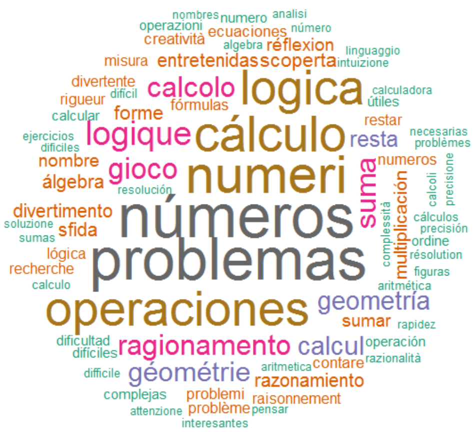
The above analysis shows a profile of the participants of each institution, taking into account their position in the diagram (Figure 7). There are what could be considered inconsistencies in the groups, due to the overlap of individual beliefs with social ones (Green, 1971). For instance, the profile of the Spanish participants clearly corresponds to a static and technical vision of maths (Group 1 of the analysis of variables), together with an emphasis on *speed* and *attention* (Group 3), which leads us to expect the attitude described in Group 3 (i) of the above-mentioned analysis of variables. This is consistent with the high weight given to *difficulty*, *reasoning* and *perseverance* by these participants in Questions 2 and 6. However, the reference to *construction* may be considered inconsistent, as well as the high weight given to *beautiful* in Question 6. This contrast leads us to expect that these participants would be inclined to provide their pupils with creative activities which they have perhaps not experienced in their own childhood.

On the other hand, it may seem strange to find *necessary to be gifted* as a part of the Italian beliefs, contrasting with other features more typical of a dynamic vision of maths, such as *fantasy*, *dialogue* and *a fun* (all of them in Group 2 of the analysis of variables). This fact, together with the weights that these participants have given to aspects such as *intuition, creativity*, *ability can be improved*, *contributes to personal growth, structures one's mind* and *essential to find a job* in Questions 2 and 6, leads us to expect that the attitude of these participants will be that described in Group 2 of the analysis of variables. This attitude is associated with a growth mindset, though the weight given to *necessary to be gifted* is difficult to explain. Most of the Italian participants are also characterized by the variables that have the highest positive coefficients in LD2, such as *creativity, attention, beautiful* and *essential to find a job.*

French participants give a high weight to those variables with negative coefficients in LD2, such as *a fun, memory, perseverance and fantasy* together with *abstraction* and *construction* to a lesser extent. The profile of the French participants is a mixture of a dynamic view of maths with an emphasis on working on tasks linked to perseverance and memory. Therefore, an attitude described in Group 3 (ii) of the analysis of variables is expected for these participants. It is surprising that French participants give a low weight to the variable *essential to find a job,* given the importance assigned to maths by the educational institutions in the process of teacher recruitment in France.

Question 1: Regarding the words that mathematics suggests to the participants according to their school experience, Figure 10 shows that most of them associate mathematics with *numeracy, calculus* and *problems.* The fact that *operations* also appear among the most repeated words reinforces the idea that most of the participants identify mathematics with arithmetic. This idea is repeated with the word *problems*, that appears mainly in the Spanish answers, whose curriculum in Primary Education only includes problems of an arithmetic nature, sometimes contextualized in measurement situations.

The Spanish word *geometría* appears less frequently than the arithmetic words. Among the Italian answers, *numeri* (numbers) and *logica* (logic) are the most frequent, followed by *gioco* (game), *calcolo* and *ragionamento* (reasoning). The word *forme* appears less frequently, as well as *scoperta* (discovery). The French participants mainly associate mathematics with *logique*, *géométrie* and *calcul.* This has to do with the French curriculum, which gives equal weight to arithmetical and geometrical contents.



	palabra	frec
números	números	43
problemas	problemas	42
numeri	numeri	37
cálculo	cálculo	35
operaciones	operaciones	34
logica	logica	33
logique	logique	21
gioco	gioco	18
suma	suma	17
calcolo	calcolo	17
ragionamento	ragionamento	17
géométrie	géométrie	16
calcul	calcul	15
geometría	geometría	14
resta	resta	11
forme	forme	10
scoperta	scoperta	10
entretenidas	entretenidas	9

Figure 10: words chosen by participants in Question 1

Question 3: The Italian participants are the only ones who name a theme corresponding to Primary level as one of the most difficult ones that they have studied in Mathematics. The Spanish participants consider that the most difficult themes are *integrales*, *derivadas* and *ecuaciones*, followed by *funciones* and *trigonometría*, none of them included in the curriculum of Primary education. The French people consider *fonctions* as the most difficult theme, followed by *matrices*, a word that is not distinguishable from the Spanish one. Italian participants emphasize themes such as *integrali*, *trigonometria* and *funzioni*, but they add *problemi*, the only theme corresponding to Primary school.

	palabra	freq
integrales	integrales	55
derivadas	derivadas	48
ecuaciones	ecuaciones	39
funciones	funciones	34
trigonometría	trigonometría	32
integrali	integrali	32
trigonometria	trigonometria	29
problemi	problemi	28
funzioni	funzioni	27
fonctions	fonctions	26
logaritmos	logaritmos	25
problemas	problemas	25
geometría	geometría	23
derivate	derivate	20
matrices	matrices	19
geometria	geometria	19
probabilidad	probabilidad	18
equazioni	equazioni	18

Figure 11: topics chosen by participants in question 3

Question 4: This question let the participants mark more than one option. The answers show that the figure of the *teacher*, both at primary and secondary level, has been the most important factor in helping them learn. A *relative* and a *classmate* are also named, followed at a certain distance by a *book*, a *game* or a *film*.

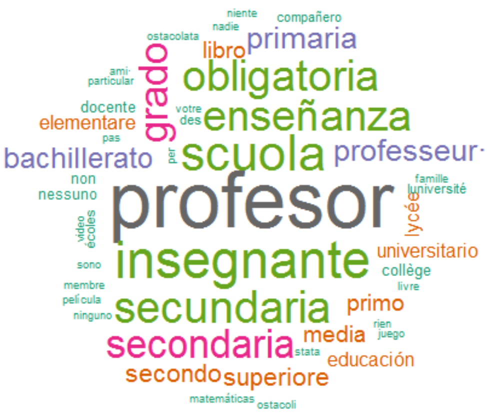
	palabra	freq
profesor	profesor	368
insegnante	insegnante	138
primaria	primaria	134
scuola	scuola	123
enseñanza	enseñanza	93
obligatoria	obligatoria	93
secundaria	secundaria	93
educación	educación	88
professeur·	professeur·	81
particular	particular	79
bachillerato	bachillerato	79
grado	grado	77
secondaria	secondaria	77
familiar	familiar	70
compañero	compañero	56
libro	libro	50
elementare	elementare	46
universitario	universitario	41

Figure 12: words chosen in relation to question 4

Question 5: It might be surprising that, when in question 5 the participants have to choose something or someone that has hindered their learning of mathematics, the *teacher* is again the option which is most often specified, as in the case of a factor which has helped them, although the teacher is mentioned less frequently than in the answers to Question 4.

The experience of students, both negative and positive, with mathematics is clearly closely linked to their teacher, i. e. the figure of the teacher has a very central role in teaching mathematics. Therefore, the teaching of mathematics cannot be considered a matter of mere technical transmission of knowledge that works or does not work independently of what the teacher does or thinks. It is clear that the interpersonal dimension of learning is an important factor in education (Orón Semper & Blasco, 2019).

It is also noteworthy that a *book* is seldom considered to be a hindrance.



	palabra	frec
profesor	profesor	173
insegnante	insegnante	108
scuola	scuola	105
secundaria	secundaria	89
enseñanza	enseñanza	88
obligatoria	obligatoria	88
grado	grado	77
secondaria	secondaria	77
professeur·	professeur·	54
primaria	primaria	52
bachillerato	bachillerato	48
secondo	secondo	42
superiore	superiore	42
media	media	35
primo	primo	35
libro	libro	32
elementare	elementare	27
universitario	universitario	26

Figure 13: words chosen in relation to question 5

5. Conclusions: reporting, recommendations and proposals for improvement

The collected data give us an idea of the beliefs and attitudes of the participants regarding the nature and teaching of mathematics in Primary Education. As expected, the answers are related to the participants' origin as well as their personal or professional situation. In this respect, it is important to remember that the ratio between in-service teachers and pre-service teachers is almost 2:1 in Rome, approximately 2:3 in Bordeaux and 0:200 in the Spanish universities.

The answers given to Question 1 support the idea that the participants mainly associate mathematics with arithmetic, especially in Spain, where words related to other areas of mathematics appear at a certain distance from the arithmetical ones in the frequency table of the answers. In contrast, both in Italy and in France, logic and geometry are also regarded as areas which are associated with mathematics.

A combined analysis of the variables appearing in Questions 2 and 6 clusters the variables in three groups, which draw two different profiles of participants, those with a static and technical view of maths and those that present a dynamic and more personal view of maths. The attitudes of the participants who have given a high value to the variables of the first group are expected to be very different to those of the participants who have assigned a high value to the variables of the second group. If the weights which are given to the variables of the third group are added to the equation, we can profile more accurately the expected attitudes towards teaching of the different groups of participants.

In addition, the technique of the discriminant analysis allows us to look for the features that best separate the participants of the different institutions. The Spanish participants' profile clearly corresponds to a static and technical vision of maths combined with a confidence in pupils' effort to improve their performance at maths. This is in keeping with the answers given by the Spanish participants to Question

1, in which they associate mathematics with arithmetic tasks, often closed tasks with little room for creativity and experimentation.

Most of the Italian participants are described by the variables associated with a dynamic and more personal view of maths together with a growth mindset with respect to their pupils. This is in keeping with the answers given by the Italian group to Question 1, where terms like *gioco* (game) or *scoperta* (discovery) are associated with mathematics. It is noteworthy that two thirds of the Italian participants are in-service teachers who attend the updating courses organized by Tokalon Association. These courses are intended to transmit the idea that mathematics is a dynamic and enjoyable subject, closely linked to human nature.

Finally, the profile of the French participants is a mixture of a dynamic view of maths with an emphasis on working on tasks linked to perseverance and memory. The answers that this group give to Question 1 show that geometry is at the same level as arithmetic in the collection of words that they associate with mathematics.

However, these profiles are not totally consistent. It is strange that the Spanish participants give a high weight to both *construction* and *maths is beautiful*, while the Italian participants think that it is *necessary to be gifted to perform well at maths*. It is also odd that French participants give a low weight to the variable *essential to find a job* given the importance assigned to maths by the educational institutions in the process of teacher recruitment in France.

The answers to Question 3, mostly themes of Secondary education, do not provide any information about difficulties in Primary school (apart from the fact that the Italian group name *problemi* as one of the most difficult themes they have studied at school). This lack of information has led us to think that this question could be removed from the general questionnaire. As a suggestion, it could be slightly changed into the way showed in Appendix Q0.

The answers given to Questions 4 and 5 show that experience with mathematics is not something merely technical. In Primary school particularly, interpersonal relationships form an integral part of this experience. At school, children do not only learn concepts; they live events (Orón, 2019). This fact can explain why the participants' experiences with maths are very connected to the relationship with their teacher. The teachers constitute both a help and a hindrance in their learning of maths.

Report of questionnaire Q1: Understanding of the arithmetic algorithms

Table of contents

1. Abstract

A questionnaire about the understanding of the arithmetic algorithms in primary schools has been designed within the framework of the Eramus+ project ANFoMAM, *Learning from children to improve primary teachers' maths-specific education* (Catalán et al., 2019). The aim is to discover the experiences of pre-service and in-service teachers with arithmetical algorithms during their school years. The questionnaire will also explore their beliefs about both the aims of teaching the traditional arithmetical algorithms at primary schools and the difficulties that children usually have with them.

After piloting the questionnaire (appendix P1) with Spanish, French and Italian participants, a mostly favourable attitude towards the teaching of the arithmetic algorithms in primary schools can be seen. In addition, the results show a relationship between the participants' own experience with the algorithms in their school years and what, in their opinion, is the most appropriate approach to take in their primary school teaching.

Finally, the analysis of the results obtained in the first implementation of the questionnaire has led us to modify it slightly for use in future investigations (appendix Q1[3]).

2. Questionnaire design: background and goals

The teaching of the arithmetic algorithms has taken up hours and hours of the school time of children for centuries. This learning has been considered useful for dealing with day-to-day life, solving problems related to money or measurements, both at home and at work. In fact, although the curricula of mathematics in primary education include other areas of mathematics such as geometry or basic statistics, it would be difficult to imagine a pupil finishing primary school without having learnt the algorithms of the four basic arithmetic operations.

However, the learning of the classical algorithms of the arithmetic operations is often considered the paradigmatic example of a routine task in mathematics. Moreover, this learning is currently being questioned now that we all carry a calculator in our pocket.

[3] Questionnaire Q1 : https://docs.google.com/forms/d/1Qa_IMwpyG9DIZkxynqi88OuvPJ93RQ0Sxo7lJQ4ICLA/copy

Nonetheless, in our view, the teaching of the classical arithmetic algorithms continues to constitute a necessary item in the mathematical curriculum of the primary school (Millán Gasca, 2018). The arithmetic algorithms are the fruit of centuries of efforts on the part of humanity to obtain the result of certain quantitative operations quickly and accurately. Moreover, each of them is an opportunity for children to become acquainted with an iterative process that always works, as is the case with the processes involved in computer programs. In addition, practising algorithms is an effective way of gaining greater insight into the decimal number system as well as a way of acquiring familiarity with the properties of numbers and their decomposition.

For these reasons, in the ANFoMAM project, a workshop about *Understanding of arithmetic algorithms* will be designed. It will include some activities to practice the four basic operations with material and graphic support, in order to facilitate the understanding of both algorithms and properties of numbers at the same time. In addition, we will seek to encourage children to make their own choices as to which strategy they use to perform the algorithm, so that children see them as more than a mere closed task. Our approach devotes less time to memorizing the routine procedures and more time to understanding the underlying dynamics. We intend to focus on 'the why' rather than on 'the how'.

As is the case with the other workshops, a questionnaire about the understanding of the arithmetic algorithms has been designed by the researchers from the *Universidad Pública de Navarra*. This questionnaire seeks to gain insight into key issues relevant to both pre-service and in-service teachers in relation to the teaching and learning of arithmetic algorithms, such as:

- The participants' experience with arithmetical algorithms during their school years: the difficulties they found, the strategies they used to follow, etc.
- The participants' beliefs about the aims of teaching the traditional arithmetical algorithms at primary schools and about the difficulties that children have with them
- Some considerations about the way in which arithmetic algorithms should be worked on at school.

A priori, we expected to find different ways of working on arithmetic algorithms at school, linked to the participants' beliefs about the aims in question:

- Teaching the classical algorithms to the children in a mechanical way, as iterative procedures that always work. The key objective would be that children gain in efficiency and in speed as soon as possible.
- Regarding the algorithms as a way for the children to better understand and put into practice the properties of numbers and their decompositions, including those related to how they are expressed in the decimal number system.

3. Data collection

The first version of the questionnaire (Appendix P1) was piloted with 185 participants from the Université Bordeaux (UB), la Universidad Pública de Navarra (Upna), Università Roma Tre (URT) and ToKalon (TK):

Table 2: Distribution of participants

	UB	UPNA	URT-TKL
pre-service teachers	22	60	67
in-service teachers	16	4	12
Doesn’t know/answer		4	

4. Data processing and analysis

The main part of the questionnaire consists of six questions (appendix P1):

1) Degree of identification with five assertions about the participants' experience with the algorithms (Likert scale from 1, the lowest, to 4, the highest)
2) Weights assigned to 5 aims of teaching arithmetic algorithms in primary education (Likert scale from 1, the lowest, to 4, the highest)
3) Weights assigned to four assertions regarding how to get students to understand algorithms (Likert scale from 1, the lowest, to 4, the highest)
4) Values assigned to the appropriateness of using traditional arithmetic algorithms for certain kinds of arithmetic operations (Likert scale from 1, the lowest, to 4, the highest)
5) Reasons why children encounter difficulties when applying arithmetic algorithms (multiple-choice)
6) Degree of agreement with some assertions regarding the teaching and practice of arithmetic algorithms in primary education (Likert scale from 1, the lowest, to 4, the highest)

In addition, there are two context questions designed to obtain information about the personal or professional circumstances of the participants.

Question 1

When the participants are asked about their personal experience with the arithmetic algorithms, the French participants assign a lower degree of identification than the rest with the assertions *They have always been boring and repetitive for me* and *I have never been interested in knowing why they are designed the way they are.*

It is the participants from the Universidad Pública de Navarra who assign the highest degree of identifications with these statements.

In addition, the Italian participants together with the French ones identify strongly with the statement *I understand fairly well the reason for each step that is given when an algorithm is applied* and the French in particular *find them easy to learn*. All the participants, especially the French ones, assert that they *have used the calculator to avoid having to apply the algorithms.*

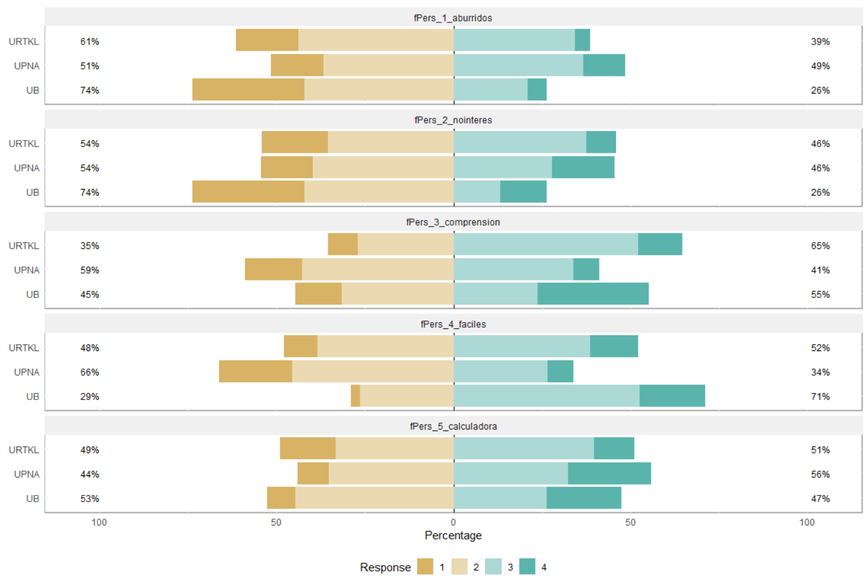


Figure 14

Key

fPers_1_aburridos	They have always been boring and repetitive for me
fPers_2_nointeres	I have never been interested in knowing why they are designed the way they are
fPers_3_comprension	I understand fairly well the reason for each step
fPers_4_faciles	I found them easy to learn
fPers_5_calculadora	As soon as I was allowed to, I used the calculator to avoid having to apply them

Question 2

The favourable attitude of the participants towards the teaching of the arithmetic algorithms in primary schools is clear. Regarding the aims of teaching arithmetic algorithms, the participants give the highest value to *being able to focus on the resolution strategy of a problem, having mastered the calculations,* followed by *helping children better understand the properties of numbers and operations,* which widens the focus beyond considering the algorithms as a mere skill towards a modern understanding of arithmetic, i. e. the study not only of numbers but of numbers included in a rich structure endowed with a series of operations. The more traditional aims of teaching the algorithms, such as *their utility for children's academic and professional future* and that *they offer the pupil the security that the calculations are right* are also highly valued by all the participants, at the same level as the contemporary reason that *they help the pupils understand the iterative processes of computer programming.*

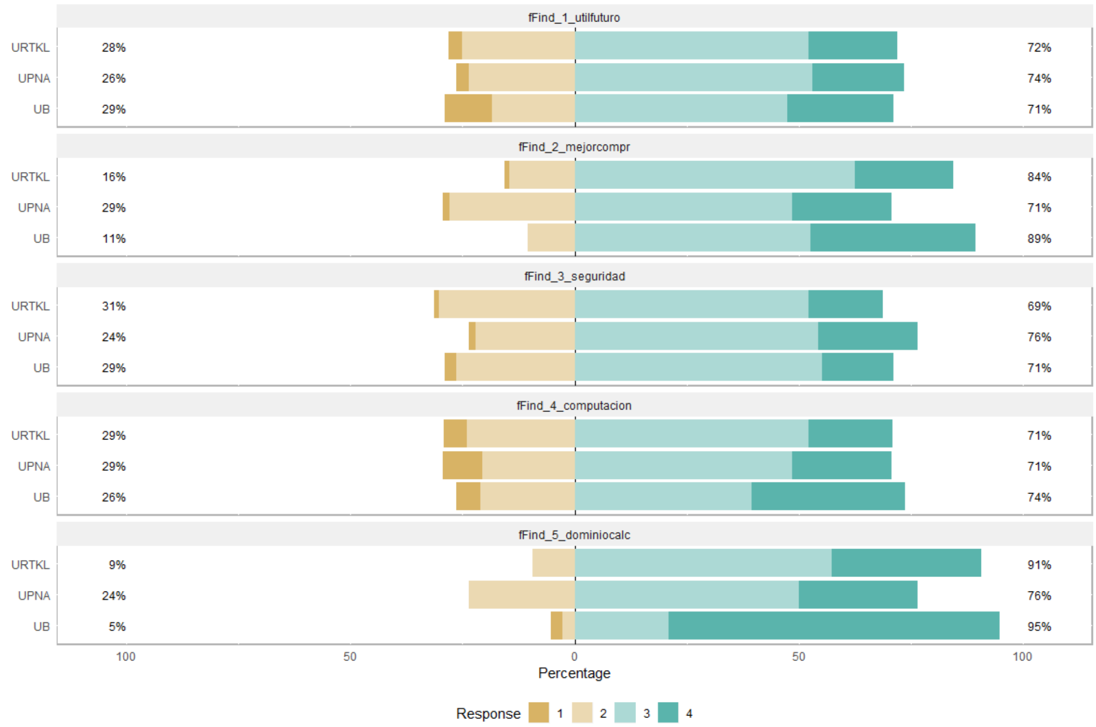


Figure 15

Key

fFind_1_utilfuturo	Useful for both the academic and professional future of the pupils
fFind_2_mejorcompr	Helps children better understand the properties of numbers and operations
fFind_3_seguridad	It offers the pupil the security that the calculations are right
fFind_4_computacion	It helps the pupil understand the iterative processes of computer programming
fFind_5_dominiocalc	The student can focus on the resolution strategy of a problem, having mastered the calculations

Question 3

In order to get students to understand, not only how, but also why the steps of the algorithms work, all the aspects and resources are positively valued by all the participants, especially manipulatives and the use of appropriate graphs, diagrams and schemes. The participants from the Universidad Pública de Navarra give less value than the rest to *encourage the children to use a precise vocabulary to name the different units involved*.

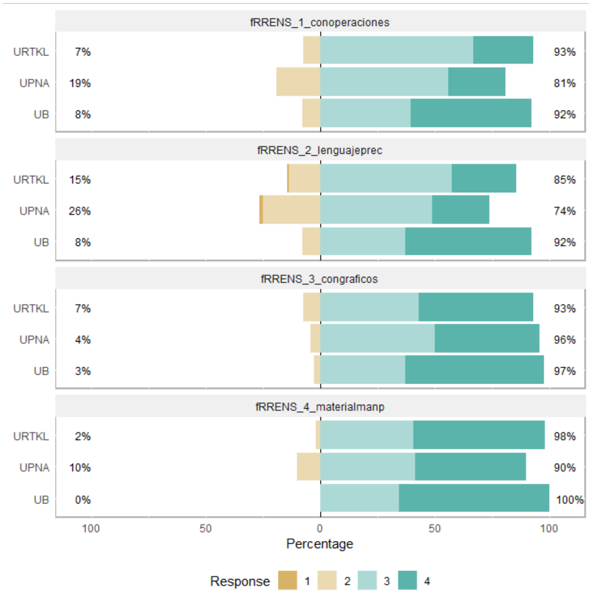


Figure 16

Key

fRRENS_1_conoperaciones	Combining their teaching with that of the properties of numbers and arithmetic operations
fRRENS_2_lenguajeprec	Encouraging the use of a precise vocabulary to name the different units involved
fRRENS_3_congraficos	Accompanying their teaching with appropriate graphs, diagrams or schemes
fRRENS_4_materialmanp	Accompanying their teaching with suitable manipulatives

Question 4

In this question, the participants had to value the appropriateness of using the traditional arithmetic algorithms to perform some kinds of arithmetic operations. The answers clearly reflect that the use of the algorithms is regarded as more necessary when the task involves numbers with more figures (the case of 234 x 346). Surprisingly, some participants, those from the Universidad Pública de Navarra, regard it as necessary to use the algorithms in the first task, a simple addition without regrouping, which can be performed easily by mental computation.

They might have thought that the alternative to the use of algorithms was the calculator and, as such, it would be more natural to use the algorithms than that device to make a simple addition. This could also explain the fact that not all the participants regard the use of the algorithms as appropriate for the third and fourth tasks.

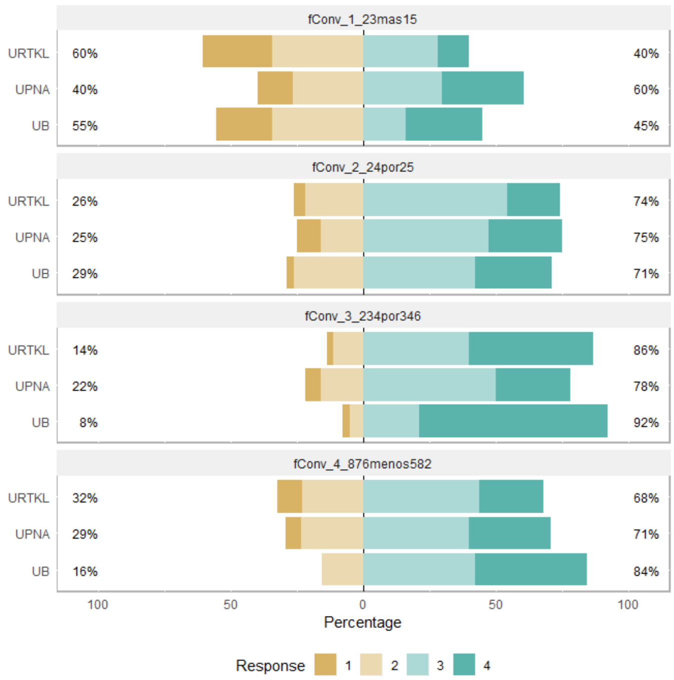


Figure 17

Key

fConv_1_23mas15	23+14 operation
fConv_2_24por25	24x25 operation
fConv_3_234por346	234x346 operation
fConv_4_876menos582	876-582 operation

The ambiguity of the answers has led us to modify this question in the following way:

Choose the most suitable way of performing each of the following types of operations:

- *a) 23+15* — *mental computation- classical algorithms – calculator*
- *b) 24x25* — *mental computation - classical algorithms – calculator*
- *c) 234 x 346* — *mental computation - classical algorithms – calculator*
- *d) 876 - 582* — *mental computation - classical algorithms – calculator*

Question 5

Regarding the reasons why children encounter difficulties when applying arithmetic algorithms, the fact that *they have learnt to apply rules without understanding their meaning* is regarded as the most important one. It is closely related to the other reasons the participants consider to be important. For instance, *the pupils are not able to make a mental representation that helps them in the calculation* or *they do not know how to use a diagram, a graph or a manipulative to support them*. All of these teaching approaches could also *help the pupils not to feel blocked*.

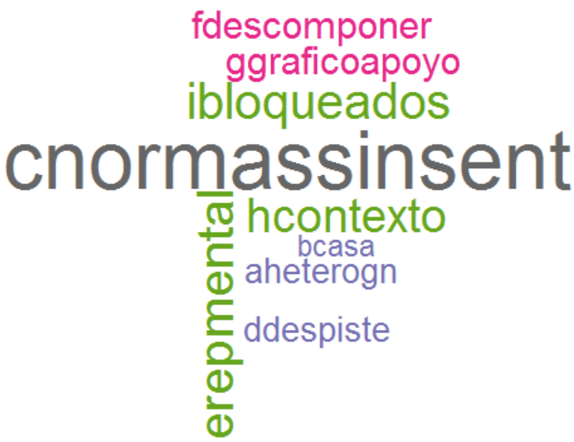


Figure 18

Key

Aheterogn	Each specific example differs in some way from the others
Bcasa	They do not practice enough at home
cnormassinsent	They have learnt to apply rules without understanding their meaning
Ddespiste	They get distracted while they are applying the algorithms
Erepmental	They are not able to make a mental representation that helps them in the calculation
Fdescomponer	They are not used to breaking down numbers
Ggraficoapoyo	They do not know how to use a diagram, a graph or a manipulative to support them
Hcontexto	They find no sense them in applying them to perform operations without a context situation
Ibloqueados	They feel blocked by a fear of making mistakes

Question 6

The participants had to express in this question their degree of agreement with the following statements regarding the teaching and practice of arithmetic algorithms in primary education.

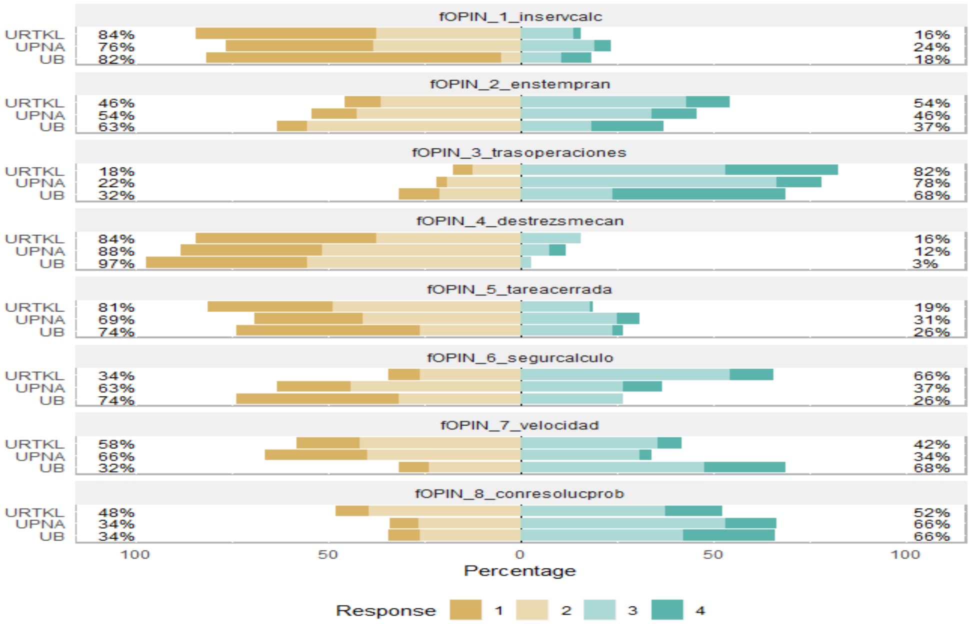


Figure 19

Key

fOPIN_1_inservcalc	Nowadays it is useless to know the arithmetic algorithms since the calculator can be used
fOPIN_2_enstempran	They should be taught at the earliest possible age
fOPIN_3_trasoperaciones	They have to be taught after the meaning of arithmetic operations has been understood
fOPIN_4_destrezsmecan	It is not appropriate that children devote time to fully understanding the arithmetic algorithms, as this delays their acquisition of mechanical skills.
fOPIN_5_tareacerrada	Practising arithmetic algorithms is a closed task that leaves no room for the pupils' initiative
fOPIN_6_segurcalculo	In primary education, the learning of arithmetic algorithms must be prioritized over problem solving, so that pupils acquire a feeling of security with calculations
fOPIN_7_velocidad	In the teaching of algorithms, the emphasis should be placed on children acquiring speed when applying them
fOPIN_8_conresolucprob	If it is performed separately from problem solving, the practising of arithmetic algorithms can result in students losing interest in mathematics

There seems to be a strong consensus about the *utility of learning the arithmetic algorithms in spite of the extended use of the calculator*, but only if this learning seeks to achieve *a full understanding of the dynamics rather than a simple acquisition of mechanical skills by children*. Related to this approach to the teaching of the algorithms, it is the participants' opinion that *this kind of exercise is a closed task*. Only the French participants *put the emphasis on children acquiring speed when applying the algorithms*, while the Italian trainee teachers *prioritize the learning of the algorithms over problem solving so that pupils acquire a feeling of security with calculations*.

However, this last statement in the questionnaire effectively comprises two assertions, and as such it is not clear whether the participants agree with one or other or both assertions.

The Italian participants also assign a slightly lower value than the rest to the assertion that *the performing of algorithms separately from arithmetic problems can result in students losing interest in mathematics*. This fact is in keeping with the emphasis of the Italian participants on *teaching algorithms at the earliest possible age*. The rest of the participants do not consider this to be appropriate. All the participants agree with *teaching the algorithms once the meaning of the arithmetic operations has been understood*. It is clear that both in-service and prospective teachers consider that understanding the meaning of each operation will give sense to the calculations, as is the case when they are done in a contextualized way, linked to arithmetic problems.

Analysis of groups of variables from Questions 1 and 6

Following Green's theory of clusters, we look for groups of variables that appear together in the collected data by using a technique of multivariate data analysis, called the principal component analysis.

By means of this technique, the data will be projected onto a plane generated by two axes, each of which will be a suitable linear combination of the variables that appear in Questions 1 and 6:

1) The first principal component (represented in the axis OX of Figure 20) collects the following variables in the positive semi-axis:

- *They have always been boring and repetitive for me* (fPers_1)
- *I have never been interested in why they are designed the way they are* (fPers_2)
- *As soon as I was allowed to, I used the calculator to avoid applying them* (fPers_5)
- *Nowadays it is useless to know the arithmetic algorithms since the calculator can be used* (fOPIN_1)
- *It is not appropriate that children devote time to fully understanding the arithmetic algorithms, as this delays their acquisition of mechanical skills* (fOPIN_4)
- *Practising arithmetic algorithms is a closed task that leaves no room for the pupils' initiative* (fOPIN_5)

2) The variables collected in the negative semi-axis OX reflect opposite beliefs:

- *I understand fairly well the reason for each step* (fPers_3)
- *I found them easy to learn* (fPers_4)

3) The positive semi-axis OY is defined by the variables from Question 6 that show the algorithms as a skill that must be learnt and mastered through practice:

- *They should be taught at the earliest possible age* (fOPIN_2)
- *In primary education, the learning of arithmetic algorithms must be prioritized over problem solving, so that pupils acquire a feeling of security with calculations* (fOPIN_6)
- *In the teaching of algorithms, the emphasis should be placed on children acquiring speed when applying them* (fOPIN_7)

4) The negative semi-axis OY is determined by the following assertion:

- *They have to be taught after the meaning of arithmetic operations has been understood* (fOPIN_3)

Figure 20

Analysing the variables that determine the positive semi-axis OX, it is clear that the participants' own experience with the algorithms during their school years are in keeping with the opinions they express about the teaching and practice of arithmetic algorithms in primary education.

The analysis shows few differences between the participants of different institutions, who are scattered around the plane, except those from Université Bordeaux (in red in Figure 20) mostly situated in the third quadrant. This means that *they find algorithms easy to learn*, *understanding the reason for each step*, and that, in their opinion, *the algorithms must be taught once the meaning of the operations has been understood by children.*

This attitude is in keeping with the other answers given by these participants. One possible inconsistency in their responses might be their emphasis *on children acquiring speed when applying the algorithms.*

Analysis of the profile of the participants of the different institutions

In order to take into account, the participants' institution/country in the analysis of the answers to questions 1 and 4, we create some virtual variables:

1) A virtual variable, called *understanding*, obtained as a result of adding the values assigned to the variables from Question 1 that define the negative semi-axis OX in Figure 20 and subtracting the values assigned to the variables that determine the positive semi-axis OX:

(+) *I understand fairly well the reason of each step* (fPers_3)
(+) *I found them easy to learn* (fPers_4)
(+) *They have always been boring and repetitive for me* (fPers_1)

(-) *I have never been interested in why they are designed the way they are* (fPers_2)
(-) *As soon as I was allowed to, I used the calculator to avoid applying them* (fPers_5)

The following diagram shows that this variable takes a positive value in the case of both French and Italian participants, while its value is negative for Spanish participants.

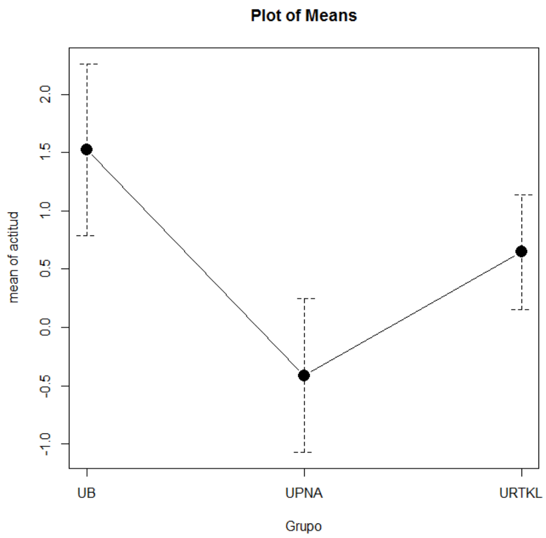


Figure 21

2) Another virtual variable, *competence*, obtained as a result of adding all the variables that define the positive semi-axis OX:
(+) They have always been boring and repetitive for me (fPers_1)
(+) I have never been interested in why they are designed the way they are (fPers_2)
(+) As soon as I was allowed to, I used the calculator to avoid applying them (fPers_5)

We observe that the students from Universidad Pública de Navarra are clearly defined by this variable, while the French participants show a markedly different profile. This variable is not the most suitable one to describe the Italian participants.

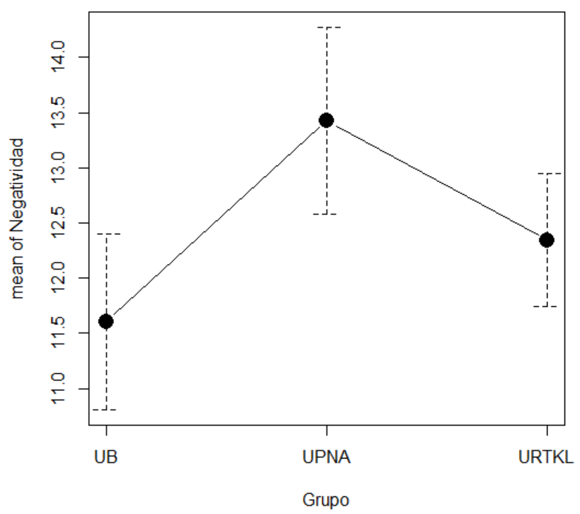


Figure 22

5. Conclusions: reporting, recommendations and proposals for improvement

In spite of the current easy access to the calculator, there seems to be a strong consensus about the usefulness of teaching the arithmetic algorithms in schools. However, working on the algorithms does not mean limiting the teaching to mechanical closed methods, but rather seeking to encourage children to understand the underlying dynamics while they acquire familiarity with numbers and their properties.

The analysis of the results makes it clear that the participants also regard working on the arithmetical algorithms at primary level as important for children's acquisition of a feeling of security with the calculations in solving problems. Moreover, they see the algorithms as an opportunity for children to become acquainted with iterative processes, such as those involved in computer programming.

Despite this general opinion, the participants recognize that children usually find difficulties with the algorithms, which they attribute to the most usual way of teaching these, i. e. as meaningless rules that do not help children form mental images of the numerical decompositions involved in the calculus. The majority of the participants think that the practising of algorithms separated from arithmetic problems can result in pupils losing interest in mathematics.

In addition, the analysis makes it clear that the participants' own experience of the algorithms during their school years are in keeping with the opinions they show about the teaching and practising of arithmetic algorithms in primary education. For instance, a boring experience with algorithms on the part of the participants correlates with the opinion that it is not appropriate for children to devote time to fully understanding them.

Regarding the participants' institutions, the French and the Italian trainee teachers seem to have more interest in understanding the algorithms' dynamics than the Spanish ones. This vision of the algorithms seems to be inconsistent with their opinions about the teaching of the topic, putting the emphasis on starting at the earliest possible age (in the case of the Italian participants) and looking for speed (in the case of the French ones).

The results obtained from the pilot experience with the questionnaire have led us to slightly modify some of the questions. In particular, we have rewritten Questions 4 and 6, which now take the form shown in Appendix Q1, in order to enable us to interpret the answers with more clarity in the future.

Moreover, the results obtained paved the way towards our design of a workshop directed towards providing present and future teachers with new experiences of the algorithms. We will work on dynamic ways of approaching this area of the teaching of mathematics, which will seek to enable the participants to discover the relationships between the algorithms and the rich properties of numbers and their operations. At the same time, we will link the practising of the algorithms to the understanding of the underlying operations, by means of the use of manipulatives, diagrams and graphs.

Report of questionnaire Q2: Solving and representing arithmetical problems

Table of contents

1. Abstract

A questionnaire about solving and representing arithmetic problems in primary schools has been designed within the framework of the Eramus+ project ANFoMAM, *Learning from children to improve primary teachers' maths-specific education*. The aim is to know the experience of pre-service and in-service teachers with arithmetic problems during their school years. The questionnaire also explores their beliefs about both the aims of working with arithmetic problems at primary schools and the difficulties that children usually have with them.

The first piloting of the questionnaire (appendix P2), with Spanish, French and Italian participants, clearly shows that the majority of them appreciate the value of working on problem solving at primary level for a wide range of objectives and aims, while at the same time recognizing that children typically find difficulties with this task. The data analysis provides us with several profiles of trainee teachers, based on their approach to teaching arithmetic problems, which correlates with the reasons they give for working on this area of mathematics.

After the analysis of the results obtained in the first implementation of the questionnaire, some slight changes are proposed for future implementations (Appendix Q2[4]).

2. Questionnaire design: background and goals

Problems constitute the heart of mathematics (Millán Gasca, 2018). Mathematics has been born, has grown and has evolved around problems that different civilizations have been obliged to face throughout the history of humanity, or from problems that have simply aroused people's curiosity. However, mathematical problems are often the main reason for children's feelings of discouragement in the learning of maths. The way in which problems are usually approached at school may be the main cause of this, a circumstance not limited to the teaching of mathematics.

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[4] Questionnaire Q2 : https://docs.google.com/forms/d/1LUEu8vL9nz-zFHwJS4AjAzpaM7wA3Rf8fXOpnIRSk5E/copy

From our point of view, the way of working on mathematical problems clearly reflects a particular way of understanding education: whether results are the only thing to be evaluated, or instead, the solving processes are also taken into account in the assessment; whether the aim is training the pupils in the performance of mechanical tasks or, on the contrary, the pupils are helped to find their own ways of accomplishing the tasks and are encouraged to communicate their strategies to each other; whether teachers promote pupils' confidence or rather make them feel limited, and so on. The way of approaching problem solving could be used as a paradigm for analysing different ways of understanding education in general.

In the ANFoMAM project (Catalán et al., 2019), a workshop about solving and representing arithmetical problems will be designed, focusing on discovering the implicit relationships which exist between the magnitudes present in the problems, both as given data and as unknowns. Despite there being many kinds of mathematical problems in primary school curricula, arithmetical ones are typically those most frequently worked on at school. For this reason, a questionnaire about solving and representing arithmetical problems was designed by the researchers from the *Universidad Pública de Navarra* that are participating in the project. This questionnaire seeks to discover the beliefs of both pre-service and in-service teachers regarding some aspects of solving arithmetical problems:

- The participants' experience with arithmetical problems during their school years: the difficulties they found, the kind of problems they liked most, the strategies they used to adopt, etc.
- The participants' beliefs about the aims of teaching arithmetical problems at primary school and about the difficulties that children usually find when solving them.
- The way in which problems should be worked on in class and how their resolution should be evaluated.

A priori, we expected to find different ways of working on arithmetic problems at school, linked to the teachers' beliefs about the aims involved:

- The application of the latest mathematical knowledge that the children have learned in class to different situations, not always related to the children's day-to-day life. The attitude that could be expected from people with these beliefs is that they will work on standard problems, problems that can be solved by repetitive procedures that lead to the right result as quickly as possible.
- Showing relationships between different mathematical concepts. It is expected that future teachers with these beliefs regarding arithmetical problems will provide children with different mathematical situations in which those relationships appear in a natural way. This is the case, for instance, with arithmetical problems regarding measurement, in which natural relationships between geometric and arithmetic aspects can be easily seen.
- An opportunity for children to put into play known strategies, or even create new ones, in situations that are different from those that they have been previously encountered. It is expected that the participants with these beliefs will provide children with problems that pose a real challenge for them.

In the initial version of the questionnaire (appendix P2), we had included, as questions, two arithmetic problems to be solved, together with two more questions about what strategies the participants had adopted in their resolution. But, after piloting the questionnaire, we saw that giving the participants problems to solve was not the best way of creating a relaxed atmosphere to help them reflect on their lived experiences with mathematics.

For this reason, the first four questions of the initial version of the questionnaire were replaced with Questions 1, 2, 3 and 4 of the latest version of the questionnaire (appendix Q2).

3. Data collection

This first version of the questionnaire (appendix P2) was piloted to 97 participants:

- Universidad de Zaragoza (27 university students and 5 in-service teachers)
- Universidad Pública de Navarra (62 university students and 3 in-service teachers)

The latest version of the questionnaire (appendix Q2) was answered by 104 participants, all of them from Rome (Università Roma Tre and ToKalon), mostly in-service teachers.

4. Data processing and analysis

The main part of the latest version of the questionnaire consists of seven questions related to:

1) Strategies most frequently used by children to solve problems (multiple-choice)
2) Difficulties that children encounter when solving arithmetical problems (multiple-choice)
3) Strategies most frequently used by the interviewed participants to solve problems (multiple-choice)
4) Difficulties that the interviewed participants encounter when solving arithmetical problems (multiple-choice)
5) Weights assigned to the participants' preferences regarding 8 kinds of arithmetic problems in primary education in the Likert scale (from 1, the lowest weight, to 4, the highest)
6) Weights assigned to 7 aims of working on solving arithmetic problems in primary education in the Likert scale (from 1, the lowest weight, to 4, the highest)
7) Weights assigned to 5 assertions about the resolution of arithmetic problems in primary education in the Likert scale (from 1, the lowest weight, to 4, the highest)

In addition, there are two questions designed to obtain information about the personal or professional situation of the participants.

Our analysis of the data begins with the questions which occur in both the initial and the latest version of the questionnaire (questions 5, 6, 7 and 8).

Question 5

76% of the participants disagree with the assertion *I do not like to solve problems*. Regarding the kind of problems preferred by the participants, 79% of them like solving problems *with several operations*. It is also notable that the majority of the participants (71%) do not like solving *problems about combinatorics*, which is probably related to the fact that 60% of participants dislike *solving non-standard arithmetic problems*, a category in which problems about combinatorics are considered to be.

It might be surprising that problems about fractions have been highly valued (66%), while problems about proportionality and percentages, very similar to those of fractions, are liked only by 45% of the participants (figure 23). A possible explanation of this fact could be that problems about proportionality and percentages are not typically explained in relation to fractions, but as if they were something different, which make them more difficult to understand.

In summary, the data seem to indicate that the participants prefer to solve standard arithmetic problems, although these require several operations, rather than problems in which they have to find their own solving strategies, such as combinatorics problems.

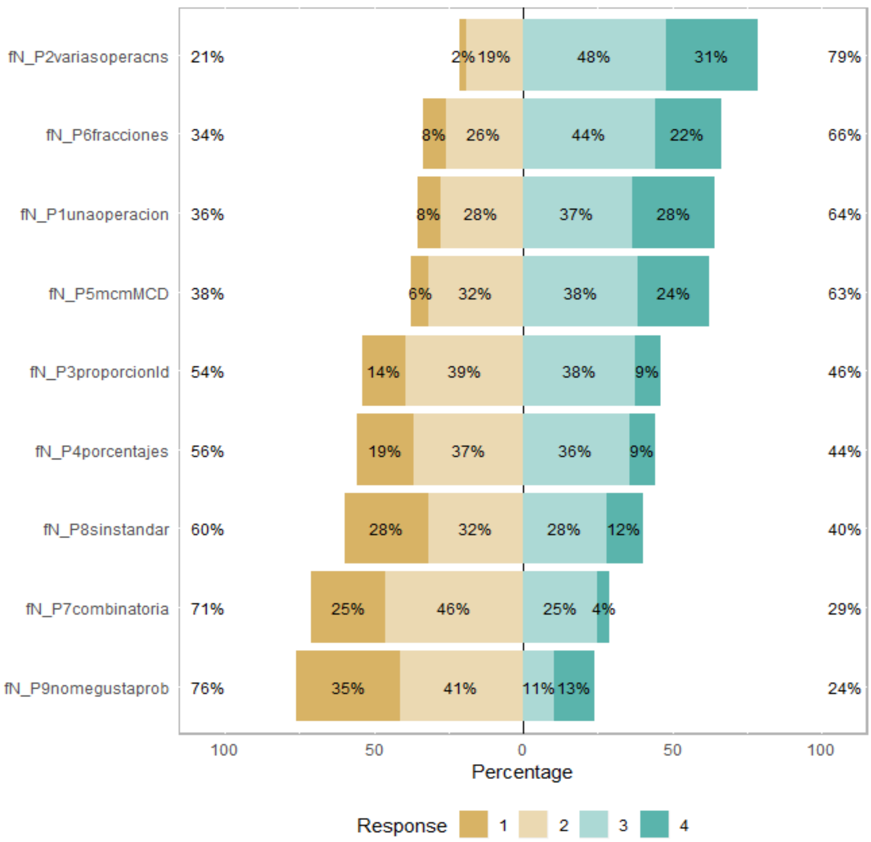

Figure 23

Key

fN_P1unaoperacion: I like problems with can be solved only one operation	fN_P6fracciones: I like problems about fractions
fN_P2variasoperacns: I like problems which requires the use of several operations	fN_P8sinstandar: I like problems that cannot be classified as standard
fN_P3proporcionld: I like problems about direct and inverse proportionality	fN_P7combinatoria: I like problems about combinatorics
fN_P4porcentajes: I like problems about percentages	fN_P9nomegustaprob: I do not like solving problems
fN_P5mcmMCD: I like problems about great common divisor and least common multiple	

Question 6

Regarding the objectives of working on solving arithmetical problems in Primary Education, there are no clear differences between the participants. There is a strong agreement on the main objective, which is, for most of them, that *children develop confidence in their own abilities.*
The fact that arithmetical problems must be worked on *to show the relationships between the different mathematical concepts* is widely accepted too, together with the aims of *showing children that mathematics may help them in their day-to-day life* as well as *letting them converse by using mathematical language and argumentation.*

The degree of agreement decreases gradually up to the aim of *preparing children for situations they will find in the future, both in academic and in professional worlds*, although even in this case the agreement is over 85%.

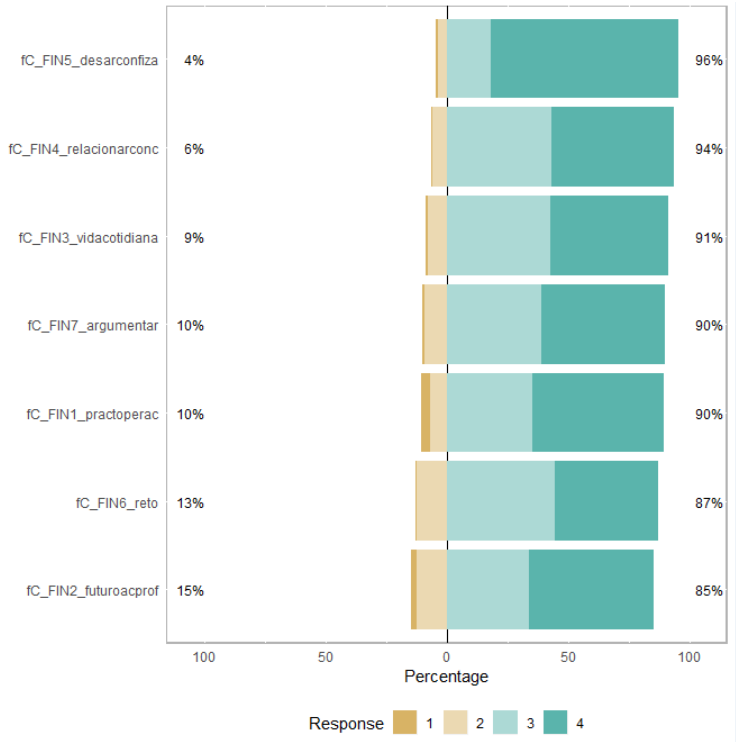

Figure 24

Key

fC_FIN1_practoperac: to practise the operation just learnt fC_FIN2_futuroacprof: to prepare children for academic or professional future fC_FIN3_vidacotidiana: to show that mathematical knowledge may help them in their day-to-day life fC_FIN4_relacionarconc: to show the relationships between the mathematical concepts worked on	fC_FIN5_desarconfiza: to help the child develop confidence in their own capacities fC_FIN6_reto: to get the child to face a challenge fC_FIN7_argumentar: to present situations for children to converse and discuss

If the study takes into account the participants' origin, then some slight differences appear between the institutions. For instance, the aim of *presenting situations for children to converse using mathematical language and argumentation* is more highly valued in Italy than in Spain, whereas the Spanish participants value more than the Italian ones the role of problems as a *preparation for children's academic and professional future*. The participants from Universidad Pública de Navarra give less weight than the rest to the aims of *showing children that arithmetic knowledge can help them in their day-to-day life* and *showing the relationships between different mathematical concepts*. Conversely, they highly value the aim of *facing a challenge that encourages them to put their mathematical knowledge and abilities into practice*.

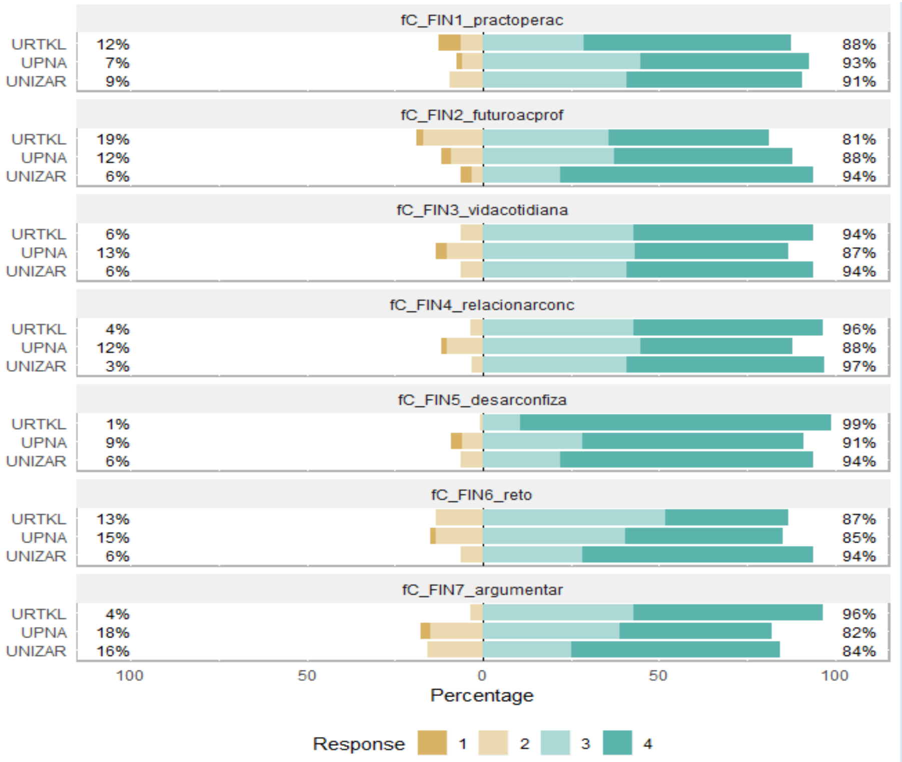

Figure 25

Key

fC_FIN1_practoperac: to practise the operation just learnt	fC_FIN5_desarconfiza: to help the child develop confidence in their own capacities
fC_FIN2_futuroacprof: to prepare children for academic or professional future	fC_FIN6_reto: to get the child to face a challenge
fC_FIN3_vidacotidiana: to show that mathematical knowledge may help them in their day-to-day life	fC_FIN7_argumentar: to present situations for children to converse and discuss
fC_FIN4_relacionarconc: to show the relationships between the mathematical concepts worked on	

Question 7

It is worth noting that the greatest agreement is given on the following sentences: *pupils' attitude towards arithmetic problems can change positively by means of a suitable teaching approach; obtaining the right answer is not the most important thing in the evaluation; it is almost impossible for a primary child to design their own strategy to solve a new problem.*

If the analysis of the answers takes into account the participants' origin then the greatest discrepancies appear around *the preference of operating techniques over problem solving in order to get the children to gain security in doing calculus*. While the Italian participants strongly agree with this statement, the Spanish ones disagree with it, especially those from Universidad de Zaragoza.

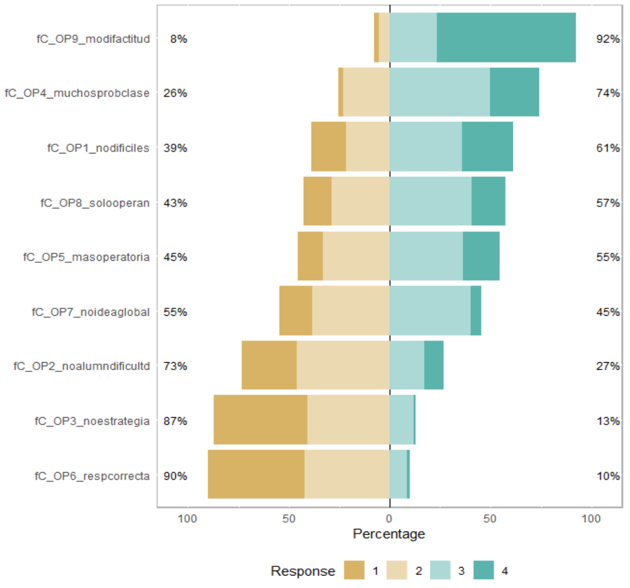

Figure 26

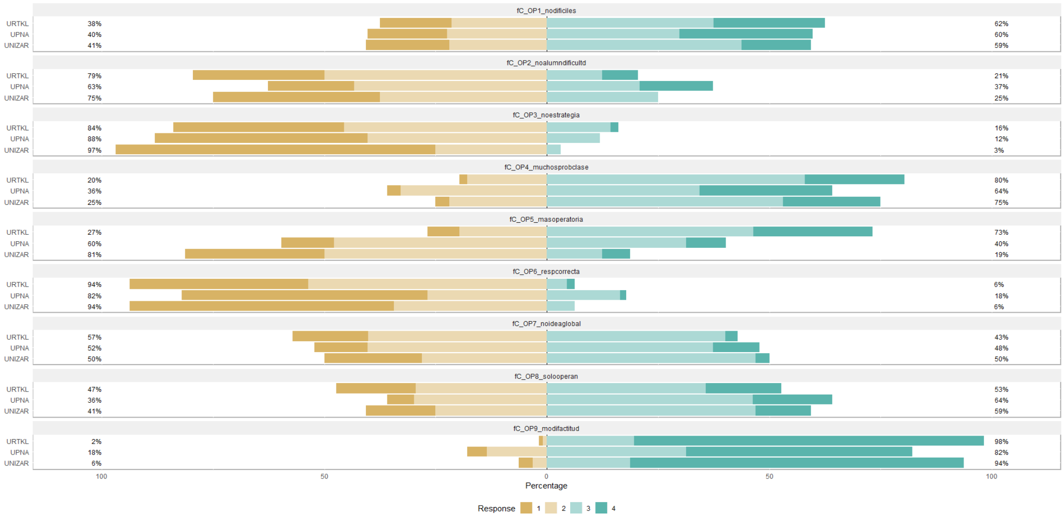

Figure 27

Key

fC_OP1nodificiles: difficult problems can make children lose interest in maths fC_OP2noalumndifficultd: it is not appropriate to present problems to children with learning difficulties fC_OP3noestrategia: it is almost impossible for a child to design their own strategy fC_OP4muchosprobclase: it is desirable to do as many problems as possible in class fC_OP5masoperatoria: preference for operating techniques over problem solving	fC_OP6respcorrecta: obtaining the right answer is not the most important thing in the evaluation fC_OP7noideaglobal: children do not try to get a comprehensive view of the problem fC_OP8solooperan: pupils think that they are only allowed to do operations fC_OP9modifactitud: pupils' attitude can change by means of a suitable teaching approach

Analysis of groups of variables from Questions 6 and 7

Following Green's theory of clusters (Green, 1971), we look for groups of variables that appear together in the collected data by using a technique of multivariate data analysis, called the principal component analysis.

Figure 28

We can distinguish three groups of variables in figure 28:

Group 1: This group (on the left of Figure 6) consists of the following variables:

- *It is almost impossible for a child to design their own strategy*
- *Presenting problems to children with learning difficulties is not appropriate*
- *Children do not try to get a comprehensive view of the problem*
- *In order to evaluate the way in which a problem has been solved, obtaining the right answer is the most important thing*

The first three sentences may be easily associated with a **pessimistic view** of children's ability to solve arithmetic problems. Indeed, the third assertion admits several interpretations. The lack of trying to get a comprehensive view of the problem might be attributed by the participants to the common forms of teaching at school, which have not been typically focused on this kind of ability, but if it is analysed together with the first two sentences, it may mean that the participants consider that such a comprehensive view is not within the children's capacities. In addition, regarding the evaluation of the way in which a pupil has solved an arithmetic problem, *obtaining the right answer* is preferred. In this group, there is no variable referring to the aims of working on solving problems in Primary Education.

The attitude that is expected from these participants when they work as teachers in a classroom is:

- To provide the pupils with standardized arithmetical problems, which they will be able to solve without great difficulty.
- To ask the pupils to check whether their solution to a problem is right or wrong, as opposed to encouraging them to analyse different strategies to solve the task.

- To train children to do problems of each kind by using a predefined strategy, instead of encouraging them to be creative.

It is hard to imagine this kind of teacher helping the pupils to trust in their own capacities to solve problems because focusing on the solution leads to students being more worried about not making mistakes than about exploring new strategies.

Group 2: Revolving around the axis of solving problems *to practise the operation that has just been learnt at class*, some variables appear in this group (on the central area of figure 28) whose common denominator seems to be the association of mathematics at Primary school with learning and practising **operating techniques** as opposed to solving 'difficult problems', such as:

- *In Primary education, the learning of operating techniques must be preferred to problem solving, to get the children to gain confidence in doing calculus*
- *Giving children arithmetic problems which are difficult for them can lead to children losing interest in mathematics*
- *Pupils believe that they are only allowed to do operations and so they do not use other kinds of strategies*
- *If we want children to learn to solve problems, it is desirable to do as many problems as possible in class*

These assertions reflect a view of solving problems as a simple competence or skill that has to be acquired (Peters, 1967). These variables, together with the belief that *it is desirable to do as many problems as possible in class if we want children to learn to solve problems,* leads us to expect that the participants who have valued these variables most highly will adopt the following attitude at school:

- Provide children with standardized problems, which can be learned by repetitive procedures.
- Use the arithmetical problems as an excuse to practise operations more than focusing on comprehension.
- Avoid giving children problems different from those that they have previously worked on in class.
- Practice problems in only one way, in order to do as many as possible, without stopping to reflect on other solving strategies or on applications to other problems of the strategy that has been used.

Group 3: In this group (on the right of figure 28), we can see the assertions that have obtained the greatest consensus. These are the remainder of the aims of doing arithmetical problems (apart from that of *practising the operation that has just been learnt at class)*, as well as the assertion that *It is possible that pupils' attitude towards arithmetic problems can change positively by means of a suitable teaching approach.*

This group of variables offers a view of problem solving not only as a mere skill, but also as an **opportunity** for children to gain a series of abilities (*understanding relationships both between mathematical ideas and between maths and real world; using mathematical language to converse)* as well as developing in terms of personal growth (*confidence in their own capacities, exploratory attitude when facing challenges, knowing how to relate school knowledge to life and vice versa, discussing with other people in order to contrast their opinions with those of others).* This way of looking at arithmetic problems is an intrinsic part of a way of understanding children's education which goes beyond the mere acquisition of some technical skills.

It is easy to imagine that teachers with these beliefs will have an attitude of trusting their pupils (Celi et al. 2019), which will lead them to:

- Give their pupils mathematical tasks that require some effort
- Promote activities that leave room for creativity
- Allow children to detect their mistakes and learn from them.

This attitude, together with a sensitive and appropriate response on the part of the teacher to children's errors, will help pupils to acquire confidence and energy (Donaldson, 1987).

Analysis of the profile of the participants of the different institutions

In order to analyse the answers to questions 6 and 7 by taking into account the possible differences between the three groups of participants (Rome, Pamplona and Zaragoza), we have created some virtual variables:

1) A virtual variable named *pessimism* as a result of adding the values assigned to the four variables appearing in Group 1:
    - *It is almost impossible for a child to design their own strategy*
    - *Children do not try to get a comprehensive view of the problem*
    - *Presenting problems to children with learning difficulties is not appropriate*
    - *In order to evaluate the way in which a problem has been solved, obtaining the right answer is the most important thing*

   The ANOVA analysis rejects the hypothesis that the three groups of participants exhibit similar behaviour with respect to this variable, with a p-value of around 0.05. More precisely, the main differences appear between Navarra, where this variable *pessimism* reaches the highest value, and Zaragoza, which is very similar to Italy with respect to this variable.

   Therefore, the participants of Universidad Pública de Navarra show a profile similar to that described in Group 1, which may be considered inconsistent with the high value assigned by these participants (in the aims mentioned in Question 6) to *facing a challenge that encourages them to put their mathematical knowledge and abilities into practice.*

2) A virtual variable named *operating skill,* obtained by adding the values assigned to the following variables, all of them appearing in Group 2:

    - *In primary education, the learning of operating techniques must be preferred to problem solving, to get the children to gain confidence in doing calculus*
    - *If we want children to learn to solve problems, it is desirable to do as many problems as possible in class*
    - *The aim of working on arithmetical problems is to practice the operation that has just been learnt at class*

   The greatest difference with respect to this variable appears between the Spanish institutions and the Italian ones, the latter institution giving the variable *operating skills* the highest value.

   As a conclusion, Italian participants' profile should be similar to that described in Group 2. There are some inconsistencies in these participants, between this emphasis on skills and the high value given by them to the aim of *presenting to the children specific situations in which they could converse using mathematical language and argumentation* (Question 6).

3) A virtual variable named *opportunity,* as a result of adding the values taken by all the objectives and aims appearing in the Group 3:

   - *To prepare children for situations that they will find in their academic or professional future*
   - *To show children that arithmetic knowledge can help them in their day-to-day life*
   - *To show the relationships between different mathematical concepts that have been worked on in class*
   - *To help the child develop confidence in their own capacities*
   - *To get the child to face a challenge that encourages them to put their mathematical knowledge and abilities into practice*
   - *To present specific situations in which children can converse using mathematical language and argumentation*

   The ANOVA analysis does not reject the hypothesis that there are differences between the three groups of participants with respect to this variable. In this case, the main differences appear between Zaragoza, where this variable *opportunity* takes the highest value, and Navarra.

We will now turn to the first four questions of the questionnaire, in which there are differences between the initial version and the latest one.

The first version of the questionnaire (appendix p2)

This first version of the questionnaire was given to 97 participants:

- Universidad de Zaragoza (27 university students and 5 in-service teachers)
- Universidad Pública de Navarra (62 university students and 3 in-service teachers)

Question 1

This question consisted of an arithmetical problem to be solved:

> *David went from home to school and, after class he went to his grandparents' house. He walked 525 meters in total. If the distance from home to school is four times longer than the distance from the school to his grandparents' house, how many metres did he walk from home to school?*

In the questionnaire, the space provided for the answer was just enough to write the numerical result, thus not allowing the participants to explain the process they had followed to resolve the problem. 79% of the participants gave the correct answer.

Question 2

It was at this moment when the participants had to explain how they had solved the problem given in Question 1. The following table collects the answers, which could be more than one per participant:

Table 3: Procedure followed to solve the problem in Question 1

ANSWER	
I have done a drawing, a diagram or a graph	69%
I have felt comfortable solving the problem	60%
I have tried to get a comprehensive view of the situation implicit in the problem	55%
I have tried to find out which operations were necessary in order to find the solution from the data	46%
I have tried to remember a similar problem that I had previously solved	27%
I have felt blocked, without knowing how to start	9%
None of the above	2%

Most of the participants (69%) have used the strategy of *doing a drawing, a diagram or a graph*, which is probably linked to the nature of the problem, a problem about measurement of length. The simple drawing of a right line makes things easier in this case. Perhaps for this reason, 60% of the participants report *having felt comfortable* solving the problem. The strategy of *getting a comprehensive view of the situation implicit in the problem* might have been carried out precisely with the help of a drawing and, for that reason, both strategies might have been used simultaneously.

Any of these two techniques, *doing a drawing, a diagram* and *a graph* or *getting a comprehensive view of the situation implicit in the problem*, might have been combined with that of *trying to find out which operations are necessary in order to find the solution from the data*. We have realized in this analysis that the intention why the latter answer has been chosen is not easy to guess. It may have been chosen either as a necessary step to solve a problem, combined with some other techniques, or as the first thing to do in order to solve the problem.

Since this question has been finally removed from the questionnaire, it is not necessary to revise the options, but it is true that the intention of the participants is not clear enough from the answers.

Question 3

This question consisted of a second arithmetical problem to be solved:

> *If the price of a product is decreased by 50% and, later, this second price is incremented by 50%, then the third price of the product is less than, equal to or greater than the first one?*

More than 65% of the participants give the right answer, *(the third price is less than the first one)*, but it is worth mentioning that about the third part of the participants consider that the price of the product remains stable:

```
                              palabra frec
inferior                     inferior   64
igual                           igual   34
precio                         precio   18
primero                       primero    9
inferiore                   inferiore    6
inicial                       inicial    5
primer                         primer    4
descuento                   descuento    3
hace                             hace    3
segundo                       segundo    3
serã¡                           serã¡    3
â‚¬                             â‚¬    3
primo                           primo    3
tercer                         tercer    2
coste                           coste    2
incrementamos       incrementamos    2
nuevo                           nuevo    2
prenda                         prenda    2
```

Figure 29

Question 4

This question asked the participants about the way in which they had solved the problem given in Question 3 and about the difficulties that they had eventually found in it. The answers are collected in the following table.

Table 4: Procedure followed to solve the problem in Question 3

ANSWER	
I have solved the problem thinking on a particular case	69%
I am sure that I have found the right answer	51%
I have answered by mere intuition	23%
I have solved the problem by means of doing a drawing, a diagram or a graph	10%
I think that I know the right answer, although I do not know how to demonstrate it	6%
This is not an arithmetical problem	4%
I have found it too difficult to solve	1%
It is not possible to solve the problem because there are not enough data	1%
I do not find this problem suitable for Primary Education	1%
None of the above	1%

As can be seen, most of the participants report having thought of a particular case to solve the problem, perhaps because the problem had no data, apart from the percentages. It is worth mentioning that the participants who have focused on a particular case have automatically generalized that particular answer to the general situation. Perhaps, the form in which the question was written led the participants to think so: "If I am asked by a problem without numerical data, all the possible examples will probably give the same answer; otherwise, it could be impossible to give a right answer".

The latest version of the questionnaire (appendix q2)

This version of the questionnaire was answered by 104 people, all of them Italian participants (from Università Roma Tre and ToKalon), mostly in-service teachers.

Questions 1 and 2

The first question deals with the strategies that children most frequently use when solving problems. The participants could choose more than one option and the responses are collected in the following table.

Table 5: strategies that children most frequently use

ANSWER	
They do a drawing, a diagram or a graph	63%
They experiment with several options	50%
They use manipulatives	49%
They discover from the data which operation to use	42%
They look for a similar problem which has been previously solved in their notebook or book	34%
They ask the teacher or a peer how to start	31%
They encounter the solution, but they do not know how they have done so	10%
They feel blocked and do not know how to start	7%
They wait for a sudden inspiration	2%

Question 2 deals with the difficulties that children most frequently find in solving arithmetic problems. The following table collects the answers. (Again, the participants could choose more than one option):

Table 6: Responses from participants on the reasons that may lead children to have difficulties in solving arithmetic problems

ANSWER	
They are used to solving exercises in a mechanical and repetitive way	79%
They feel blocked by a fear of making a mistake	42%
They do not understand the text	39%
They get confused because there is either too much information or too many conditions to be considered	25%
They try to guess which operations are needed	22%
They do not practise enough at home	16%

The differences between the answers to questions 1 and 2 are worth analysing. When the participants think about the strategies that children most frequently use, in Question 1, the option of *feeling blocked without knowing how to start* is seldom chosen. We have realized in the analysis that indeed this is not a strategy per se but rather a response. When they reflect (in Question 2) on the difficulties that children find, the option of *feeling blocked* is chosen by a significant number of participants. Also, the factor *of being used to solving exercises in a mechanical and repetitive way* is viewed in Question 2 as the principal impediment. Interestingly, the kind of strategies that most frequently appear in the answers to Question 1 are not mechanical at all: *doing a drawing of a diagram, trying possible options, using manipulatives*, etc.

The reason for these differences between the answers may be that, in Question 1, the participants are considering, unconsciously, the most frequently used strategies that lead children to success in solving problems, while in Question 2, they focus on the cases of failure. For instance, *discovering from the data which operation to do* is very highly valued in the case of the strategies most frequently used by children, while this answer is less valued when the participants think about children's difficulties.

Questions 3 and 4

Question 3 asks about the strategies which were most frequently used by the participants when they solved arithmetical problems at school. The following table shows the answers. Again, each participant was allowed to choose more than one option:

Table 7: Responses on participants' problem-solving strategies

ANSWER	
Do a drawing, a diagram or a graph	74%
Discover from the data which operation to do	51%
Look for a similar problem that has been previously solved in their notebook or book	46%
Experiment with several options	38%
Use manipulatives	25%
Feel blocked and do not know how to start	8%
Ask the teacher or a peer how to start	7%
Encounter the solution, but they do not know how they have done so	6%
Wait for a sudden inspiration	2%

Question 4 asks about the difficulties that the participants most frequently found when they had to solve arithmetical problems at school:

Table 8: Responses from participants on the reasons why they encountered difficulties in problem-solving

ANSWER	
I was used to solving exercises in a mechanical and repetitive way	39%
I felt blocked by a fear of making a mistake	29%
I got confused because there was either too much information or too many conditions to be considered	27%
I did not understand the text	22%
Some of my classmates were better than me and I was demoralized	20%
I tried to guess which operations were needed	15%
Each new problem was different from the others	15%
I never had difficulties solving problems	14%
I did not practise enough at home	12%
It was the teachers' fault	7%

Comparing the answers to Questions 3 and 4, we find the same kind of differences that have been observed between Questions 1 and 2. While *doing a drawing, a diagram or a graph* is considered the most frequently used technique, which is a non-mechanical way of solving problems, the highest valued difficulty is *having used to solving exercises in a mechanical way.* However, the second and third most popular answers to Question 3, *discovering from the data which operation to do* or *looking for a similar problem that has been previously solved in their notebook or book,* may be considered mechanical forms of solving problems.

In addition, comparing the answers given to Questions 4 and 2, it is clear that the difficulties that the participants, mostly Italian in-service teachers, find in their pupils are the same that they themselves found at school. Nevertheless, when asked about the children (question 2), some of the options stand out while, when they think of themselves (question 4), all the options are more similarly valued. However, the answers regarding the difficulties in *understanding the text of the problem,* which is related to *there was either too much information or too many conditions to be considered,* have similar values irrespective of whether the participants think of themselves or when they think of their pupils.

In addition, in Question 4, some personal answers were not available in the case of the children's difficulties, for instance, *some of my classmates were better than me and I was demoralized.* This answer, chosen by 20% of the participants, was not included in the options of Question 2 because the feeling in question can only be observed introspectively.

5. Conclusions: reporting, recommendations and proposals for improvement

The analysis of the results clearly shows that the participants appreciate the value of working on problem solving at primary level for a variety of objectives and aims, although they recognize that children typically find difficulties with that task. Also, there is a significant consensus regarding the assertion that *pupils' attitude towards arithmetic problems can change positively by means of a suitable teaching approach.* The main differences between the participants become evident when they are asked about the most suitable approach for achieving this positive change.

On the one hand, there are some participants who see the arithmetical problems as a way of practising the operating techniques more and more, either because they think that this performance is more important in mathematics than solving problems, or because of their pessimism about children's capacity to discover their own strategies to solve them, pessimism that increases when they focus on children with some kind of learning difficulty. On the other hand, there are other participants who regard the teaching of problem solving as an opportunity to develop in terms of personal growth, for instance, gaining confidence, having an exploratory attitude when facing challenges, acquiring a capacity to communicate with others and a better understanding of both themselves and the world they live.

The latter group of participants consider that children are capable of discovering by themselves relationships between different mathematical concepts and of creating and sharing their own strategies. It is also worth noting, nevertheless, that all of the participants see the need to evaluate the processes that the children follow when they try to solve a problem, instead of just focusing for the evaluation, on the final results they obtain.

This analysis has led us to design a workshop for prospective and in-service teachers, specifically devoted to arithmetical problem solving. This workshop will be designed to show the participants new ways of dealing with this aspect of the teaching of mathematics, a real challenge for them. In the workshop, we will avoid presenting mechanical ways or repetitive procedures to solve problems. Instead, we will put the emphasis on discovering the relationships which are implicit in every arithmetic problem, working on how to represent these relationships, either with manipulatives or by means of graphics or drawing.

Regarding the questionnaire itself, in the latest version (Appendix Q2) Question 5 has been slightly changed in order to enhance clarity. In addition, the initial sentences in Questions 1 and 3 have also been reformulated so that they include the cases in which people get blocked when facing an arithmetic problem.

Report of questionnaire Q3: Integrated arithmetic and geometry

Table of contents

1. Abstract

Within the framework of the ANFoMAM project, Learning from children to train teachers in the area of mathematics, a questionnaire on the Relationship between arithmetic and geometry has been designed. The questionnaire aims to analyze the knowledge, beliefs and attitudes of teachers in training, both initial and continuous, about this aspect of mathematics and its teaching in the Primary Education stage.

This report analyzes the responses provided by a sample of students from the Bachelor's Degrees in Early Childhood and Primary Education in a first research experience with the questionnaire (annex Q3[5]). The results indicate that, from a theoretical point of view, around half of the respondents agree with the relevance of geometry over arithmetic. We also collect their responses on the importance they attach to the need for students to know arithmetic techniques before studying geometric relationships or to the convenience of combining both forms of mathematics teaching at this stage.

These theoretical data are complemented by the results that the participants give to the questions or exercises of the questionnaire in which they have to put both types of techniques into practice.

2. Questionnaire's design: background and goals

The goal of this study is to analyze the relevance of the disciplines of arithmetic and geometry in the teaching of mathematics at the level of Early Childhood and Primary Education from the point of view of the teacher, especially the teacher in training. In particular, this questionnaire has tried to analyze the practical point of view of the teacher so that the answers reflect their own vision, from their personal experience. The work is completed with a round of theoretical questions on the role of geometry in the teaching of mathematics.

We start from the reality that arithmetic is prevalent in mathematics curricula in general, and in Early Childhood and Primary Education in particular (Millán Gasca 2012, Monari 1998, Monari-Benedetti 2011). This prevalence in the objectives makes us ask ourselves two relevant questions for the project:

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[5] Questionnaire Q3:https://docs.google.com/forms/d/1WavTkg-_TC_wVIqgUaahITLnF2mfWxXEXiIfFHN3tMM/copy

- The prevalence of arithmetic is due to the conviction of the educational system that its number-based methods (counting, calculation, concept of binary operation, addition, subtraction, multiplication and division, grouping) are the most appropriate for the introduction of mathematics or the most basic and essential.
- The teacher is more inclined or more familiar with the methods of arithmetic due to his previous training and personal experience.

The results of this study will serve as a basis to check to what extent the convictions and vision of arithmetic and/or geometric methods is susceptible to change as a result of the training sessions that are planned within the project.

3. Data collection

In order to identify the reason for this prevalence and the degree of practical affinity of the teacher with the methods of arithmetic and geometry, we will carry out a qualitative/quantitative study of Questionnaire Q3 (see Annex Q3). This questionnaire has been prepared within the ANFoMAM project co-financed by the Erasmus + program of the European Union and we have obtained 122 responses from students of the teaching degree.

The survey was carried out during the first semester of the 2018/19 academic year. The survey is conducted in Spanish and English in Google Forms format and was answered on Android platforms or desktop computers. In order to complete the questionnaire, it was requested to have a pencil and paper on hand to be able to draw pictures if necessary, a graduated ruler, a compass and a calculator. The written material was collected at the end of each questionnaire and is available.

In this context of elementary education we understand by geometric techniques or methods those rudiments of mathematics that are specific to geometry and refer to models and shapes in the line, the plane and space, their properties, decomposition, their movements and their representations in recognizable objects in our environment. Somehow we distinguish these techniques from arithmetic techniques or methods, which are those that have a direct relationship with the integer, cardinal or ordinal, rational or irrational number, their representation, operations and relationships between them, and their interpretation and application in activities in our environment. Of course, it can be argued that the count of the sides of a polygon, the measure of these or their area is in common territory. Thus, we want to highlight as a geometric technique or method not that which is alien to number, but does not rest on it. For example, we will consider the activity counting the number of sides of a polygon as an arithmetic technique (as we could count apples and we would not speak of a biology activity), while recognizing the pentagons in a series of figures will be considered here as a geometric technique since we do not need to know how to count to five to recognize a pentagon. Thus, the five sides could be recognized by a process of subitization or by identification with some experience associated with the number five.[6]

[6] For example, the song "Close Encounters of the Third Kind" that has five notes (Re-Mi_Do_do-Sol)

The questionnaire focuses on asking the teacher to solve certain problems, analyze properties or apply concepts of a general nature in mathematics and, based on their own answers, analyze the methods used to solve them. In our study of the questionnaire we will distinguish between the arithmetic and geometric techniques used for the proposed activities.

4. Data management and analysis

As a general result we can observe the following points:

1) Although in theory it is considered that both disciplines (arithmetic and geometry) are basic in mathematics training in Early Childhood and Primary Education, 66% of those surveyed choose an arithmetic-type problem as representative of the knowledge to be acquired by a primary and secondary student and a 78% say they are more comfortable working with numbers than with geometric shapes.
2) This is supported by the answers to the practical questions in which the tendency to the preference and primary use of arithmetic over geometric methods is found in a ratio of 3 to 1.
3) The use of basic geometric techniques and methods is more imprecise and even erroneous in a high percentage (between 20% and 25%). This percentage shoots up for somewhat more complicated problems.

Next, we detail the results obtained by questions.

4.1. Problem (see questions 1.1. and 1.2.)

a) A 66% of those surveyed acknowledge having stated an arithmetic problem, while 24% acknowledge that the statement is geometric. The rest is divided equally between both and neither.

b) *A large number of the problems considered to be of a purely geometric nature are doubtfully classified by the respondent since they refer to arithmetic techniques, even if geometric terminology is used.*

- *Find how many sides this triangle has.*
- *Calculate the number of sides that certain geometric figures have. (3 answers)*
- *Draw 2 shapes that have four sides. (4 answers)*
- *If we know that the side of a square measures 2cm, what will be the perimeter of said square?*

c) The precision in the statement of problems of a geometric nature is lower. For example:

- Differentiate and classify round bodies, prisms and pyramids.
- Calculate the area of different polyhedra with reusable materials. (It is likely that in this context the respondent refers to polygons instead of polyhedra that are three-dimensional objects and whose area is not the object of study in 3rd grade of Primary).

d) The only purely geometric concepts used are referring to polygon classification (2 answers) and parallelism (1 answer). There is no reference to other concepts or techniques such as symmetries, regions separated by lines or curves, decomposition of a figure into parts, irregular shapes (pieces that fit together, kaleidoscopes), directions (to describe movements or displacements), strokes (use of the rule, the compass).

4.2. Properties (see questions 2.1., 2.2., and 2.3.)

a) 30% of the respondents are not able to find the four properties requested and of the answers given, 9% are wrong and 6% are imprecise.

b) *As in question 1 (see 1.2.), a certain discrepancy is observed between the classification as a geometric property and the geometric nature of this property. For example:*

- *Their weights are all multiples of 3. (Arithmetic technique since the property of numbers is being emphasized).*
- *Dimensions are multiples of 3 and 2. (Ambiguous use of the concept dimension and reference to the property of numbers)*
- *The weight of the figures is doubled successively.*
- *Their heights are 10.*

c) There is a more imprecise and even erroneous use of geometric concepts is observed, in total 15% of the answers are of this type. For example:

- Same perimeter (12 answers). (This is incorrect).
- It is the same figure placed differently (7 answers) ... they are similar (3 answers). (Both are imprecise or directly incorrect, depending on the definition of similar).
- (Same) distance to the center.
- They have right angles (3 answers). (This is incorrect).
- (Same) figures that can be extracted from each one. (You are probably trying to describe the fact that all three are made up of two equal trapezoids.)
- Equal apothems. (Not being regular figures, the concept of apothem is not adequate).
- Equal two to two. (3 answers) (Inaccurate or incorrect concept).
- If we split the figure in half, we see that it is the same figure but with a mirror effect. (This is probably a somewhat imprecise description of symmetry.)

4.3. Concepts: longitudes and rational numbers (see questions 3.1., 3.2., 3.3., and 3.4.)

a) A 20% of the respondents do not use any mathematical method (by visual estimation), a 4% do not know how to carry out the requested construction. Out of the remaining answers, a 75% use arithmetic methods (measuring, using numbers that indicate exact measurements, using numbers that indicate approximate measurements). The rest use geometric methods (using a ruler and compass, without using numbers).

4.4. Concepts: surface area and rational numbers (see questions 4.1., 4.2., 4.3., and 4.4.)

a) Similar to question 3, around 75% of the students who respond adequately use methods that rely on measuring with the ruler. We draw attention to the fact that between 20% and 21% of the answers are wrong, all of them of the following kind:

- [In order to draw two surfaces such that one is twice as long as the other ...] I draw a 1cm x 1cm square and another 2cm x 2cm.

4.5. Concepts: longitudes and irrational numbers (see questions 5.1., 5.2., and 5.3.)

a) This is a somewhat more elaborate geometric problem. The data of previous answers 3 and 4 are sharpened, resulting in that 90% of the answers are wrong and the rest all use arithmetic methods.

b) Despite this, 46% of those surveyed have the impression that they have solved the problem correctly and only 35% say they did not know how to do it.

4.6. Concepts: surface areas and longitudes (see questions 6.1. and 6.2.)

a) A 25% of the students answer this question inaccurately or wrongly. Of the rest, 15% admit having used geometric techniques that involve visualizing the figure. A 66% use the formula for area or numbers as an example. The rest do not acknowledge having used either of these two techniques.

4.7. Surface areas and longitudes (see questions 7.1. and 7.2.)

a) As in the case of question 5, this is a problem similar to the previous one, but somewhat more elaborate than question 6. In this case, 65% of the answers are wrong. Out of the correct answers, 50% claim to have used the area formula while the other 50% claim to have used other reasoning.

The answers to practical questions 3 to 7 can be summarized in the following table that collects an evaluation of those answers that use arithmetic methods, those that use geometric methods (see section 3) and those that are imprecise or erroneous. Questions 3, 4 and 6 correspond to basic geometric problems, while questions 5 and 7 correspond to somewhat more elaborate geometric problems.

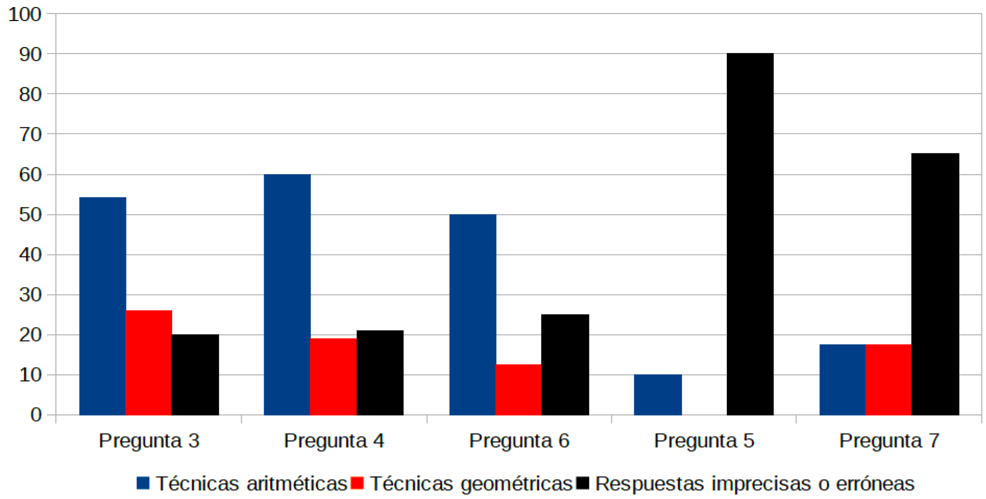


Figure 30

4.8. The treasure map. A practical challenge (see questions 8.1. and 8.2.)

a) This is an activity in which they must describe instructions for a movement from one point to another where a Cartesian diagram and North, South, East, West coordinates are provided.

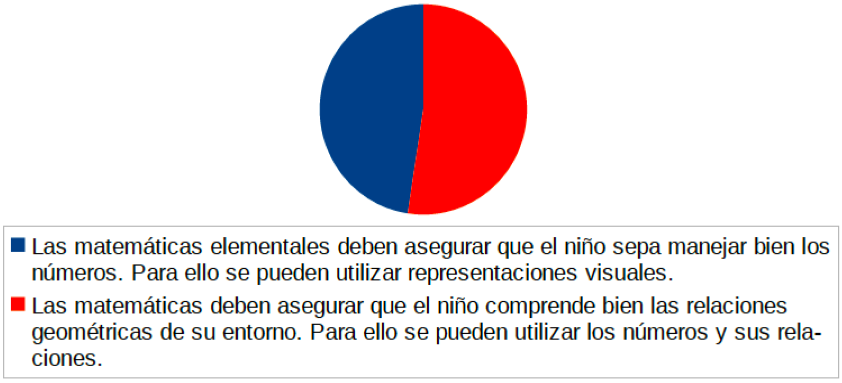
The question asked for precise instructions for someone who cannot see the map. 21% of those surveyed do not respond adequately to the question, either because their indications are wrong or because they are not sufficiently precise, for example:

• *Walk in a diagonal line 10 steps (2 answers).* (For someone who doesn't see the map, diagonal doesn't make sense.)

• *10 steps right and 10 down (22 answers).* (It is not clear what is right and what is below).

4.9. Teaching of mathematics (see questions 9.1.a. - 9.4.b.)

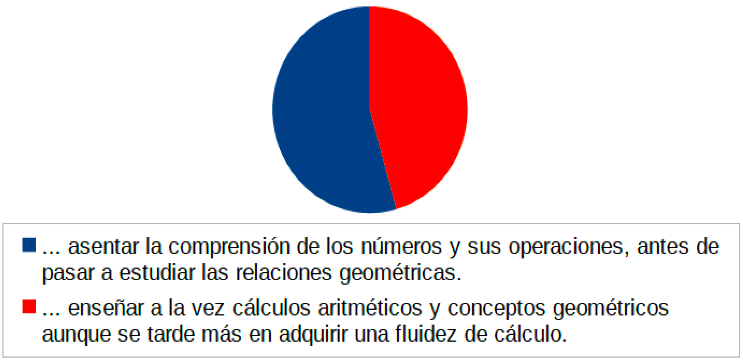
a) 52.4% of the respondents are more in agreement that mathematics should ensure that the child understands well the geometric relationships of their environment, for which numbers and their relationships can be used. On the other hand, 47.6% of those surveyed believe that elementary mathematics should ensure that the child knows how to handle numbers well and for this, visual representations can be used.



Legend. Blue: elementary math should ensure that the child can handle numbers well and visual representations can be used for this. Red: mathematics should ensure that the child understands well the geometric relationships of his environment, for which numbers and their relationships can be used

Figure 31

b) Regarding teaching, the majority (54.4%) believe that it is preferable to establish arithmetic techniques before studying geometric relationships.



Legend. Blue: Establish arithmetic techniques before studying geometric relationships. Red: Teaching both arithmetic and geometric concepts, although it takes longer to become fluid in calculus.

Figure 32

c) The vast majority (92.6%) believe that mathematics is understood in different ways by children and that is why the different methods (arithmetic and geometric) help at all ages. On the other hand, 7.4% think that visual representations make the transition to abstraction difficult and that is why it is better to dispense with them progressively.

d) A vast majority of respondents (78.2%) feel more comfortable working with numbers than with geometric shapes (21.8%).

Legend. Blue: I feel more comfortable working with numbers. Red: I feel more comfortable working with geometric shapes.

Figure 33

5. Conclusions: reporting, recommendations and proposals for improvement

The analysis of the results shows the importance that the participants attach to learning geometric techniques, along with arithmetic, at school. On the one hand, from a theoretical point of view, around half of the respondents (52.4% versus 47.6%) agree with the relevance of geometry compared to those who consider arithmetic to be more relevant. Similarly, there is a distribution in terms of teaching methods (54.4% versus 45.6%) who believe that it is preferable to establish arithmetic techniques before studying geometric relationships.

There is almost unanimity (92.6%) in the idea that mathematics is understood in different ways by children and that is why the different methods (arithmetic and geometric) help at all ages.
Despite this, a vast majority of respondents (78.2%) feel more comfortable working with numbers than with geometric shapes (21.8%).

These theoretical data are complemented by the results of the practical answers, according to which between 20% and 25% of the answers to basic geometric problems are imprecise or incorrect. Of the remainder, around 75% of the techniques used to solve these problems are of the arithmetic type compared to 25% that use purely geometric techniques. When the problem is slightly more complicated, the wrong answers are a great majority that can reach up to 90% of the answers.

The results obtained reaffirm the need to design a workshop, intended for teacher training, in which the relationship between arithmetic and geometry in the teaching of mathematics in primary education is worked on. Rather than stress the theoretical importance of working on this relationship, practical activities will be developed that provide participants with concrete resources that they themselves can easily transfer to a primary school classroom.

Report of questionnaire Q4: Mental calculus and use of calculator

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1. Abstract

Within the framework of the Eramus+ program ANFoMAM, *Learning from children to improve primary teachers' specific formation in mathematics*, a questionnaire focusing on "Mental computation and the use of the calculator" has been designed. One of the aims of designing and delivering the questionnaire is to explore relationships between the experiences that preservice and in-service teachers have regarding both mental computation and the calculator, and their beliefs about the appropriateness of combining these two ways of working on arithmetic at school. For the design of the questionnaire (appendix Q4[7]), some relevant studies about the usefulness of the calculator in elementary school and about the most propitious time for its introduction have been taken into account. Both the classical vision, which regards the calculator as a mere tool that might hinder the learning of the algorithms, and the modern vision of the calculator as a source of interesting exercises are considered.

The analysis of the answers given to the questionnaire shows some differences between the participants from the different institutions regarding the aims of working on mental computation in schools as well as the way in which it should be combined with the use of the calculator.

2. Questionnaire design: background and objectives

In the design of this questionnaire, we drew on several documents on computation in elementary school (see bibliography), including the French literature on mathematics education; we also drew on our experiences of the initial training and professional development of primary school teachers.

This questionnaire consists of seven questions, grouped into three blocks:
- Three questions on mental computation;
- Three questions on the calculator;
- A final question, where the participants are given three specific computation tasks to perform and are asked in each case which means they would use to carry out the task: mental computation, written computation or calculator.

In (Celi, 2017), we provide the results of a survey to identify the knowledge and beliefs (as defined by Vause, 2011) of prospective teachers regarding teaching and learning mental computation in elementary

[7] Questionnaire Q4:https://docs.google.com/forms/d/1bXDuJcWb3zPvXaxBflW-O5HKk4XD7F7BFoGK9dejbIU/copy

school. In a first phase of this study, the prospective teachers interviewed were asked to complete the following sentence: "For me, mental computation is ... ". From the responses provided, we have chosen the terms which appear in question 1 of the present questionnaire Q4. In the posterior analysis it seemed interesting to us to combine the answers given to some groups of two/three terms, which has been grouped either by similarity or by opposition between them:

- strategy, initiative, reasoning;
- automatism and speed;
- automatism and training;
- reasoning and automatism;
- dislike and play.

The teaching of mental computation, which has appeared for several decades in French elementary school mathematics programs, has pursued, over time, different objectives. Until the 1970s, **memorization** and **speed** were favored, mental computation being considered as **educational** and **necessary in daily life**. Later, and still today, the phrase "mental computation" has incorporated other types of computation: reasoned computation, memorized computation**, oral and/or written** computation, with a time limit or otherwise. Moreover, its role has changed, probably due to both technological progress and the evolution in the way of understanding the teaching of mathematics. Thus, mental computation has become important in order to **explore numbers, operations and their properties**, as well as to **promote reasoning** (Butlen, 2007). Therefore, it has acquired an important role in relation to other types of computation, including computation written in columns. Mental computation is necessary to improve the learning of operational techniques and, according to some researchers, it should be given more importance than computation written in columns (CNESCO, 2015).

Learning mental computation can also be useful in finding different ways of performing the same computation; in checking a result achieved in a different way (in columns or using a calculator, for example); in evaluating an order of magnitude (Assude, 2007; Charnay, 1993-1994). It has also importance in relation to problem solving. These aspects, together with the more recent vision of mental computation, have been taken into account in the design of the first three questions of this questionnaire, given the fact that they are present in both French and Spanish elementary school culture.

It also seemed important to us to consider other elements in the questionnaire on mental calculus, namely,

- Behavior of the students: applying the established computation techniques in one's head; using the fingers to find the result of an additive computation, etc.
- Teacher beliefs: using fingers to perform additive computations is not good; learning of the addition and multiplication tables is facilitated by their ritual verbal repetition, in ascending order (predominance of memorized computation over written computation).

As the calculator has become an object of common consumption, the question arises whether or not, in elementary school learning, its use can be complementary to that of mental and written computation.

How useful can the calculator be in primary school classrooms?

The Italian, French and Spanish school curricula[8] refer to this tool, but in different ways. The Italian programs simply mention the calculator as an alternative to mental computation, whereas the Spanish and French curricula also regard the calculator as a tool to explore numbers and their properties.

[8] We use here the official texts of the school programs of Navarre, due to the fact that the mathematical contents of the programs published in the diverse Spanish regions are similar to each other.

The Italian curriculum mentions the calculator in tasks such as doing computations (with numbers which are large); checking the result of a computation that has already been done by other means; solving problems in which the student has to focus more on the procedure than on the computations, or in which a great amount of quantities is involved. However, there are several additional details that we find in the programs of Spain and France that have led us, in **question 4**, not to limit ourselves to the tasks mentioned in the Italian curriculum, but to also take into account others in which the calculator is seen as a tool that can be combined effectively with mental computation, being used both to explore numbers and to provide arithmetical problems to work on (MEN, 2003). This will help to identify to what extent respondents to the questionnaire have a classical or modern view of the use of this tool.

In Brouillard (1994), the author reports a research about the beliefs of a group of French school teachers regarding the use of the calculator. Although at the time of the survey, the calculator was already a socially integrated tool, the results of the research show its use was rather occasional. Almost three quarters of the respondents think that the calculator should be used in schools, but the majority think that it should not be used until the age of 8-9 years.

At what stage of the learning process should the calculator be allowed: after the introduction to the algorithms, while they are being learned or after their mastery has been achieved? Or should the use of the calculator remain independent from learning (or even mastering) the operational techniques?

We selected and included these questions in our questionnaire (**question 5**): the first two statements in question 5 reflect a less favorable opinion of the use of the calculator in school than the last two.

Moreover, there are a number of mostly unfavorable beliefs about the use of the calculator by children (in school or out of school). These are taken into account in question 6. To write this question, in addition to the Niebla Survey (ibidem, 1994), we used the work of Schaub (2009) where the author wonders both about the usefulness of the calculator in elementary school and the arguments in favor of or against its introduction at this stage, and also speculates about the most propitious time for its introduction.

Several items that have been included in the **first six questions** of this questionnaire were also developed in accordance with the work of Assude (2007), who studies the changes induced by the integration of this instrument in elementary school, as well as the resistance found among teachers.

With regard to the **first three questions**, we find it interesting to combine certain issues. In particular, it could be fruitful to link the words "initiative", "reasoning" and "strategy" (**question 1**) with the possible aims of working on mental computation in primary school, such as "develop reasoning and argumentation skills", "build and strengthen knowledge about the properties of numbers and operations" or "identify the different ways of carrying out the same computation" (**question 2**) and with some statement in question 3 such as "in carrying out the computations, procedure should be prioritized over speed". This linkage could allow us to highlight (or otherwise) the current view of mental computation, as opposed to the classical view of it.

Regarding **questions 4 to 6**, in order to highlight a vision favorable to the use of the calculator in elementary school and its complementarity with mental calculus, we find it interesting to combine some issues, in particular: "the calculator is useful to explore numbers" and is "a source of exercises" (question 4); "its use can be allowed in simultaneously while the pupils are learning the operational techniques or without they necessarily knowing them (**question 5**); "the calculator can be used to provide students with interesting learning situations" (**question 6g**).

Question 7 focuses on mathematical knowledge, with the aim of analyzing how respondents (students or teachers) use their knowledge of numbers and operations to carry out computations on some given natural integers: by mental computation, by using algorithms (in which the computation is placed in columns), or by using the calculator?

We present below a mathematical analysis, done a priori, of the tasks proposed in Question 7, in order to highlight the rich variety of possibilities they offer when using mental computation. (It should be noted that, in this case, the use of the calculator is taken as an alternative to the other forms of computation proposed, in a classical view of the calculator as a tool to perform computations, replacing mental computation).

Addition of three terms: 657 + 95 + 48. The presence of three terms, one of which is a three-digit term, could lead to the use of column computation or the calculator. However, if appropriate decompositions are performed, the result can be obtained by mental computation (reflection and/or memorization), i.e.: (650 + 5 + 2) + 95 + 48 = 650 + (95 + 5) + (48 + 2) = 650 + 100 + 50 = (650 + 50) + 100 = 800

Subtraction of two numbers: 3456 – 897. By using the "constant difference" property, the following transformation could be carried out:
(3456 + 3) – (897 + 3) = 3459 – 900; subtracting now 9 hundred from 34 hundred, we obtain 25 hundred, i.e. 2500 units, which, added to 59 units, give the result of 2559.

Multiplication of two numbers: 12 x 19. Two procedures of mental computation are possible:
1) 12 x (20 – 1) = 240 – 12 = (240 – 2) – (12 – 2) = 238 – 10 = 228.
2) (10 + 2) x 19 = 190 + 38 = 190 + 10 + 28 = 228.

Euclidean division: 10008 ÷ 9
10008 is equal to 10000 + 8 = 9999 + 9. Now, 9999 ÷ 9 = 1111 and 9 ÷ 9 = 1, where 10008 ÷ 9 =1111 +1 = 1112.

The answers given do not allow us to know which specific procedure the respondents use if they choose the option of mental computation. We find it more interesting here to see which method they spontaneously opt for.

3. Data collection

The questionnaire Q4 regarding mental computation and the use of the calculator, was delivered in ToKalon Assocation (TKL) and also in the universities of Bordeaux (UB), Pamplona (UPNA) and Rome (UR).

The students in the University of Bordeaux were offered the questionnaire by email to be answered online. This mail was sent to the students of the MeEF Master (first year) as well as to some in-service primary teachers and to some university teachers. In the UPNA the questionnaire was delivered to the students of the Primary teachers' degree, prior to studying any subject of mathematics and the didactics thereof. In UR, the questionnaire was given to the students of the 'Dipartimento di Scienze della Formazione', who are teachers in courses of professional development offered by ToKalon Association. They had to answer it in a limited time, before listening to a talk about mental computation (by Valentina Celi from UB).

4. Data management and analysis

For the analysis, the questions have been grouped according to their form. In each group, we will follow a different kind of analysis:

1. Group 1 T[9] –**question 1**– value assigned to several terms in relation to mental computation (likert scale from 1 to 4)
2. Group 2 CM[10]–**question 3**– degree of agreement with some assertions regarding mental computation (likert scale from 1 to 4).
3. Group 3 CA, **question 6**: degree of agreement with some assertions regarding the use of the calculator (likert scale from 1 to 4).
4. Group 4, **questions 2, 4 y 5**: multiple-choice questions regarding the learning of mental computation in primary school, the usefulness of the calculator in primary school and the most propitious time for its introduction.
5. Group 5, **question 7**: the choice of a method for some specific computations; only one choice being possible in each case.

Group 1 T–question 1

In the first analysis of all the answers, which does not take into account the participants' origin, the words that are most frequently associated with mental computation are "attention" and "usefulness". The remaining terms are highly valued by a large number of the participants, with the exception of "initiative", which was associated with mental computation by only 62%; and "dislike", which only 29% of the participants viewed as related.

In a second analysis, which groups the participants in accordance with their origin, the greatest discrepancy between institutions appears with regard to the term "dislike", which is highly valued by 42% of the respondents in UPNA, a percentage significantly higher than the other institutions. There are also discrepancies in relation to the terms "usefulness", which is assigned a higher value in UPNA, and "automatism", more highly valued by the group from UB.

With regard to the term *entraînement*, the discrepancy between UPNA and the rest of the groups may be due to how the term was translated into Spanish: *adiestramiento,* a term similar to "training" but with a more negative nuance (implying blind obedience) than the terms used in Italian (*allenamento*) and French (*entraînement*).

[9] Some terms have only been included in one or two versions of the questionnaire. The terms marked with letter T are included in all of the versions.
[10] Some assertions are not formulated in the same way in all the versions of the questionnaire. In a first analysis of all the answers, which does not take into account the participants' origin, we consider only those expressions that appear in all the versions. However, in a second analysis, which groups the participants in accordance with their origin, we consider all the different formulations of each assertion.

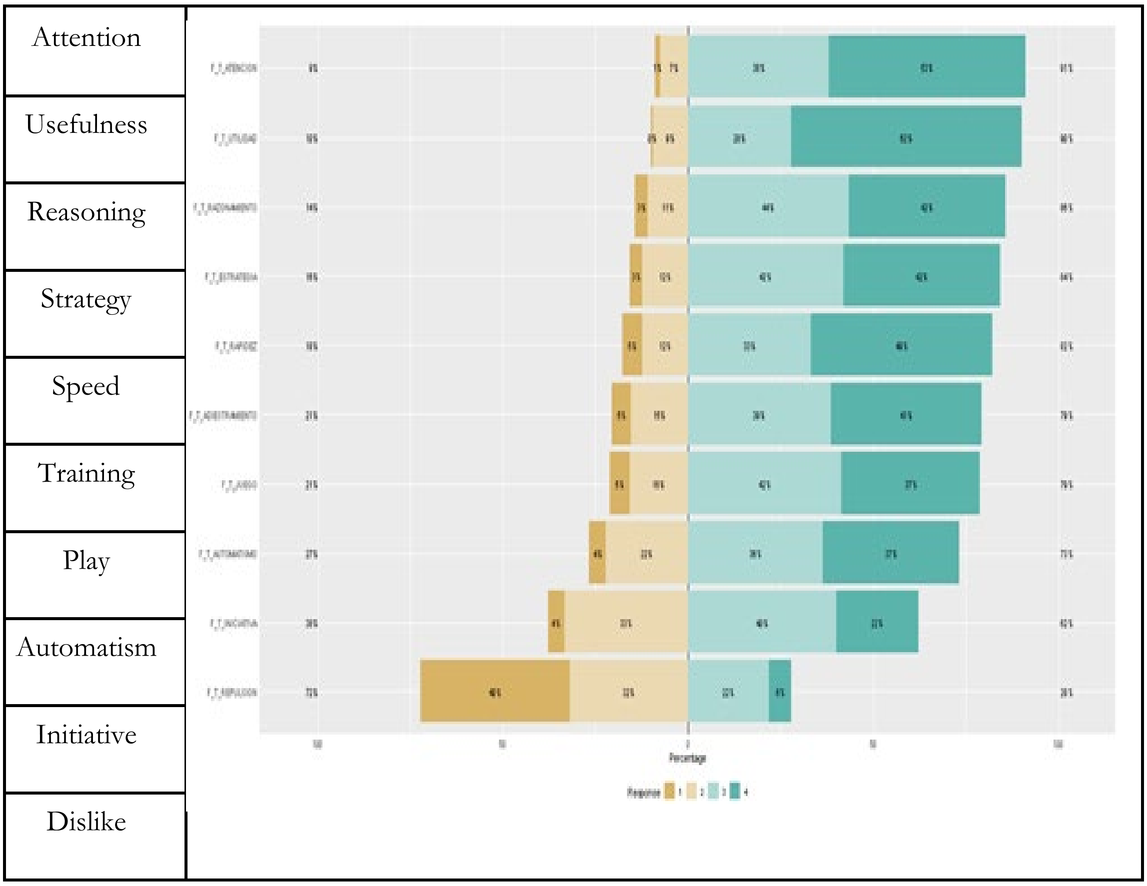
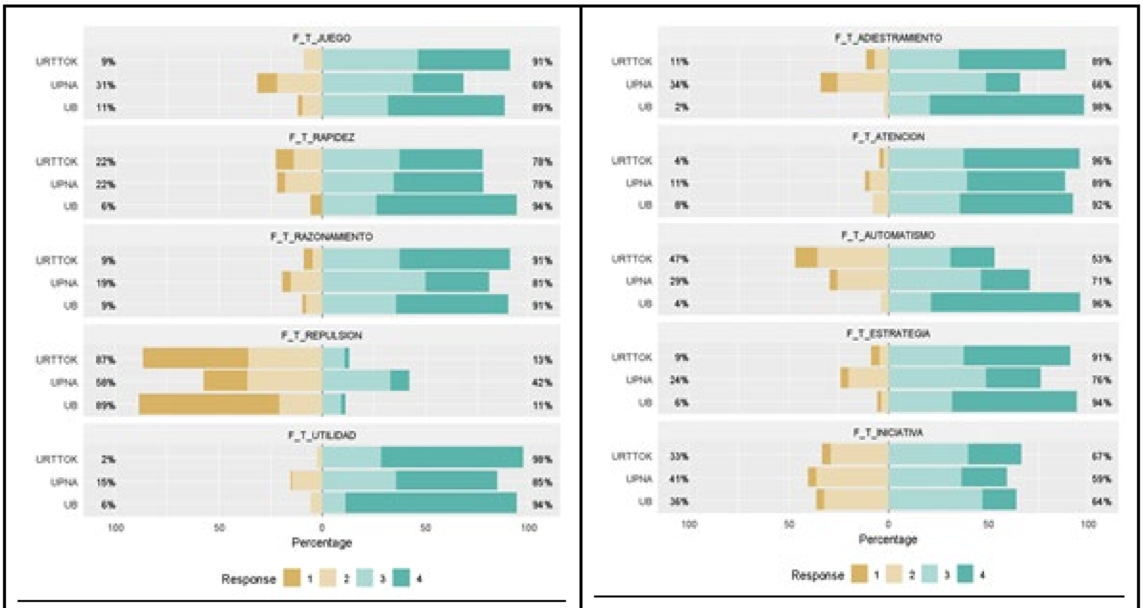


Figure 34: Graph of the first analysis of the answers given to Question 1



From top to bottom: Play- Speed – Reasoning – Dislike – Usefuness	From top to bottom: Training- Attention – Automatism – Strategy – Initiative

Figure 35. Answers to Question 1 (in accordance with the participants' institutions)

We find it interesting to combine the answers given to some terms, either on account of an analogy between them, or some kind of opposition:

- Strategy, initiative, reasoning

```
                  F_T_ESTRATEGIA
F_T_RAZONAMIENTO     1    2    3    4 Total Count
               1 28.6 28.6 14.3 28.6 100.1     7
               2  4.5 45.5 36.4 13.6 100.0    22
               3  2.2 11.2 55.1 31.5 100.0    89
               4  2.3  3.5 32.6 61.6 100.0    86
           Total 3.4 12.3 42.2 42.2 100.0

        Pearson's Chi-squared test

data:  .Table
X-squared = 61.977, df = 9, p-value = 5.566e-10
```

Vertical axis: Reasoning. Horizontal axis: Strategy

```
                  F_T_INICIATIVA
F_T_RAZONAMIENTO     1    2    3    4 Total Count
               1 28.6 42.9 28.6  0.0 100.1     7
               2  9.1 68.2 13.6  9.1 100.0    22
               3  3.4 37.1 48.3 11.2 100.0    89
               4  2.3 19.8 39.5 38.4 100.0    86
           Total 4.4 33.3 40.2 22.1 100.0

        Pearson's Chi-squared test

data:  .Table
X-squared = 48.636, df = 9, p-value = 0.0000001942
```

Vertical axis: Reasoning. Horizontal axis: Initiative

```
              F_T_INICIATIVA
F_T_ESTRATEGIA    1    2    3    4 Total Count
             1 14.3 42.9 28.6 14.3 100.1     7
             2  8.0 68.0 16.0  8.0 100.0    25
             3  3.5 29.1 47.7 19.8 100.1    86
             4  3.5 26.7 40.7 29.1 100.0    86
         Total 4.4 33.3 40.2 22.1 100.0

        Pearson's Chi-squared test

data:  .Table
X-squared = 23.033, df = 9, p-value = 0.006122
```

Vertical axis: Strategy. Horizontal axis: Initiative

Figure 36: Relation between the answers referring to: “strategy”, “initiative” and “reasoning”

The double-entry tables show the relation between the variables, taken two by two. If the *p-value* is less than 0.05, it means that there exists some correlation between the values that the participants assign to these two variables.

The vertical axis shows the values (1 to 4) given to a first variable, whilst the horizontal axis depicts the values (1 to 4) given to a second variable.

The table therefore represents in percentages how the participants have responded to the variables in question. Thus, for instance, 28.6% of people who assigned the value 1 to the term “reasoning” also assigned 1 to “strategy”. This percentage is compared with that in the last row, which indicates that 3.4% of the total participants assigned a value 1 to the term “strategy”. This discrepancy between the behavior of the participants in general and that of those who assigned the value 1 to “reasoning” shows a certain tendency to assign a low value to the term “strategy” (in relation to mental computation) on the part of those who value the term “reasoning” the least. It is also worth mentioning that 61.6% of the participants who assigned a value of 4 to “reasoning”, also assigned a value of 4 to the term “strategy”. By comparing this percentage with the 42.2% of all the respondents who assigned a value of 4 to “strategy”, we conclude that there exists a certain tendency to value highly “strategy” on the part of those who assigned a high value to “reasoning”. Therefore, there exists some correlation between the terms “reasoning” and “strategy”: the higher “reasoning” is valued, the higher “strategy” is valued too.

The tables make evident the inter-relation between all the pairs of variables which appear, as can be seen in Figure 36.

It is important to highlight that most of the participants assign a high value to the term "reasoning", which means that only 29 people do not do so. It is also clear that the people who assign a low value to "reasoning" have a tendency to do the same with "initiative" and that this kind of interrelation is greater when the pair "reasoning" and "strategy" is considered. Regarding the pair "strategy" and "initiative", those who assign a low value to "strategy" also do so with "initiative". However, those who assign a high value to "strategy" do not necessarily assign either a higher or a lower value to "initiative" than the participants as a whole.

- **Speed and automatism**

```
Row percentages:
                      F_T_RAPIDEZ
F_T_AUTOMATISMO      1    2    3    4 Total Count
              1 22.2 22.2 44.4 11.1  99.9     9
              2  8.9 22.2 35.6 33.3 100.0    45
              3  4.1 16.2 36.5 43.2 100.0    74
              4  2.7  1.3 28.0 68.0 100.0    75
          Total  5.4 12.3 33.5 48.8 100.0

        Pearson's Chi-squared test

data:  .Table
X-squared = 31.774, df = 9, p-value = 0.0002178
```

Vertical axis: Automatism. Horizontal axis: Speed

Figure 37: Relation between the answers referred to "speed" and "automatism"

There is a certain inter-relation between these two terms. The table shows that those who assign a low value to "automatism" also does so with "speed". Conversely, people who highly value "automatism", tend to do the same with "speed", further than the generality.

- Automatism and training

```
                      F_T_ADIESTRAMIENTO
F_T_AUTOMATISMO      1    2    3    4 Total Count
              1  0.0 22.2 55.6 22.2   100     9
              2 13.3 22.2 37.8 26.7   100    45
              3  5.4 17.6 45.9 31.1   100    74
              4  1.3  8.0 30.7 60.0   100    75
          Total  5.4 15.3 38.9 40.4 100.0

        Pearson's Chi-squared test

data:  .Table
X-squared = 27.028, df = 9, p-value = 0.001384
```

Vertical axis: Automatism. Horizontal axis: Training

Figura 38: Relation between the answers referred to "automatism" and "training"

The dependence between the variables in this table is not reflected as clearly as in the case of the variables in the previous tables: those participants who assign a high value to "automatism" also assign a high value to “training”; those who assign a low value to “automatism” also assign a low value to “training”. However, those who assign a low value to “automatism” tend to assign a high value to “training”.

- Reasoning and automatism

```
Row percentages:
                   F_T_RAZONAMIENTO
F_T_AUTOMATISMO   1    2    3    4 Total Count
              1 0.0 11.1 22.2 66.7 100.0     9
              2 4.4 20.0 51.1 24.4  99.9    45
              3 2.7  8.1 45.9 43.2  99.9    74
              4 4.0  8.0 40.0 48.0 100.0    75
          Total 3.4 10.8 43.8 41.9 100.0

        Pearson's Chi-squared test

data:  .Table
X-squared = 12.298, df = 9, p-value = 0.197
```

Vertical axis: Automatism. Horizontal axis: Reasoning

Figure 39: Relation between the answers referred to “reasoning” and “automatism”

In the previous table, the lack of dependence between both terms seems clear, i.e. the value assigned to “automatism” gives no indication of what value might be assigned to “reasoning”. In the main, no matter which value they assigned to “automatism”, the participants consistently assign a high value to “reasoning”.

- Dislike and play

```
                 F_T_JUEGO
F_T_REPULSION    1    2    3    4 Total Count
            1  0.0  4.9 46.3 48.8 100.0    82
            2  6.2 18.5 36.9 38.5 100.1    65
            3  8.9 20.0 48.9 22.2 100.0    45
            4 25.0 58.3  8.3  8.3  99.9    12
        Total  5.4 15.7 41.7 37.3 100.0

        Pearson's Chi-squared test

data:  .Table
X-squared = 47.859, df = 9, p-value = 0.0000002714
```

Vertical axis: Dislike. Horizontal axis: Play

Figure 40: Relation between the answers referred to “dislike” and “play”

There is an evident inter-relation between these terms. In fact, those participants who assign a low value to the term “dislike” tend to assign a high value to “play”, while those who assign a high value to the term “dislike” tend to assign a low value to the term “play”.

Group 2 CM –question 3

There is a strong consensus (92%) against the prohibition of the use of fingers in additive computations and there is also a rejection, albeit not so strong, of the non-use of a written medium. In addition, the degree of agreement with the other assertions is around 50%, which indicates that there are differences of opinion with regard to these statements.

Lack of mastery of mental computation hinders learning algorithms
Children tend to put numbers in columns in their head
Learning of tables is facilitated by repeating them in ascending order
Mental computation should be prioritized over computation in columns
The focus should be on procedure rather than speed
No written support should be provided
The use of fingers should be prohibited

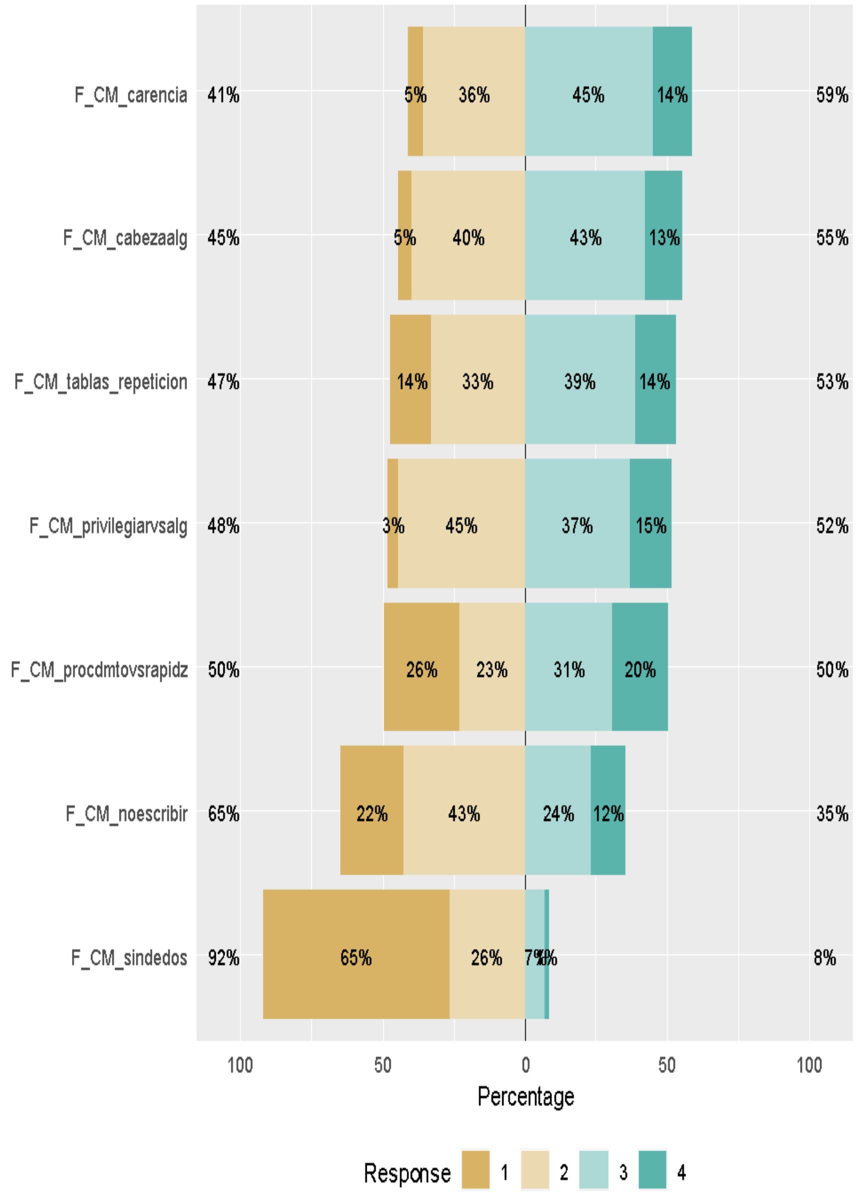


Figure 41: Answers to Question 3, analyzed without taking into account the participants' origin

In the second analysis, which groups the answers in accordance with the participants' origin, there are different answers regarding whether *the lack of mastery of mental computation makes it difficult to learn the arithmetic operations*. While there are marked differences in opinion with regard to this statement in UPNA and also in URTKL, 70% of UB participants agree with this assertion.

There are also different views on the extent to which, in carrying out the computations, procedure should be prioritized over speed: while the participants in UB prioritize procedure (81%), those in UPNA give priority to speed. In the case of the participants in URTKL, opinions are mixed, the results indicating a lower predilection for procedure than in UB (60 %).

The participants in UPNA and UB are respectively 62% in favor of and 62% against the ritual verbal repetition of the addition and multiplication tables in an ascending order. Again there are differences of opinion in the case of the respondents in URTK.

Children tend to put numbers in columns in their heads
Lack of mastery of mental computation makes it difficult to learn algorithms
No written support should be provided
Mental computation should be prioritize over computation in columns
The focus should be on procedure rather than speed
When learning basic additive computations, the use of fingers should be prohibited
Learning of the tables is facilitated by repeating them in ascending order

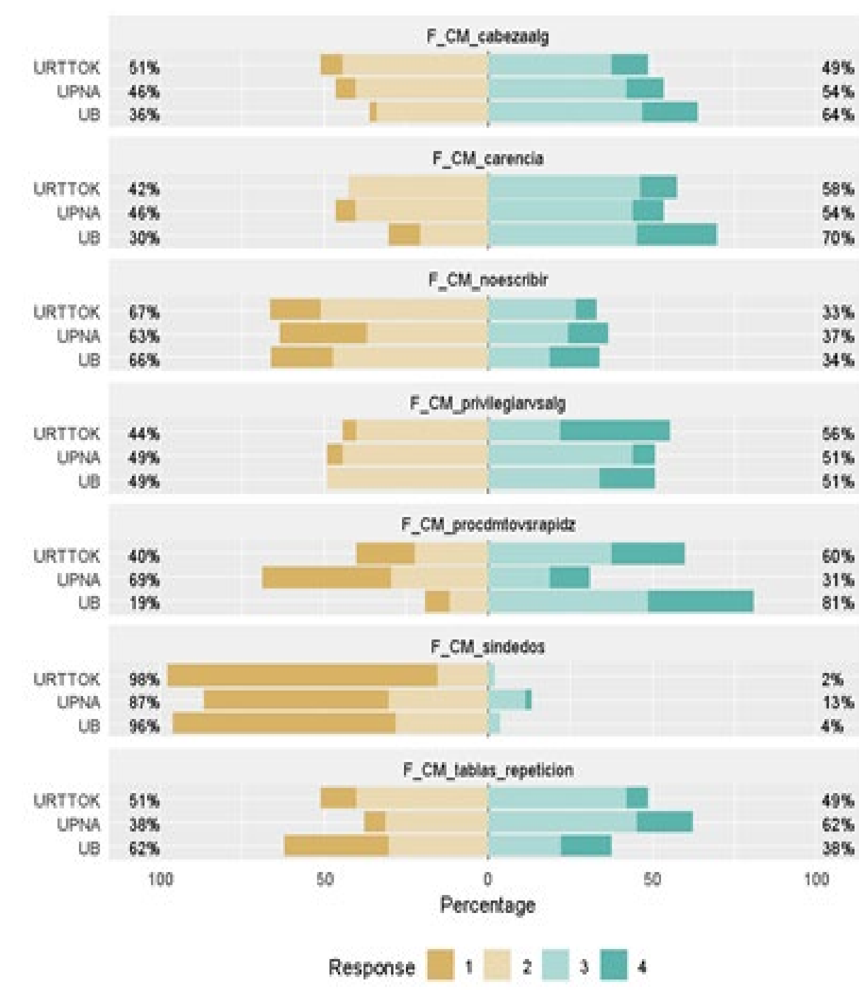


Figure 42: Answers to Question 3, analyzed in accordance with their origin

Group 3 CA –question 6

Regarding the use of the calculator, there is a tendency to agree with all the statements in question 6, with the exception of the assertion that the calculator hinders thinking, to which the agreement is only 44%. There are also differences of opinion (with 50% of agreement) regarding the statement that the calculator severely hinders mental computation. In addition, there different degrees of agreement with the assertion that a high dependence on the calculator is an indicator of a deficient mathematical knowledge.

The agreement with the other assertions regarding the calculator is marked, reaching percentages of between 68% and 81% in the first five statements. This means that the calculator is associated with its use at home rather than at school as well as considered as a source of interesting exercises. At the same time, there exists a strong consensus on the statements that frequent use of the calculator indicates a lack of resources and that the calculator does not teach children how to calculate.

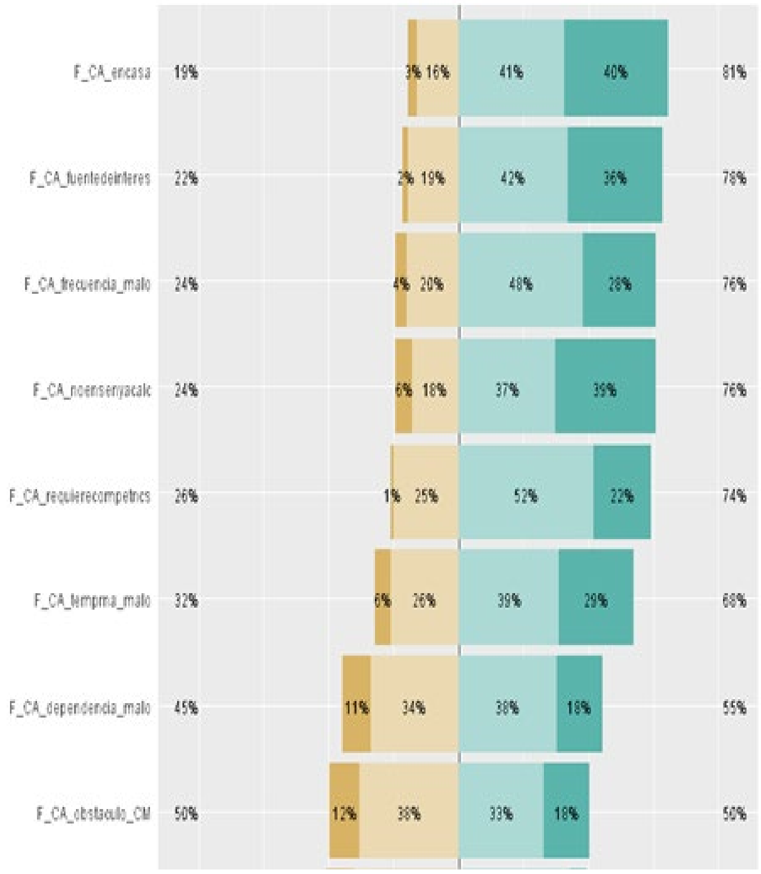
About the calculator	
It is more often used at home than at school	
It can provide the pupils with interesting learning situations	
Too frequent use of it hinders maths skills	
It won't teach pupils how to calculate	
Digital skills are required to use it	
Its early introduction hinders maths skills	
Over-reliance on it is a sign of maths knowledge deficiency	
It prevents children from thinking	

Figure 43: Answers to Quest. 6, analyzed without taking into account the participants' origin.

When the analysis groups the participants in accordance with their origin, the greatest divergence between the groups appears on the belief that "an over-reliance on the calculator is indicative of a lack of mathematical knowledge". The group from URTKL disagrees with this statement, while the other groups agree with it (60%). Something similar occurs with the statement "the mathematical skills are hindered if the calculators are too frequently used". In addition, the UPNA group is the one that agrees most with the assertions "the calculator hinders thinking" and "the calculator severely hinders mental computation".

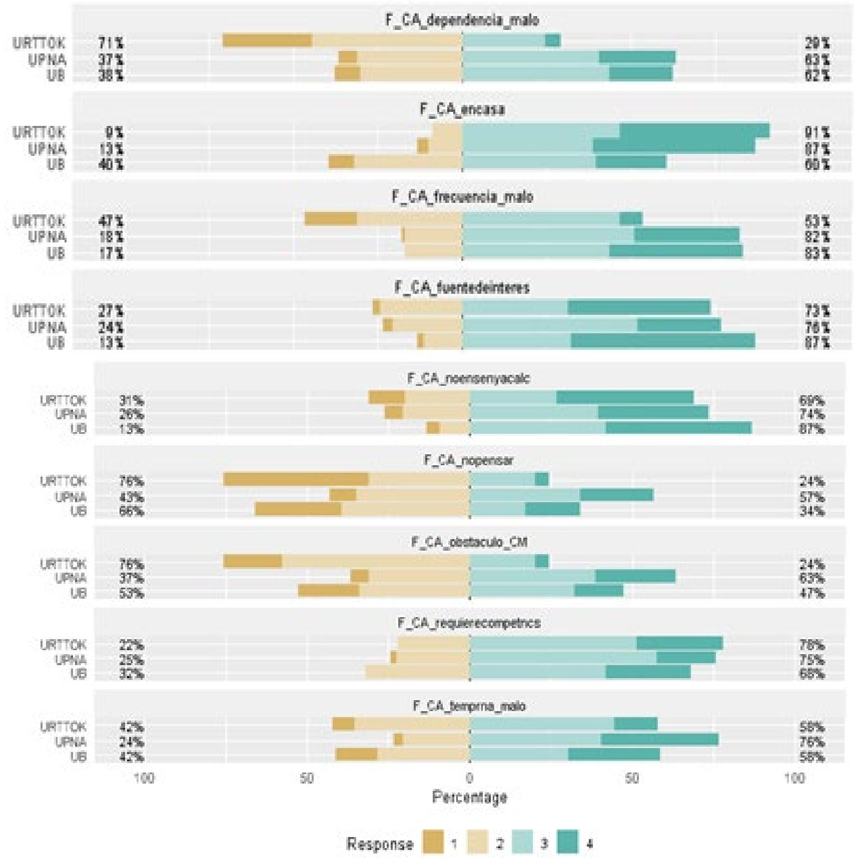
<table>
<tr><th>About the calculator</th><th></th></tr>
<tr><td>Over-reliance on it is a sign of maths knowledge deficiency</td><td rowspan="9">
</td></tr>
<tr><td>It is more often used at home than at school</td></tr>
<tr><td>Too frequently use of it hinders maths skills</td></tr>
<tr><td>It can provide the pupils with interesting learning situations</td></tr>
<tr><td>It won’t teach pupils how to calculate</td></tr>
<tr><td>It impedes thinking</td></tr>
<tr><td>It hinders mental computation</td></tr>
<tr><td>Numerical skills are required to use it</td></tr>
<tr><td>Its early introduction hinders mathematical skills</td></tr>
</table>

Figure 44: Answers to Question 6, analyzed in accordance with the participants’ origin

Group 4 –questions 2, 4 and 5

Question 2: Why must mental computation be learnt in Primary School?

Around 65% of the participants choose the options c, d, g and b, i.e. “mental computation builds and strengthens knowledge of the properties of numbers and operations”; “identifies different ways of carrying out the same computation”; “assists in solving problems and develops reasoning and argumentation skills.”

When the analysis is made in accordance with the participants’ origin, the results are similar to those obtained without taking this origin into account, with the exception of the group from UB; for whom the benefits of mental computation for “evaluating the order of magnitude of a result” are given a different degree of agreement.

Table 9: Participants answers to question 2

Together (204)		UB (53)		UPNA (106)		URTKL (45)	
palabra	frec	palabra	frec	palabra	frec	palabra	frec
construiryreforzar	142	construiryreforzar	41	construiryreforzar	71	razonamiento	32
identificarymaneras	130	calculoyaprox	38	resolproblemas	71	construiryreforzar	30
resolproblemas	124	memorizar	33	identificarymaneras	68	identificarymaneras	29
razonamiento	123	identificarymaneras	33	razonamiento	63	resolproblemas	23
memorizar	106	resolproblemas	30	memorizar	59	memorizar	14
calculoyaprox	56	razonamiento	28	entenderynociones	32	calculoyaprox	6
entenderynociones	32	controlycalculadora	15	calculoyaprox	12	controlycalculadora	4
controlycalculadora	31			controlycalculadora	12		

KEY	
Construir y reforzar	Build and strengthen knowledge of the properties of numbers and operations
Identificar maneras	Identify different ways of carrying out the same computation
Resolproblemas	Assist in problem solving
Razonamiento	Develop reasoning and argumentation skills
Memorizar	Develop memory skills
Calculo aproximaciones	Evaluate the order of magnitude of a result
Entender nociones	Understand arithmetical concepts
Control calculadora	Check the result displayed by a calculator

Question 4. Why do you think that the calculator is useful in primary school?

The most frequently answered option is that the calculator is a tool to check the result and solve problems in which a great amount of quantities is involved. The three groups of participants are aligned with the first column (below), in which all the answers are considered together, although the group from URTKL also gives a higher importance to the term "explore".

Table 10: Participants answers to question 4

Together (204)		UB (53)		UPNA (106)		URTKL (45)	
palabra	frec	palabra	frec	palabra	frec	palabra	frec
verificar	130	verificar	48	verificar	71	verificar	29
grandesoperaciones	116	grandesoperaciones	40	grandesoperaciones	70	explorar	24
calcular	66	resolverprob	28	calcular	45	grandesoperaciones	16
resolverprob	66	despreocupar	21	explorar	36	despreocupar	15
explorar	62			aprfuncionesupna	36	resolverprob	11
despreocupar	45			resolverprob	35	fuente	10
aprfuncionesupna	36						
reducirmemorizcn	32						

KEY	
Verificar	A tool to check the result of a computation
Grandes operaciones	A tool for performing computations with large (or a lot of) numbers
Calcular	A tool for computing
Resolver problemas	A tool for solving problems that require a lot of testing
Explorar	A support to explore numbers
Despreocupar	A tool for students to solve a problem without worrying about computations
Fuente	A source of mathematical problems and exercises
Reducir memorización	A tool to reduce memorization

Question 5. It is assumed that the pupils are allowed to use the calculator. It can be allowed…

In all the groups, the option which is chosen most is “using the calculator after the algorithms have been mastered”. The second most popular one is “at the same time as the operational techniques are being learnt”. The third ranked option, “after the operational techniques have been taught" (as opposed to mastered), is chosen less frequently, while “before teaching the algorithms” is the least chosen in all the institutions apart from UB.

Table 11: Participants answers to question 5

Together (204)		UB (53)		UPNA (106)		URTKL (45)	
palabra	frec	palabra	frec	palabra	frec	palabra	frec
despues	90	despues	22	despues	42	despues	26
mientras	71	mientras	18	mientras	39	mientras	14
trasfaseinicial	33	sinconocer	8	trasfaseinicial	24	trasfaseinicial	4
sinconocer	10	rasfaseinicial	5	sinconocer	1	sinconocer	1

KEY	
Después	After the algorithms are mastered
Mientras	At the same time as the operational techniques are being learnt
Tras fase inicial	After the operational techniques have been taught (as opposed to mastered)
Sin conocer	Before teaching the algorithms

Group 5: Question 7

In order to carry out the division task, the preferred option in all groups is that of the calculator. In the case of the subtraction task, the preferred option in all groups is that of the algorithm. In the case of the addition task, the option of mental computation is the most frequently chosen. The same occurs with the multiplication task, with the exception of UPNA, which tends to prefer the calculator over mental computation (a general tendency exhibited by UPNA, which contrasts markedly with the UB group, which tends to favour mental computation).

Table 12: Participants answers to question 7

ADDITION: 657+95+48

Together		UB		UPNA		URTKL	
palabra	frec					palabra	frec
calculomental	64	calculomental	51	calculomental	25	calculomental	21
algrtm	57	algrtm	40	algrtm	24	algrtm	21
calculadora	21	calculadora	20	calculadora	4	calculadora	3

SUBTRACTION: 3456-897

Together		UB		UPNA		URTKL	
palabra	frec					palabra	frec
algrtm	77	algrtm	31	algrtm	20	algrtm	26
calculadora	34	calculomental	14	calculadora	14	calculadora	12
calculomental	32	calculadora	8	calculomental	11	calculomental	7

MULTIPLICATION:12*19

Together		UB		UPNA		URTKL	
palabra	frec					palabra	frec
calculomental	72	calculomental	39	calculadora	17	calculomental	22
algrtm	37	algrtm	8	algrtm	16	algrtm	13
calculadora	33	calculadora	6	calculomental	11	calculadora	10

DIVISION: 10008/9

Together		UB		UPNA		URTKL	
palabra	frec					palabra	frec
calculadora	79	calculadora	29	calculadora	28	calculadora	22
algrtm	40	algrtm	16	algrtm	13	calculomental	12
calculomental	23	calculomental	8	calculomental	3	algrtm	11

KEY	
Calculadora	Calculator
Algoritmos	Algorithms
Cálculo mental	Mental computation

Some further analyses

We also make some further analyses of the answers by combining some terms and statements from the first three questions. In particular, we combine the terms “initiative”, “reasoning” and “strategy” (question 1) with some of the aims of mental computation in question 2 ("develop reasoning and argumentation skills", "build and strengthen knowledge of the properties of numbers and operations" and "identify different ways of carrying out the same computation") and the statement that "procedure should be prioritized over speed" (question 3).

```
              fconstruirYreforzar
F_T_INICIATIVA    0    1 Total Count
             1 22.2 77.8   100     9
             2 36.8 63.2   100    68
             3 20.7 79.3   100    82
             4 40.0 60.0   100    45
         Total 30.4 69.6 100.0

        Pearson's Chi-squared test

data:  .Table
X-squared = 7.1702, df = 3, p-value = 0.06667
```

Build and strengthen knowledge of the properties of numbers and operations

```
              fidentificarYmanera
F_T_INICIATIVA    0    1 Total Count
             1 44.4 55.6   100     9
             2 48.5 51.5   100    68
             3 32.9 67.1   100    82
             4 22.2 77.8   100    45
         Total 36.3 63.7 100.0

        Pearson's Chi-squared test

data:  .Table
X-squared = 8.9194, df = 3, p-value = 0.03038
```

Identify different ways of carrying out the same computation

```
              fdesarr_raznto
F_T_INICIATIVA    0    1 Total Count
             1 55.6 44.4   100     9
             2 41.2 58.8   100    68
             3 45.1 54.9   100    82
             4 24.4 75.6   100    45
         Total 39.7 60.3 100.0

        Pearson's Chi-squared test

data:  .Table
X-squared = 6.3885, df = 3, p-value = 0.09416
```

Develop reasoning and argumentation skills

```
              F_CM_procdmtovsrapidz
F_T_INICIATIVA    1    2    3    4 Total Count
             1 33.3 22.2 22.2 22.2  99.9     9
             2 39.7 20.6 29.4 10.3 100.0    68
             3 17.1 26.8 36.6 19.5 100.0    82
             4 22.2 20.0 24.4 33.3  99.9    45
         Total 26.5 23.0 30.9 19.6 100.0

        Pearson's Chi-squared test

data:  .Table
X-squared = 17.672, df = 9, p-value = 0.03918
```

Prioritize procedure over speed

Figure 45: Relation between “initiative” and some statements (questions 2 and 3)

The tables reflect independence between “initiative” and both "build and strengthen the properties of numbers and operations" and "develop reasoning skills", because the choice of the latter statements is the same regardless of the value assigned to “initiative”.

There is a slight dependence between the term “speed” and the statements “identify different ways of carrying out a computation” and “prioritize procedure over speed”.

<table>
<tr>
<td><pre>
                  fconstruirYreforzar
F_T_RAZONAMIENTO   0   1 Total Count
               1 71.4 28.6   100   7
               2 36.4 63.6   100   22
               3 28.1 71.9   100   89
               4 27.9 72.1   100   86
           Total 30.4 69.6 100.0

        Pearson's Chi-squared test

data: .Table
X-squared = 6.417, df = 3, p-value = 0.093
</pre></td>
<td><pre>
                  fdesarr_raznto
F_T_RAZONAMIENTO   0   1 Total Count
               1 42.9 57.1   100   7
               2 63.6 36.4   100   22
               3 39.3 60.7   100   89
               4 33.7 66.3   100   86
           Total 39.7 60.3 100.0

        Pearson's Chi-squared test

data: .Table
X-squared = 6.5837, df = 3, p-value = 0.08642
</pre></td>
</tr>
<tr>
<td>Build and strengthen knowledge of the properties of numbers and operations</td>
<td>Develop reasoning and argumentation skills</td>
</tr>
<tr>
<td><pre>
                  fidentificarYmanera
F_T_RAZONAMIENTO   0   1 Total Count
               1 71.4 28.6   100   7
               2 31.8 68.2   100   22
               3 38.2 61.8   100   89
               4 32.6 67.4   100   86
           Total 36.3 63.7 100.0

        Pearson's Chi-squared test

data: .Table
X-squared = 4.5882, df = 3, p-value = 0.2046
</pre></td>
<td><pre>
                  F_CM_procdmtovsrapidz
F_T_RAZONAMIENTO   1   2   3   4 Total Count
               1 42.9 28.6 28.6 0.0 100.1   7
               2 45.5 27.3 13.6 13.6 100.0   22
               3 28.1 27.0 33.7 11.2 100.0   89
               4 18.6 17.4 32.6 31.4 100.0   86
           Total 26.5 23.0 30.9 19.6 100.0

        Pearson's Chi-squared test

data: .Table
X-squared = 21.32, df = 9, p-value = 0.0113
</pre></td>
</tr>
<tr>
<td>Identify different ways of carrying out the same computation</td>
<td>Prioritize procedure over speed</td>
</tr>
</table>

Figure 46: Relation between the term reasoning and some assertions (questions 2 and 3)

There is independence between the term “reasoning” and the three aims regarding mental computation (“build and strengthen knowledge of the properties of numbers and operations”; “develop reasoning and argumentation skills” and “identifying different ways of carrying out the same computation”), although the few participants who assign a low value to “reasoning” tend not to choose “develop reasoning and argumentation skills” as an aim of mental computation. In addition, the more “reasoning” is valued, the more “procedure over speed” is chosen.

The value assigned to “strategy” is independent of whether or not one of the options, “build and strengthen knowledge of the properties of numbers and operations” or “procedure over speed”, are chosen. However, those who assign a low value to “strategy” tend not to choose “develop reasoning and argumentation skills” and “identify the different ways of carrying out the same computation” as aims of mental computation.

```
               fconstruirYreforzar
F_T_ESTRATEGIA     0    1 Total Count
             1 42.9 57.1   100     7
             2 36.0 64.0   100    25
             3 36.0 64.0   100    86
             4 22.1 77.9   100    86
         Total 30.4 69.6 100.0

        Pearson's Chi-squared test

data:  .Table
X-squared = 4.9854, df = 3, p-value = 0.1729
```

Strategy & Build and strengthen knowledge of the properties of numbers and operations

```
               fdesarr_raznto
F_T_ESTRATEGIA     0    1 Total Count
             1 57.1 42.9   100     7
             2 68.0 32.0   100    25
             3 32.6 67.4   100    86
             4 37.2 62.8   100    86
         Total 39.7 60.3 100.0

        Pearson's Chi-squared test

data:  .Table
X-squared = 11.308, df = 3, p-value = 0.01017
```

Strategy & Develop reasoning and argumentation skills

```
               fidentificarYmanera
F_T_ESTRATEGIA     0    1 Total Count
             1 71.4 28.6   100     7
             2 48.0 52.0   100    25
             3 38.4 61.6   100    86
             4 27.9 72.1   100    86
         Total 36.3 63.7 100.0

        Pearson's Chi-squared test

data:  .Table
X-squared = 7.9977, df = 3, p-value = 0.04606
```

Strategy & Identify different ways of carrying out the same computation

```
               F_CM_procdmtovsrapidz
F_T_ESTRATEGIA    1    2    3    4 Total Count
             1 42.9 28.6 28.6  0.0 100.1     7
             2 44.0 24.0 12.0 20.0 100.0    25
             3 25.6 27.9 30.2 16.3 100.0    86
             4 20.9 17.4 37.2 24.4  99.9    86
         Total 26.5 23.0 30.9 19.6 100.0

        Pearson's Chi-squared test

data:  .Table
X-squared = 13.693, df = 9, p-value = 0.1337
```

Strategy & Prioritize procedure over speed

Figure 47: Relation between the term "strategy" and some statements (questions 2 and 3)

Figure 48 groups the participants in accordance with their answers. The red circle (on the upper right side of the figure) groups those participants who assign a low value (1-2) to reasoning/strategy/initiative/procedure, and the light brown circle (on the upper left side) groups those participants who assign a high value (4) to these terms. Those participants who choose some specific aims of mental computation ("build and strengthen the knowledge of the properties of numbers and operations"; "identify different ways of carrying out the same computation"; "develop reasoning and argumentation skills") and agree with "prioritizing procedure over speed" figure, in a group, in the yellow circle (in the center of the figure), whereas those participants who do not choose these options figure in the blue circles (not grouped), near the red circle.

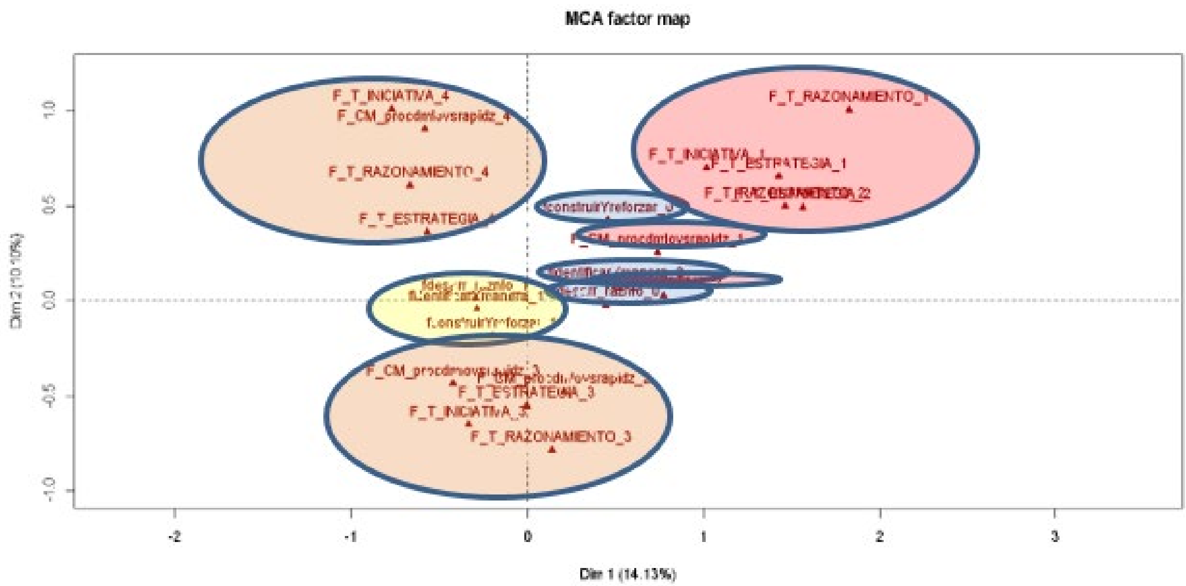


Figure 48: Multiple correspondence analysis of the answers

With regard to questions 4 to 6, in order to analyse the relationship between perspectives favorable to the use of the calculator in primary school and other viewpoints which favor a combined use of the calculator and mental computation at school, we combine the answers to some of the statements, in particular: the calculator is useful to "explore numbers" and is a "source of exercises" (question 4); "the use of the calculator may be allowed when the children are learning the operational techniques or without necessarily having learnt them" (question 5); "the calculator can be used to provide children with interesting exercises" (statement 6g).

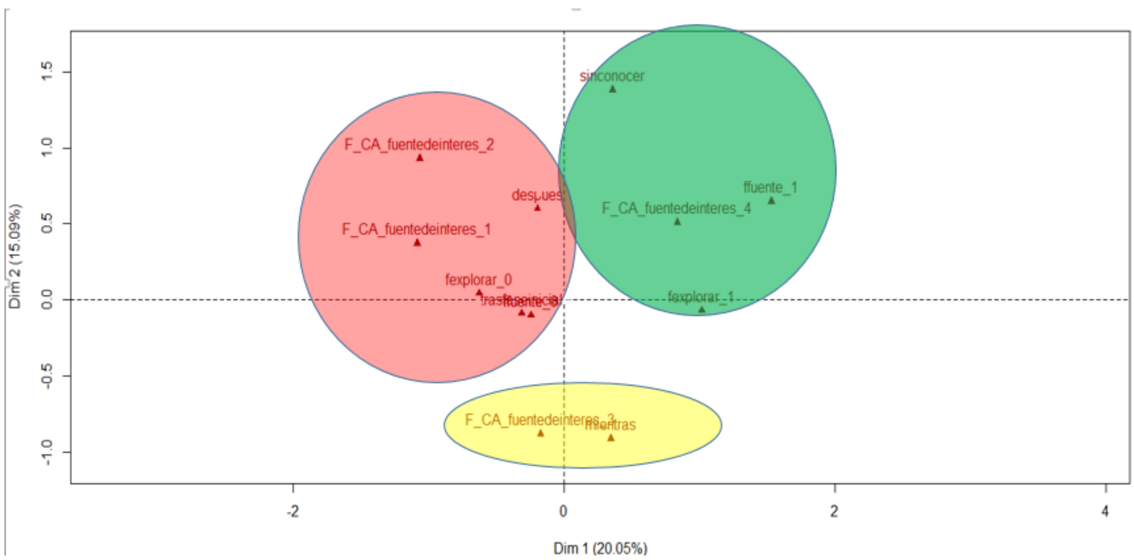


Figure 49: Multiple correspondence analysis (questions 4, 5 and 6)

There are three groups of participants: those who view the calculator as "a source of exercises" tend to agree with it "being used without necessarily having learnt the algorithms" and with using the calculator "to explore the numbers". A second group does not consider the calculator as a "source of exercises" and, at the same time, thinks that it should be used after the algorithms have been mastered. A third group views the calculator as a "source of exercises", but tends to think that it should be used while the algorithms are being learnt.

We make a new analysis by combining the items in pairs (questions 4, 5 and 6):

```
                   ffuente
F_CA_fuentedeinteres     0    1 Total Count
                   1 100.0  0.0   100     5
                   2  92.3  7.7   100    39
                   3  90.7  9.3   100    86
                   4  77.0 23.0   100    74
               Total 86.3 13.7 100.0

        Pearson's Chi-squared test

data:  .Table
X-squared = 8.7591, df = 3, p-value = 0.03267
```

```
                   fexplorar
F_CA_fuentedeinteres    0    1 Total Count
                   1 80.0 20.0   100     5
                   2 89.7 10.3   100    39
                   3 65.1 34.9   100    86
                   4 41.9 58.1   100    74
               Total 61.8 38.2 100.0

        Pearson's Chi-squared test

data:  .Table
X-squared = 26.416, df = 3, p-value = 0.000007805
```

calculator can be used to provide children with learning situations & useful as a source of exercises	calculator can be used to provide children with learning situations & useful to explore properties

```
        fexplorar
ffuente     0    1 Total Count
      0 67.6 32.4   100   176
      1 25.0 75.0   100    28
  Total 61.8 38.2 100.0

        Pearson's Chi-squared test

data:  .Table
X-squared = 18.575, df = 1, p-value = 0.00001633
```

source of exercises & useful to explore properties

```
                    cal_cuando
F_CA_fuentedeinteres despues mientras sinconocer trasfaseinicial Total Count
                   1    60.0     20.0        0.0            20.0 100.0     5
                   2    56.4     20.5        5.1            17.9  99.9    39
                   3    37.2     43.0        2.3            17.4  99.9    86
                   4    44.6     33.8        8.1            13.5 100.0    74
               Total    44.1     34.8        4.9            16.2 100.0

        Pearson's Chi-squared test

data:  .Table
X-squared = 10.347, df = 9, p-value = 0.3232
```

calculator can be used to provide children with learning situations & point in time prefered for using the calculator at school

Figure 50: Relationships between certain variables referred to the use of the calculator

There is independence between "viewing the calculator as a source of exercises" and which point in time (before, at the same time or after learning algorithms) is preferred for using the calculator at school. In addition, unsurprisingly, those who consider the calculator as "a source of exercises" tend to also see it as a tool "to explore numbers".

```
                         F_T_ESTRATEGIA[Grupo == "UB"]
F_T_RAZONAMIENTO[Grupo == "UB"]   1    2    3     4 Total Coun
                               1 0.0  0.0  0.0 100.0 100.0
                               2 0.0 25.0 50.0  25.0 100.0
                               3 0.0  0.0 57.9  42.1 100.0
                               4 3.4  3.4 13.8  79.3  99.9
                           Total 1.9 3.8 32.1 62.3 100.0

        Pearson's  Chi-squared test

data:  .Table
X-squared = 17.773, df = 9, p-value = 0.0379
```

```
                         F_T_INICIATIVA[Grupo == "UB"]
F_T_RAZONAMIENTO[Grupo == "UB"]   1    2    3    4 Total Cou
                               1 0.0  0.0 100.0  0.0   100
                               2 0.0 50.0  25.0 25.0   100
                               3 5.3 36.8  47.4 10.5   100
                               4 3.4 27.6  48.3 20.7   100
                           Total 3.8 32.1 47.2 17.0 100.0

        Pearson's Chi-squared test

data:  .Table
X-squared = 3.3853, df = 9, p-value = 0.947
```

```
                          F_T_RAPIDEZ[Grupo == "UB"
F_T_AUTOMATISMO[Grupo == "UB"]   1   2    3    4 Total
                              1 NaN NaN  NaN  NaN   NaN
                              2 0.0   0 50.0 50.0   100
                              3 9.1   0 36.4 54.5   100
                              4 5.1   0 23.1 71.8   100
                          Total 5.8 0 26.9 67.3 100.0

        Pearson's Chi-squared test

data:  .Table
X-squared = NaN, df = 9, p-value = NA
```

```
                         F_T_ADIESTRAMIENTO[Grupo ==
F_T_AUTOMATISMO[Grupo == "UB"]   1   2   3    4 Total Co
                              1 NaN NaN  NaN  NaN   NaN
                              2   0 0.0 50.0 50.0   100
                              3   0 9.1 27.3 63.6   100
                              4   0 0.0 17.9 82.1   100
                          Total 0 1.9 21.2 76.9 100.0

        Pearson's Chi-squared test

data:  .Table
X-squared = NaN, df = 9, p-value = NA
```

```
                         fconstruirYreforzar[Grupo ==
F_T_INICIATIVA[Grupo == "UB"]    0     1 Total Count
                            1  0.0 100.0   100     2
                            2 23.5  76.5   100    17
                            3 20.0  80.0   100    25
                            4 33.3  66.7   100     9
                        Total 22.6 77.4 100.0

        Pearson's Chi-squared test

data:  .Table
X-squared = 1.28, df = 3, p-value = 0.7339
```

```
                         F_T_JUEGO[Grupo == "UB"]
F_T_REPULSION[Grupo == "UB"]   1     2    3    4 Total Cou
                            1  0   5.6 30.6 63.9 100.1
                            2  0   9.1 36.4 54.5 100.0
                            3 20  20.0 40.0 20.0 100.0
                            4  0 100.0  0.0  0.0 100.0
                        Total 1.9 9.4 32.1 56.6 100.0

        Pearson's Chi-squared test

data:  .Table
X-squared = 22.064, df = 9, p-value = 0.008677
```

```
                         F_T_RAZONAMIENTO[Grupo == "UB"]
F_T_AUTOMATISMO[Grupo == "UB"]   1   2     3    4 Total Count
                              1 NaN NaN   NaN  NaN   NaN     0
                              2 0.0 0.0 100.0  0.0   100     2
                              3 0.0 9.1  27.3 63.6   100    11
                              4 2.6 7.7  35.9 53.8   100    39
                          Total 1.9 7.7 36.5 53.8 100.0

        Pearson's Chi-squared test

data:  .Table
X-squared = NaN, df = 9, p-value = NA
```

```
                         fdesarr_raznto[Grupo == "UB"]
F_T_INICIATIVA[Grupo == "UB"]    0    1 Total Count
                            1 50.0 50.0   100     2
                            2 47.1 52.9   100    17
                            3 52.0 48.0   100    25
                            4 33.3 66.7   100     9
                        Total 47.2 52.8 100.0

        Pearson's Chi-squared test

data:  .Table
X-squared = 0.932, df = 3, p-value = 0.8177
```

```
                         fidentificarYmanera[Grupo == "UB"]
F_T_INICIATIVA[Grupo == "UB"]     0    1 Total Count
                            1 100.0  0.0   100     2
                            2  52.9 47.1   100    17
                            3  28.0 72.0   100    25
                            4  22.2 77.8   100     9
                        Total 37.7 62.3 100.0

        Pearson's Chi-squared test

data:  .Table
X-squared = 6.9032, df = 3, p-value = 0.07505
```

```
                         F_CM_procdmtovsrapidz[Grupo =
F_T_INICIATIVA[Grupo == "UB"]    1    2    3    4 Total C
                            1  0.0  0.0 50.0 50.0   100
                            2  5.9  5.9 64.7 23.5   100
                            3 12.0 16.0 52.0 20.0   100
                            4  0.0 11.1 11.1 77.8   100

Total 7.5 11.3 49.1 32.1 100.0

        Pearson's Chi-squared test

data:  .Table
X-squared = 13.823, df = 9, p-value = 0.1288
```

```
                         fconstruirYreforzar[Grupo == "UB"
F_T_RAZONAMIENTO[Grupo == "UB"]    0     1 Total Count
                              1  0.0 100.0   100     1
                              2 25.0  75.0   100     4
                              3 21.1  78.9   100    19
                              4 24.1  75.9   100    29
                          Total 22.6 77.4 100.0

        Pearson's Chi-squared test

data:  .Table
X-squared = 0.36985, df = 3, p-value = 0.9464
```

```
                         fdesarr_raznto[Grupo == "UB"]
F_T_RAZONAMIENTO[Grupo == "UB"]     0    1 Total Count
                              1 100.0  0.0   100     1
                              2  75.0 25.0   100     4
                              3  42.1 57.9   100    19
                              4  44.8 55.2   100    29
                          Total 47.2 52.8 100.0

        Pearson's Chi-squared test

data:  .Table
X-squared = 2.6226, df = 3, p-value = 0.4535
```

```
                         fidentificarYmanera[Grupo == "UB"
F_T_RAZONAMIENTO[Grupo == "UB"]     0     1 Total Count
                              1 100.0   0.0   100     1
                              2   0.0 100.0   100     4
                              3  36.8  63.2   100    19
                              4  41.4  58.6   100    29
                          Total 37.7 62.3 100.0

        Pearson's Chi-squared test

data:  .Table
X-squared = 4.2445, df = 3, p-value = 0.2362
```

```
                         F_CM_procdmtovsrapidz[Grupo =
F_T_RAZONAMIENTO[Grupo == "UB"]    1    2     3    4 Total C
                              1  0.0  0.0 100.0  0.0 100.0
                              2  0.0  0.0  25.0 75.0 100.0
                              3 10.5 10.5  68.4 10.5  99.9
                              4  6.9 13.8  37.9 41.4 100.0
                          Total 7.5 11.3 49.1 32.1 100

        Pearson's Chi-squared test

data:  .Table
X-squared = 10.687, df = 9, p-value = 0.2978
```

```
          fconstruirYreforzar[Grupo == "UB"]
F_T_ESTRATEGIA[Grupo == "UB"]    0     1 Total Count
                            1  0.0 100.0   100     1
                            2 50.0  50.0   100     2
                            3 35.3  64.7   100    17
                            4 15.2  84.8   100    33
                        Total 22.6  77.4 100.0

        Pearson's Chi-squared test

data:  .Table
X-squared = 3.7581, df = 3, p-value = 0.2888
```

```
                  fdesarr_raznto[Grupo == "UB"]
F_T_ESTRATEGIA[Grupo == "UB"]     0    1 Total Co
                            1 100.0  0.0   100
                            2 100.0  0.0   100
                            3  35.3 64.7   100
                            4  48.5 51.5   100
                        Total  47.2 52.8 100.0

        Pearson's Chi-squared test

data:  .Table
X-squared = 4.345, df = 3, p-value = 0.2265
```

```
                  fidentificarmanera[Grupo == "UB"]
F_T_ESTRATEGIA[Grupo == "UB"]     0    1 Total Count
                            1 100.0  0.0   100     1
                            2  50.0 50.0   100     2
                            3  41.2 58.8   100    17
                            4  33.3 66.7   100    33
                        Total  37.7 62.3 100.0

        Pearson's Chi-squared test

data:  .Table
X-squared = 2.1359, df = 3, p-value = 0.5447
```

```
                        F_CM_procdmtovsrapidz[Grupo ==
F_T_ESTRATEGIA[Grupo == "UB"]    1     2    3     4 Total Co
                            1  0.0 100.0  0.0   0.0 100.0
                            2  0.0   0.0  0.0 100.0 100.0
                            3 11.8  11.8 47.1  29.4 100.1
                            4  6.1   9.1 54.5  30.3 100.0
                        Total  7.5  11.3 49.1  32.1 100.0

        Pearson's Chi-squared test

data:  .Table
X-squared = 13, df = 9, p-value = 0.1626
```

Figure 51: Relationships between some answers from the participants in UB referred to the use of the calculator

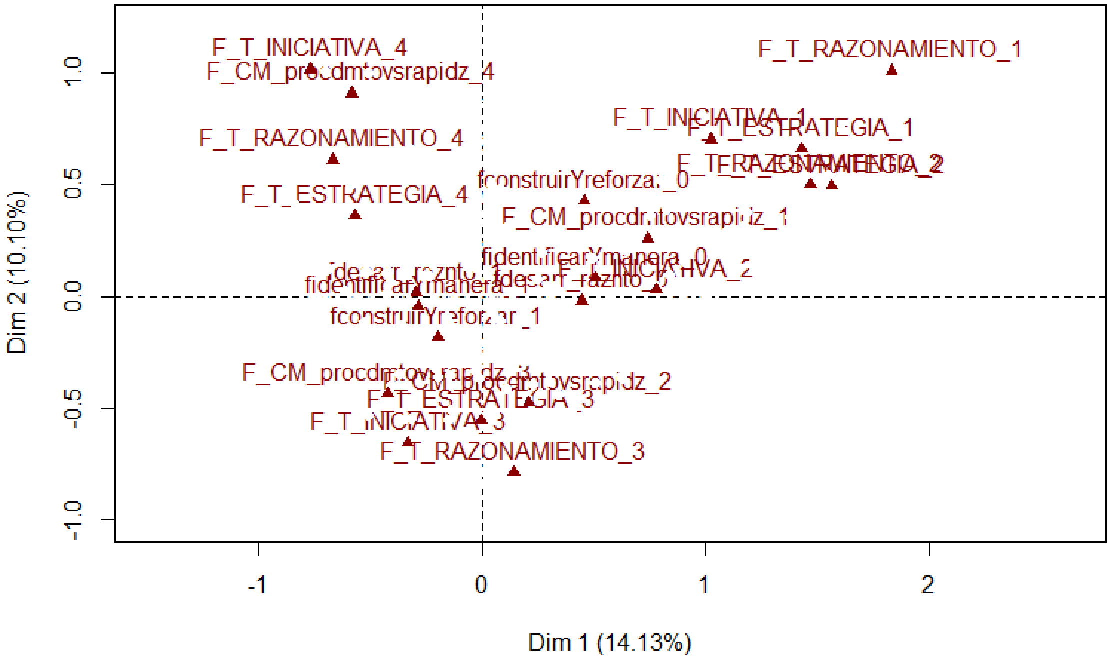


Figure 52: Relationships between some answers from the participants in UB referred to the use of the calculator

If we focus the analysis on the participants from UB, we observe that the more “reasoning” is valued, the more “strategy” is valued. And the same occurs with the terms “automatism” and “training”. In addition, the more “dislike” is valued, the less “play” is valued. There seems to be independence between the other pairs of terms studied.

5. Conclusions: observations, recommendations and proposals for improvement

The classical vision of mental computation, which focuses on speed and memorization, seems to be reflected in the answers of some participants. This is particularly notable in the case of the Spanish respondents, who are in favor of speed rather than procedure when working on mental computation at school and who also approve of the ritual verbal repetition of the addition and multiplication tables in an ascending order. These participants also show a marked “dislike” of mental computation.

A more modern vision of mental computation is reflected in a set of variables which are strongly correlated, namely the terms reasoning, strategy and initiative, and the belief that procedure should be prioritized over speed when children are working on mental computation. Both the French and Italian participants show this mindset, but there are differences between them. The French responses reflect this mindset together with a preference for speed and automatism, whereas the Italian preservice and in-service teachers show this modern vision of mental computation in conjunction with the notion of mental computation as play.

These visions of mental computation are coherent with the preferences which the participants express when certain operations have to be carried out. The Spanish participants tend to prefer the calculator over mental computation, while the other groups indicate a preference for mental computation.

While all the participants regard mental computation as useful, they have different perspectives of the aims of teaching it in primary school. The Spanish participants regard mental computation as an aid in solving problems, although they also consider some modern aims of mental computation, such as better knowing the properties of numbers and operations or developing reasoning and argumentation skills. For their part, the French participants place importance on the benefits of mental computation for evaluating the order of magnitude of a result, while the answers of the Italian participants regarding the aims of mental computation are more varied.

Despite these differences between the participants, there is a strong consensus against the prohibition of the use of fingers and also the use of a written medium in computations, which reflects a modern vision of teaching mental computation.

Regarding the use of the calculator in primary school, the Italian curriculum simply mentions the calculator as an alternative to mental computation, which reflects a classical vision of its use. The Spanish and French curricula, however, also regard the calculator as a tool to explore numbers and their properties, reflecting a more modern perspective which envisages the use of the calculator in combination with mental computation.

Analysing the answers which the participants have given to the questionnaire, the view which most frequently occurs is one which sees the calculator as a tool to check the result and solve problems in which a great amount of quantities is involved. A vision of the calculator as a source of mathematical problems and exercises or as a support to explore numbers is much less common.

There is a strong consensus on the belief that children use the calculator at home rather than at school. Moreover, most of the respondents believe the calculator will not teach children how to calculate. With the exception of the Italian participants, it is also believed that an over-reliance on the calculator indicates a lack of mathematical knowledge and hinders both thinking and the acquisition of mathematical skills. The above-mentioned curricula differences regarding the use of the calculator at school do not seem to be reflected in the participants' beliefs.

Regarding the point in time when the calculator should be introduced, there are three distinct viewpoints. Firstly, there are those who view the calculator as a source of exercises and tend to agree with it being used as an aid to explore numbers without necessarily having learnt the algorithms. A second group views the calculator as a source of exercises, but tends to think that it should be used while the algorithms are being learnt. Finally, a third group does not consider the calculator as a source of exercises and therefore thinks that it should not be used until after the algorithms have been mastered. This might mean that, while some participants can imagine a more creative use of the calculator at school, others regard the calculator as something used in a lazy way, to avoid either thinking or performing the algorithms.

This analysis has led us to design a single workshop devoted to mental computation and the use of the calculator. The activities will enable the participants to discover how to use both resources in primary school to explore numbers and their properties. Moreover, they will work on how to engage children in creative activities which allow the use of the calculator in combination with mental computation tasks.

Report of questionnaire Q5: History of Mathematics and its teaching

Table of contents

1. Abstract

This report describes the process followed in designing the *Questionnaire on History of Mathematics and its teaching*, which has already been delivered to both in-service teachers and students of the BA degree in Education of the institutions associated with the ANFoMAM Project (appendix P5). The analysis of the collected data shows clear differences between the participants of the Spanish and Italian institutions, both in terms of knowledge of History of Mathematics and of their beliefs about the nature of the mathematical discipline per se. At the end of the report, some amendments to the questionnaire are proposed for the future (appendix Q5[11]).

2. Questionnaire design: background and objectives

Mathematics has become a purely technical subject at school; its contents are presented as a set of automatic procedures (writing and reading numbers, solving vertical operations, etc.). Its language is reduced to a set of technical terms that serve essentially to classify, instead of facilitating the abstract-quantitative thinking typical of mathematics. Even solving problems is learnt in a procedural way, using methods that always provide the right solution, without time being invested in reflecting on why those methods always "work" or why they need to be learnt, nor is there an awaken of the desire to respond to the challenge being faced.

The abovementioned approach to teaching mathematics is justified by the supposed requirement of 'rigor', while the roots of the issue are completely neglected, that is, the anchoring of mathematical concepts and relationships in human perception, movement, intention, and action (humanising mathematics). In addition, teaching mathematics in a purely technical way entails transmitting the idea that mathematics constitutes a closed, already finished, existing body of knowledge in itself, without origin or evolution, something that we only have to learn and apply in different exercises and problems. Fostering an awareness that mathematics has evolved throughout history, and that it has not always been as we know it nowadays, is an excellent way of giving a human meaning to both the subject and its

[11] QuestionnaireQ5: https://docs.google.com/forms/d/1YGUvdI1SDH51RMJPk9PIBdflGl6kEjhiLCqHN54KWlQ/copy

teaching. Mathematics will become more meaningful to children the more clearly we show its relationship to humanity's concerns throughout history.

The ANFoMAM Project will design a workshop on the History of Mathematics and its teaching for teachers in both pre-service and in-service training programs. Prior to commencing the activities, participants may be invited to fill out a questionnaire that allows them to reflect on their convictions and knowledge about the evolution of mathematics throughout history, as well as the importance of incorporating the History of Mathematics in the teaching process.

A priori we expect to find different participant profiles:

- Pre-service and in-service teachers who see mathematics as a closed and finished body of knowledge which the teacher conveys in the same way s/he received it, without there being much room for personal creativity. For these participants, the History of Mathematics will be, at most, just one more skill to be trained, a mere tale of successful discoveries and famous characters.

- Participants with a dynamic idea of mathematics who see the discipline as a subject that has evolved throughout history, linked to the ways of understanding the life and concerns of different civilizations. This vision could be linked to a greater knowledge of the History of Mathematics and would also lead to a more active way of teaching mathematics, a teaching style that involves students in the activities in a way that the students themselves, with the help of the teacher, make their own "mathematical discoveries".

3. Data collection

The questionnaire has been delivered to the following participants:

- 38 undergraduate students in both Infant and Primary Education at the Public University of Navarre.
- 21 undergraduate students (from third to fifth year) in Scienze della Formazione Primaria la Universitá Roma Tre, who were enrolled in the course of "Matematica e didattica della matemática". In this course the History of Mathematics is present, as well as the history of elementary and particularly infant mathematical education.
- 19 Italian infant and primary school teachers who have come into contact with the project through the website and social channels of the ToKalon Association.

4. Data management and analysis

The collected data have been processed in the following way:

1. For all questions requiring two or three words, the following simplifications have been made:
   - Words in their plural and singular forms: one of the two forms has been chosen (for example, instead of distinguishing between *calcolo* and *calcoli,* they have been unified in the *calcolo* form, or *cálculo* in Spanish).

- Words present as a noun or as a verb: one of the two forms has been chosen (for example, the words *conteggio* and *contare* have been unified in the *contare* form, or *contar* in Spanish).

- Words present as a noun or as an adjective: one of the two forms has been chosen (for example, in the case *of creativo* and creatività, creatività has been chosen, or *creatividad* in Spanish).

- Some groups of words referring to a single recognisable concept have been unified in a single form, preferably the one most cited (e.g. for *trigonometria, seno* and *coseno* and *funzioni goniometriche,* the word *trigonometria* has been chosen).

2. Numbers 1 to 4 are related to the following qualitative expressions:
   1= Nothing at all; 2 = Little; 3 = Quite a lot; 4 = A lot

3. The analysis distributes the data by separating Italian questionnaires (also distinguishing between students and teachers) from those from Spain, in order to compare them.

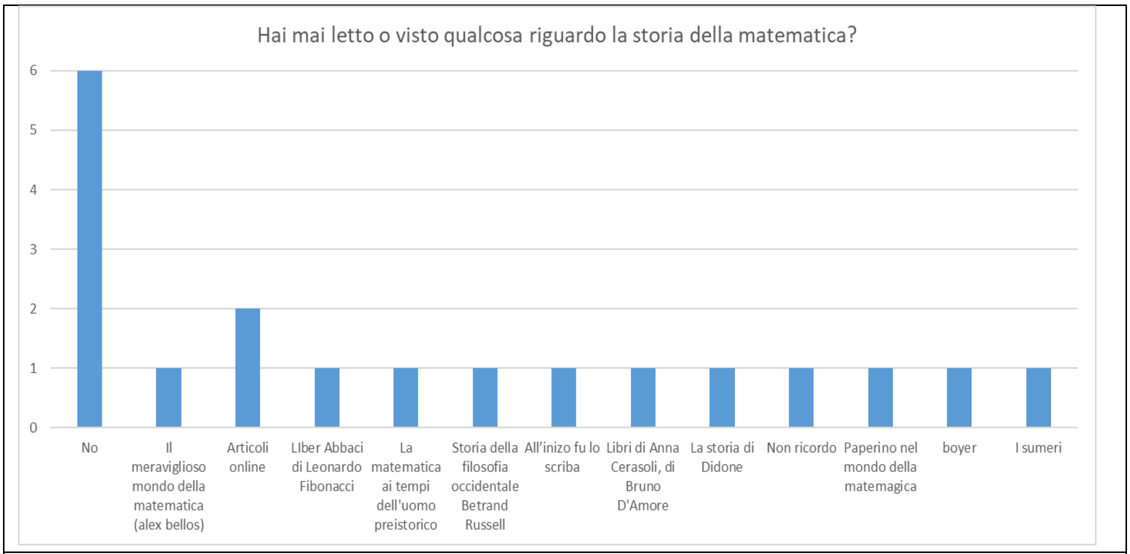
   a) The data charts related to the Italian participants sometimes bring together students and teachers; at other times, students and teachers are dealt with separately.
   b) The data have been prepared graphically. After that, the researchers have carried out a qualitative analysis based on a collective discussion of the different perspectives.

We analyse the answers obtained question by question:

4.1. Question 1: Book about history of mathematics

Have you ever read or watched anything about the history of mathematics? If so, which one?

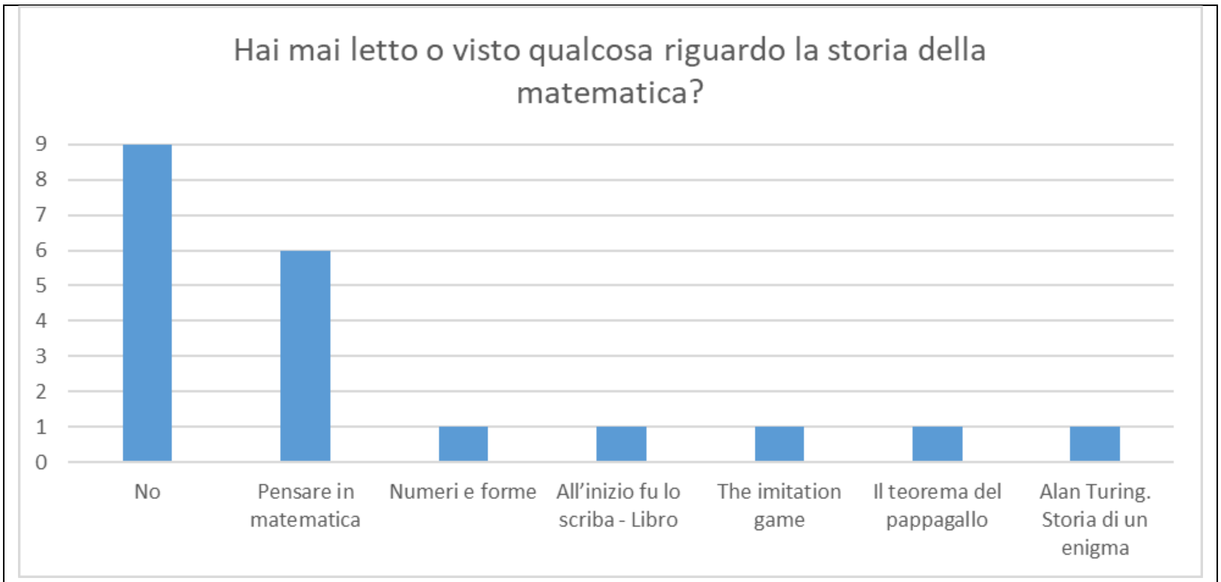
In the case of the Italian teachers, 32% of participants claim to have never read anything related to the History of Mathematics. The books mentioned by the rest of the teachers are listed in the following graph. Some of them are books intended for a young audience, such as those by Anna Cerasoli or Alex Bellos. Others are more systematic treatises, such as Carl B. Boyer's book *History of Mathematics* or Bertrand Russell's *History of Philosophy*.



Have you ever read or watched anything about the History of Mathematics?

Figure 53: Books read by the Italian teachers

In the case of the Italian students, the percentage of participants who have not had contact with the History of Mathematics was 43%, slightly higher than in the case of the teachers. Among the books mentioned, there are several written by the researcher Ana María Millán Gasca, such as *Pensare in matematica, Numeri e forme* and *All'inizio fu lo scriba,* together with novels related in some way to mathematics.



Have you ever read or watched anything about the History of Mathematics?

Figure 54: Books read by the Italian university students

Figure 55 shows the marked contrast between the answers given by the Spanish and the Italian students to this question. Only one of the Spanish students had read a book related to the History of Mathematics, *Mr. Cuadrado*, by the Italian author Anna Cerasoli.

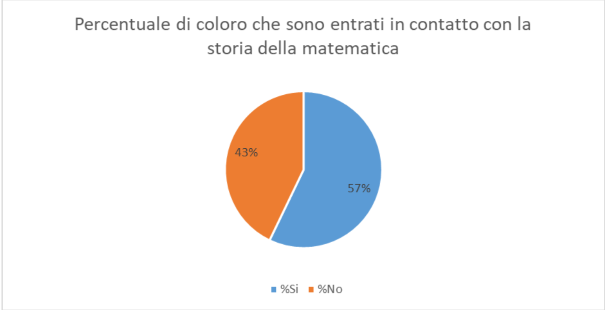
	
Percentage of Italian students who have come into contact with the history of mathematics (57%)	Percentage of Spanish students who have come into contact with the history of mathematics (2.6 %)

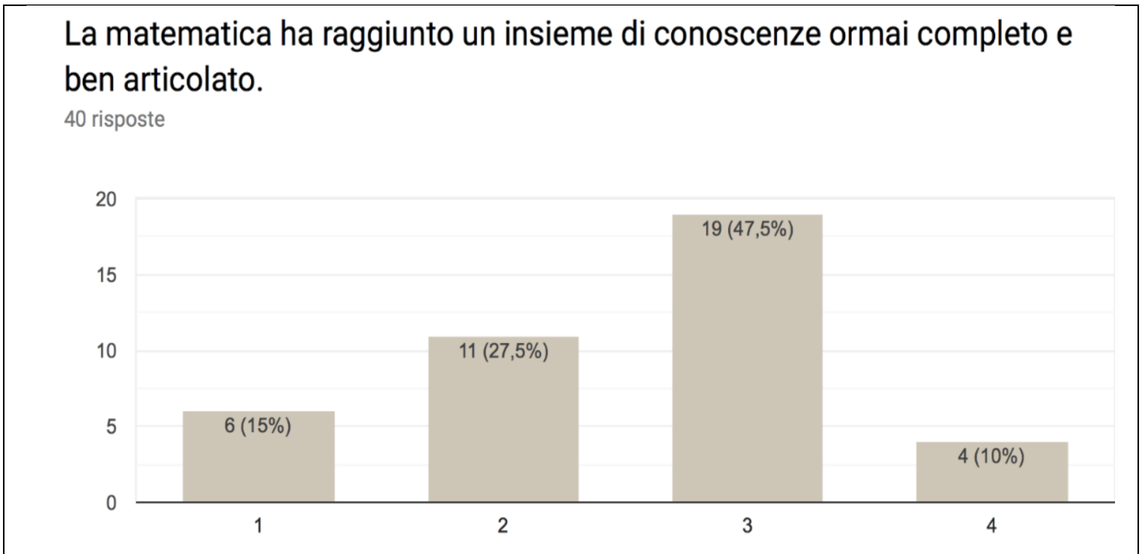
Figure 55: Answers to Question 1 (Italian students vs. Spanish students)

4.2. Question 2:

Assign a value from 1 to 4 to indicate your degree of agreement with the following assertion:

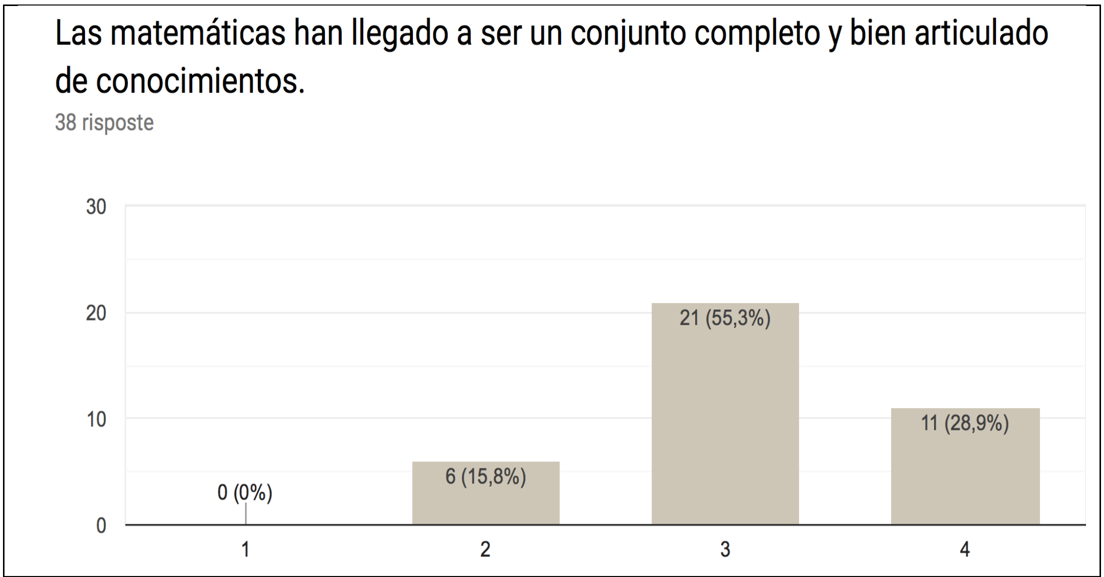
Mathematics has become a complete well-structured body of knowledge

As the two following graphs depict, a large majority of Spanish participants (84.2%) agreed strongly or very strongly with this statement, whereas, in the case of the Italian participants, the percentage is 57.5%. Relating these answers to those given to Question 1, it would appear that there is a close relationship between a lack of knowledge about the History of Mathematics and a more rigid conception of Mathematics per se.



Mathematics has become a complete well-structured body of knowledge

Figure 56: Answers from Italian teachers and students to Question 2



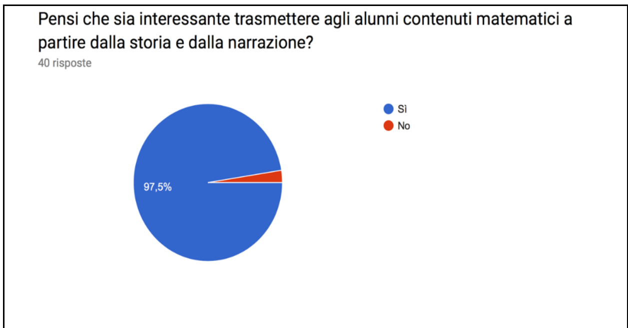
Mathematics has become a complete well-structured body of knowledge

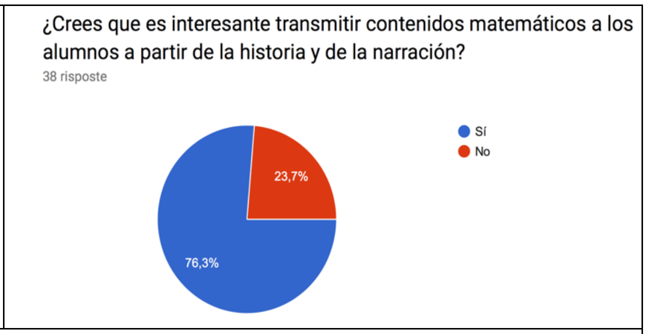
Figure 57: Answers from the Spanish students to Question 2

4.3. Question 3: With regard to History…

Do you find it appropriate to use history and stories as a starting point for transmitting mathematical contents to pupils?

When studying the data provided by the Italian participants (97.5% answered in the affirmative), the question might be considered redundant. However, for the Spanish students, the issue is not so clear. There are 23.7% of Spanish university students who do not find it *appropriate* to transmit mathematical contents to students through history and storytelling.





Do you find it appropriate to use history and stories as a starting point for transmitting mathematical contents to pupils?

Answers from the Italian participants to Question 3 (97.5%)	Answers from the Spanish participants to Question 3 (76.3%)

Figure 58: Answers from the Italian participants vs. the Spanish ones to Question 3

4.4. Question 4: Systems of numeration different from ours

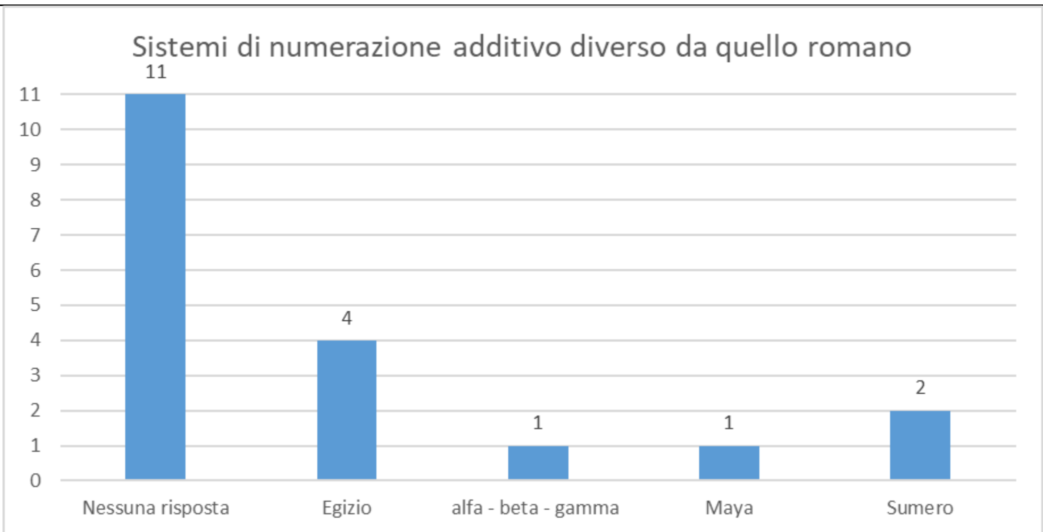
Are there systems of numeration that are different from our decimal and positional system?

The unanimity that exists in the responses, both among Italian participants (97.5%) and Spanish participants (97.4%), makes us wonder whether this question should continue to be put in the questionnaire, at least in the way it is written.

4.5. Question 5:

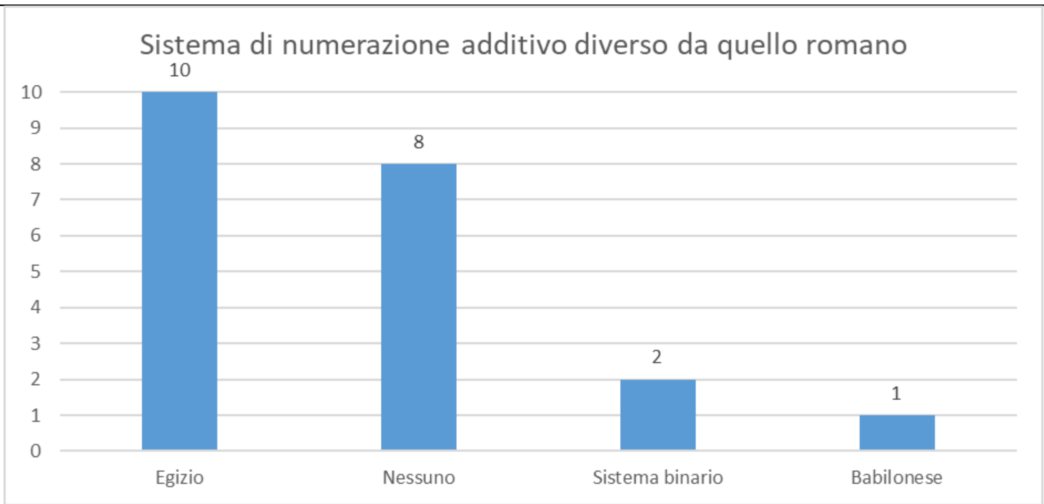
Do you know any additive numerical system apart from the Roman one? If so, which one?

It is striking that a relatively high percentage of teachers, 58%, do not know any additive system of numeration different from the Roman one. Participants who do indicate one in the main refer to the Egyptian system, although the Mayan and Sumerian systems are also cited. The Italian university students cite the same examples, although there are almost 40% of Italian students who do not know any additive system different from the Roman one. The percentage is higher in the case of the Spanish participants, 80% of participants claiming not to know any additive system different from the Roman one. Moreover, the systems named by the remaining 20%, with the exception of the Egyptian one, are not additive systems.



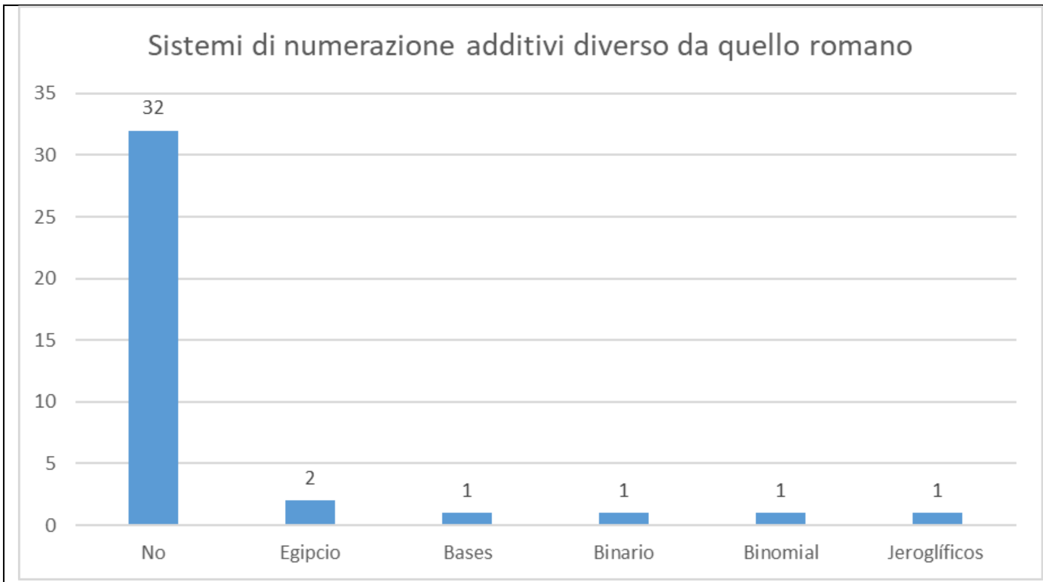
Additive systems of numeration different from the Roman one

Figure 59: Answers from the Italian teachers to Question 5



Additive systems of numeration different from the Roman one

Figure 60: Answers from the Italian students to Question 5



Additive systems of numeration different from the Roman one

Figure 61: Answers from the Spanish students to Question 5

4.6. Question 6:

We all know famous Greek mathematicians: Pythagoras, Euclid, Archimedes, and Tales. Do you know any other ancient mathematician? If so, write down who s/he is:

Given that each participant could write three names, there could have appeared a high number of different mathematicians. However, several participants left the answer blank or indicated fewer than three names of mathematicians. Therefore, we have decided to represent in the following histogram how many participants mentioned none, one, two or three mathematicians.

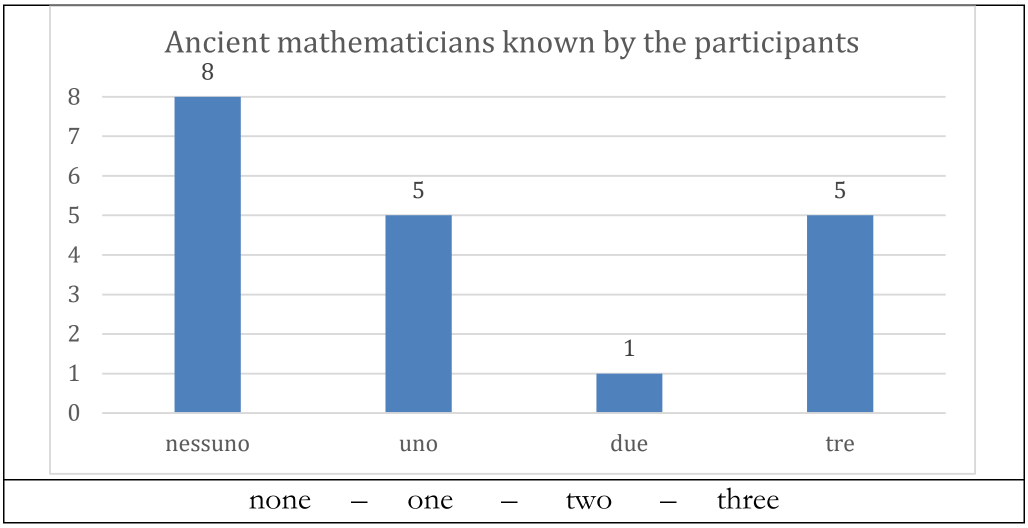


Figure 62: Number of ancient mathematicians given by each Italian teacher in response to Question 6

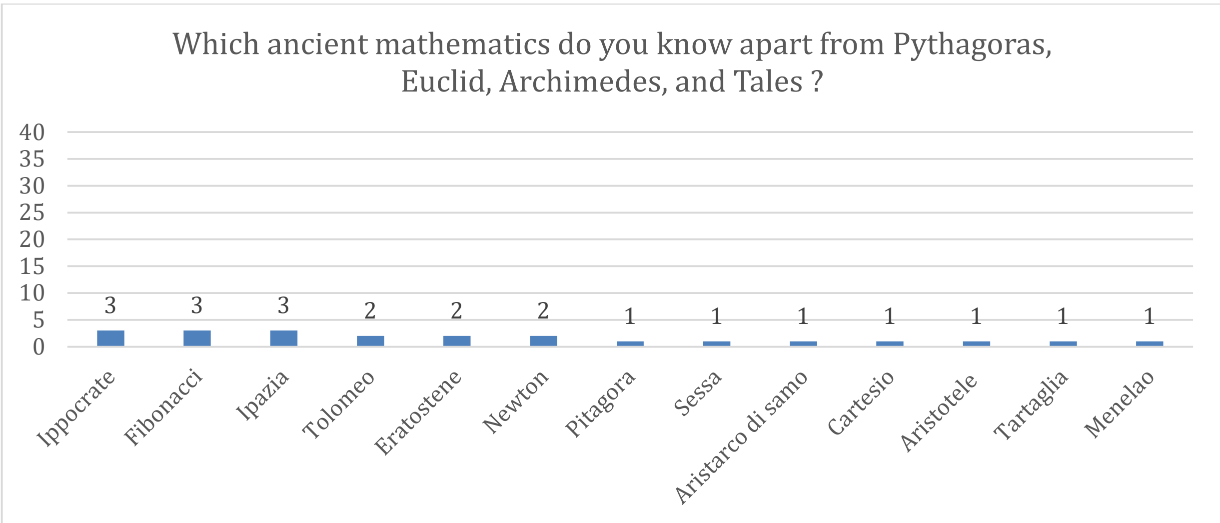


Figure 63: Names of the ancient mathematicians given by the Italian teachers in response to Question 6

It is remarkable that 8 of the 19 in-service teachers were not able to indicate the name of any ancient mathematician apart from Pythagoras, Euclid, Archimedes, or Tales.

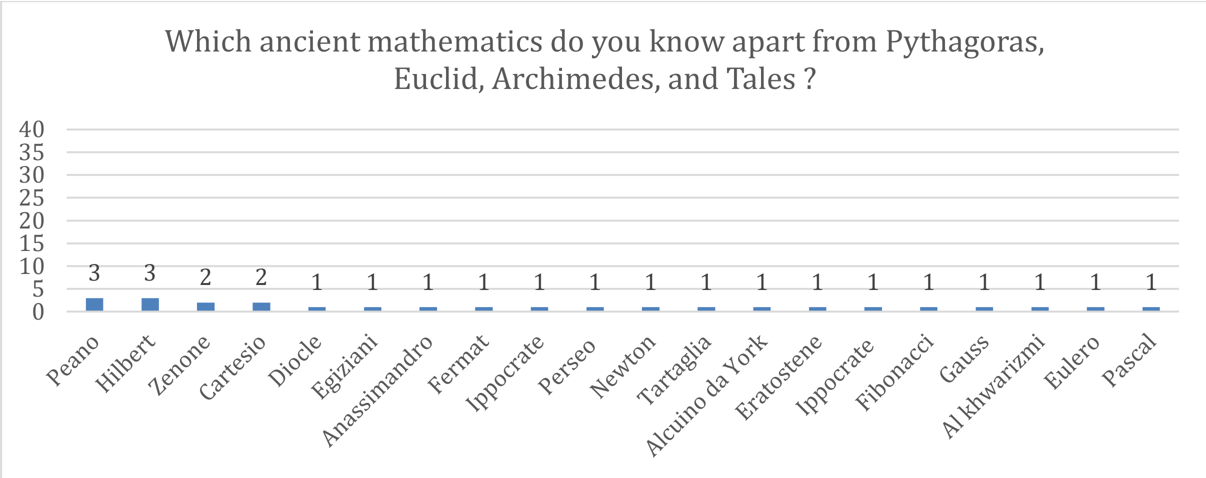
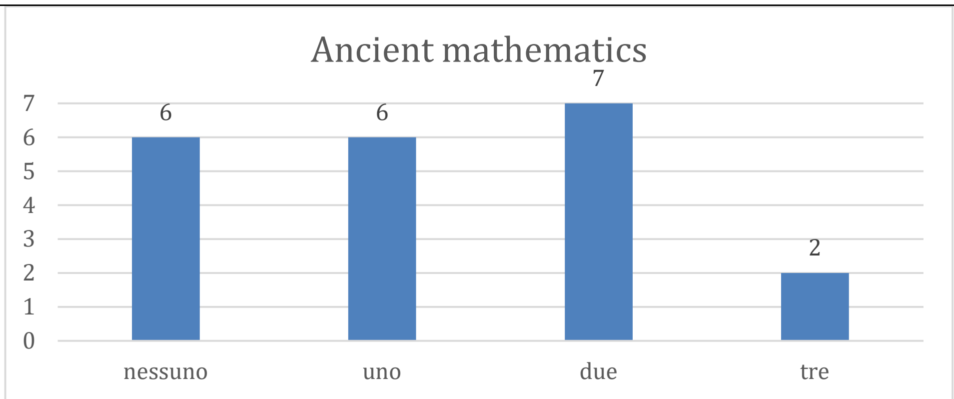


Figure 64: Answers from the Italian students to Question 6



none – one – two – three

Figure 65: Answers from the Italian students to Question 6

Nearly a third of the 21 Italian students were not able to indicate the name of any ancient mathematician apart from the ones given in the question. Of the names they wrote down, as seen in Figure 64, some are repeated and others do not belong to Antiquity.

In the Spanish version of the questionnaire, it was not specified that the mathematicians they had to list should exclude Pythagoras, Euclid, Tales or Archimedes. For that reason, all the Spanish participants gave at least two names. Pythagoras and Tales were the most mentioned, along with other 'intruders' who did not live in Antiquity.

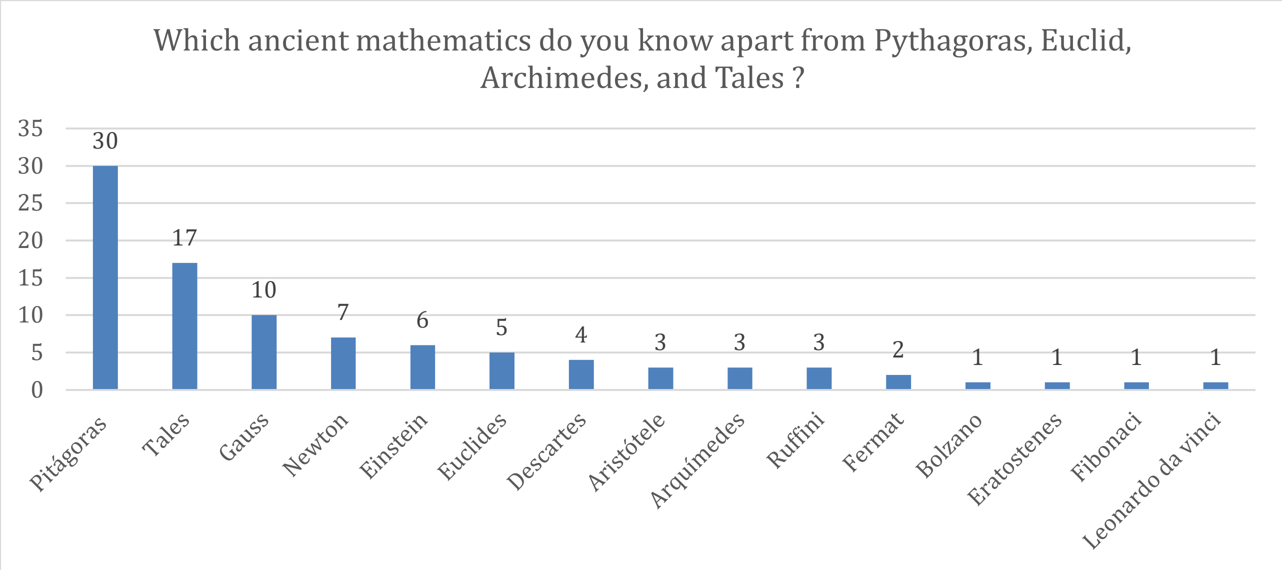


Figure 66: Answers from the Spanish students to Question 6

4.7. Question 7:

Write down the names of three non-Italian mathematicians who lived after the Middle Ages

Nearly half of the teachers were not able to name a single non-Italian mathematician who lived after the Middle Ages.

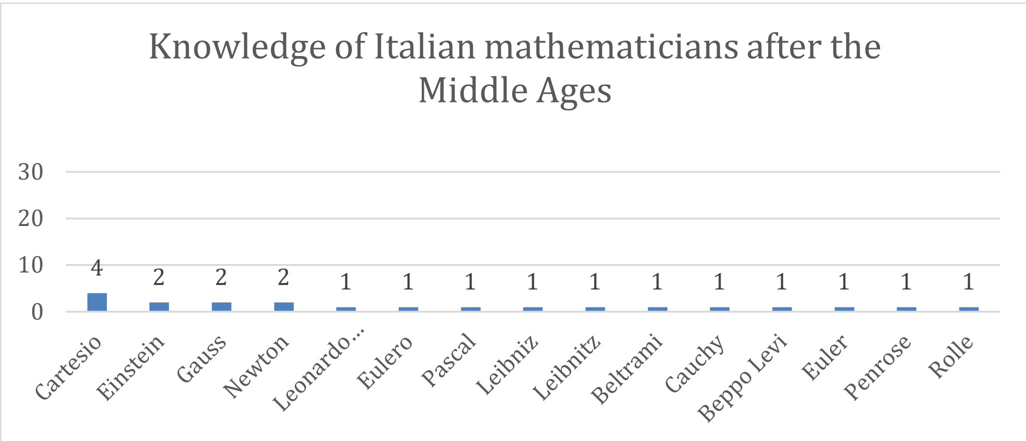


Figure 67: Answers from the Italian teachers to Question 7

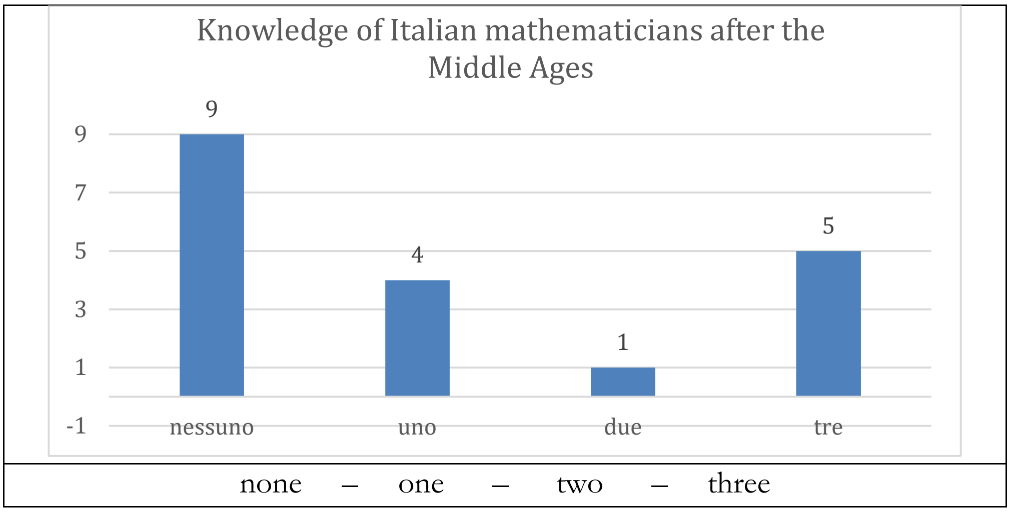


Figure 68: Answers from the Italian teachers to Question 7

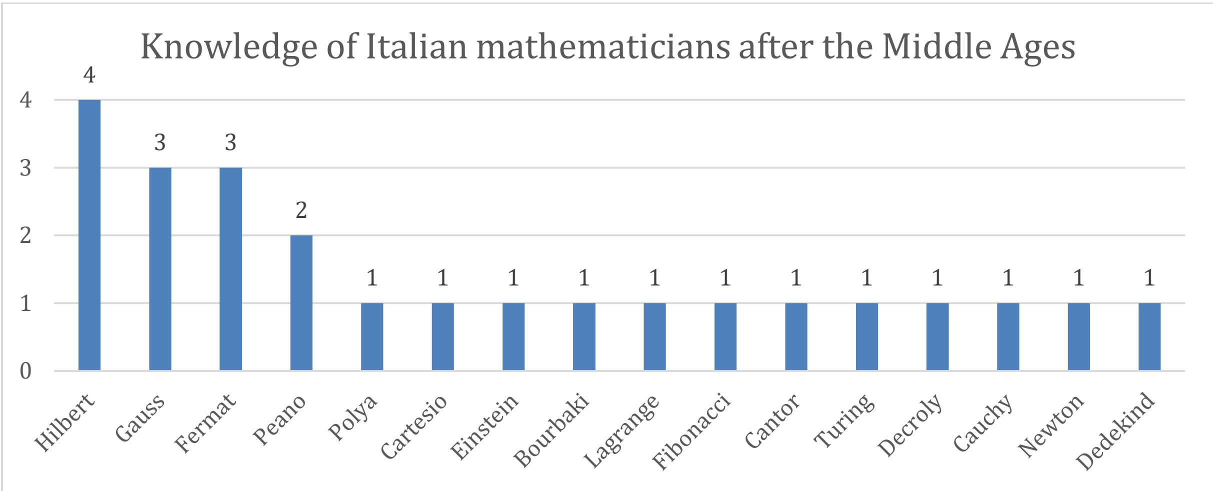


Figure 69: Answers from the Italian students to Question 7

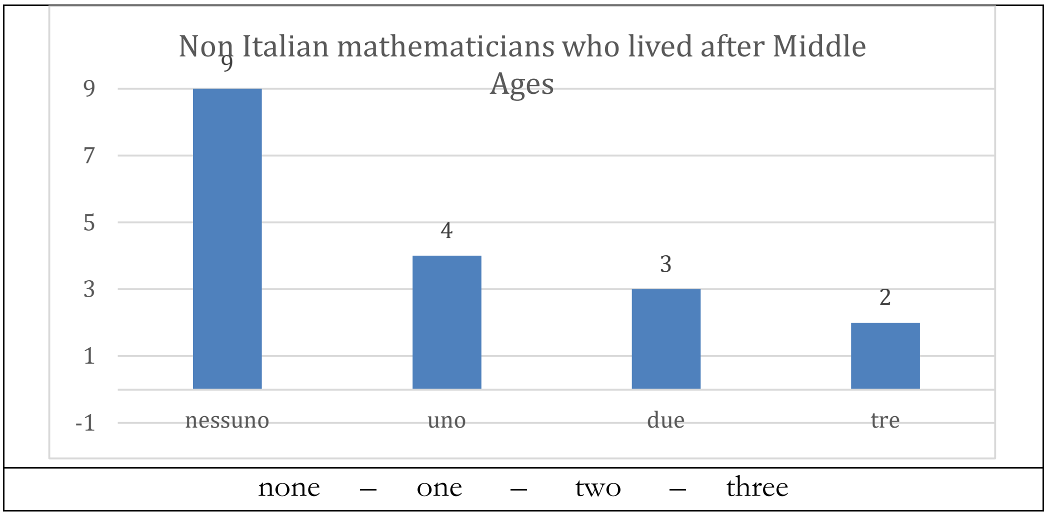


Figure 70: Answers from the Italian students to Question 7

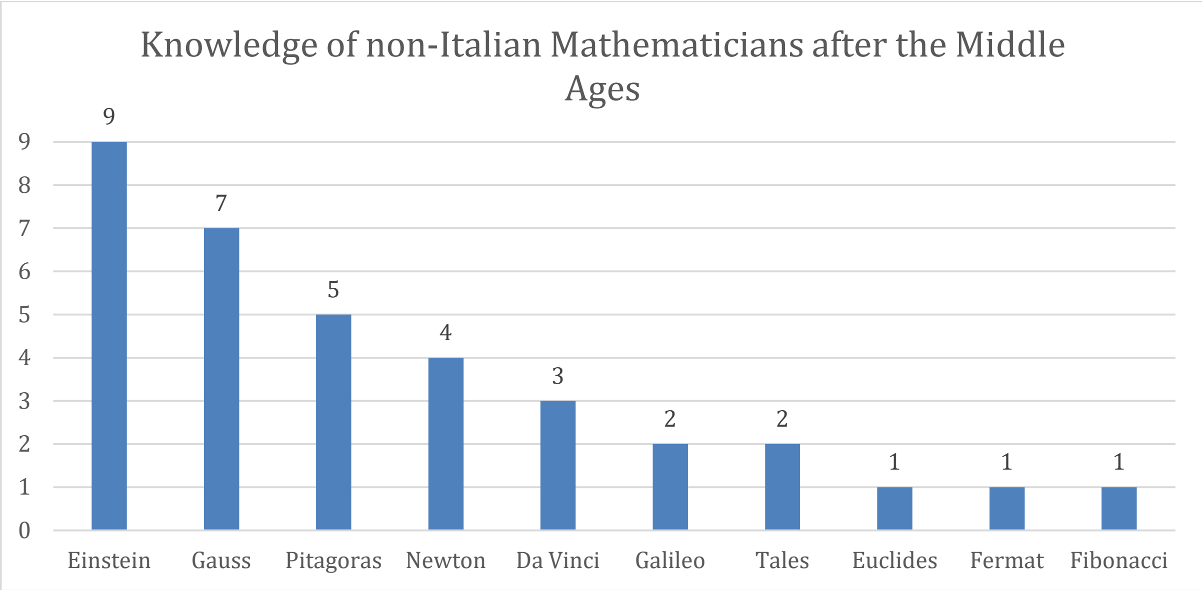


Figure 71: Answers from the Spanish students to Question 7

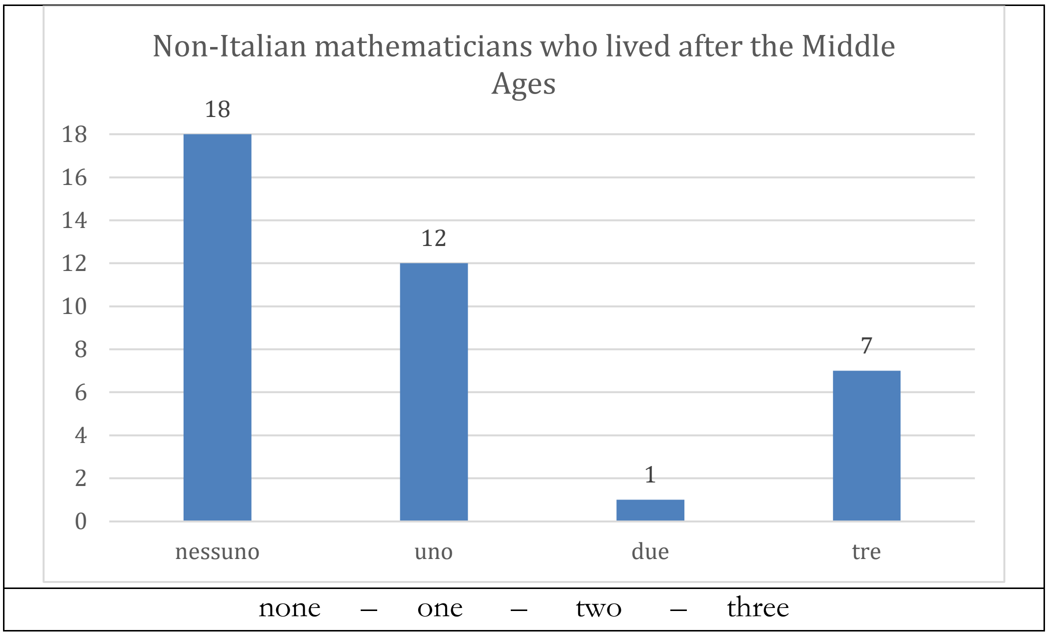
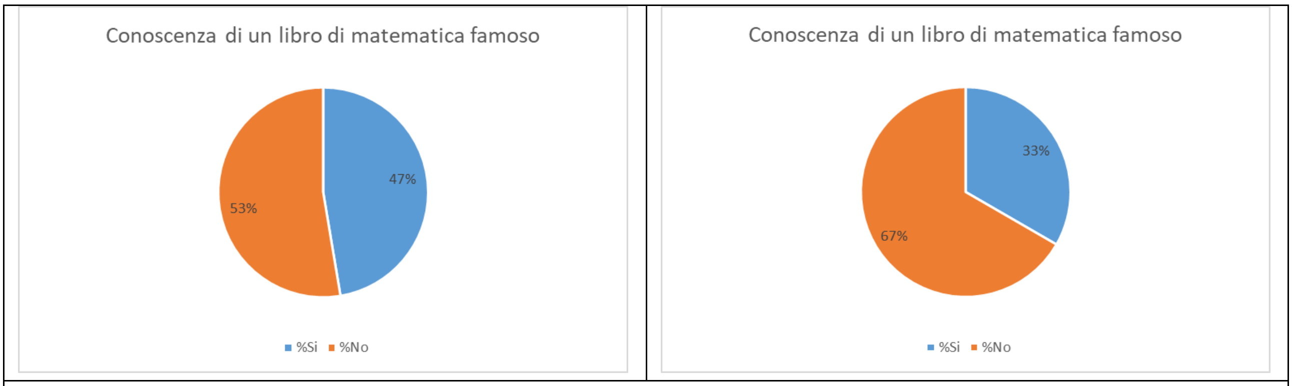


Figure 72: Answers from the Spanish students to Question 7

4.8. Question 8: *If you know any famous mathematics books, write them here*

Only 47% of teachers know a famous mathematics book. Some of the books mentioned coincide with those quoted in Question 1 as books about the History of Mathematics, such as those by Anna Cerasoli or Bruno D'Amore. Interestingly, they cite in this section the book of Chiara Valerio, *Human History of Mathematics,* the book *Didactics of Mathematics* by Emma Castelnuovo, the famous book by Fibonacci, *Liber Abbaci,* and a problem-solving classic book, *How to solve it* by G. Polya.

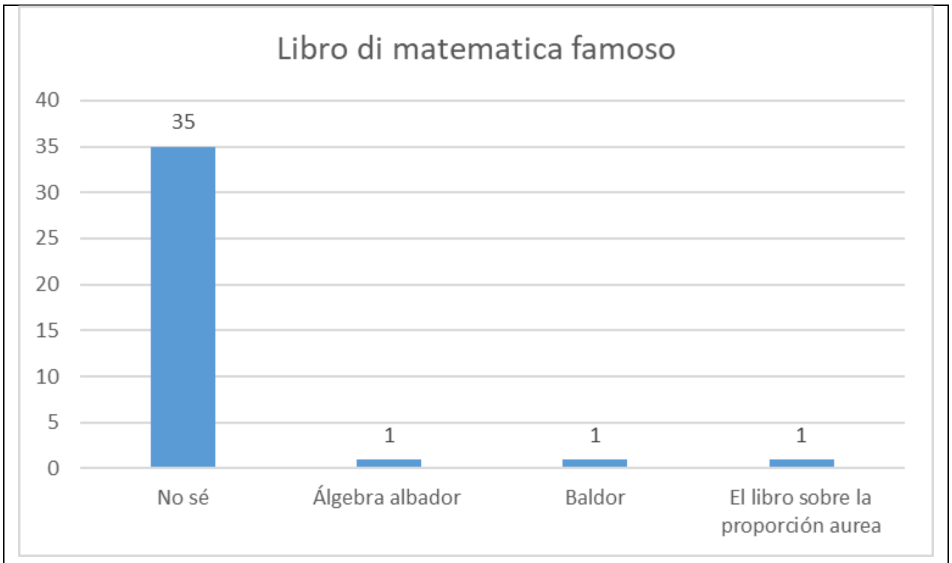
The percentage of those who know a famous book of mathematics drops to 33% if we have a look at the Italian students, who mention *Elements,* in addition to other much lighter books such as *The Wizard of Numbers* or *The Parrot´s Theorem.* The book *Pensare in matematica,* by Giorgio Israel and Ana Millán Gasca, reappears on this list.



Acquaintance with a famous mathematics book

Figure 73: Italian teachers´ vs. Italian students´ answers to Question 8

In the case of the Spanish students, the lack of knowledge is very striking, as shown in Figure 74. No famous mathematics book is known apart from three titles (the name "Algebra Albador" surely refers to the same book as "Baldor"), whereas, in the case of the Italian participants, almost half of the teachers and a third of the students had been able to mention a famous mathematics book.



Knowledge of any famous book of mathematics

I don't know – Algebra albador – Baldor – The book about golden ratio

Figure 74: Answers from the Spanish students to Question 8

4.9. Question 9:

In mathematics at school, the text book is: (maximum two answers)

- *A starting point*
- *All you need*
- *A guide*
- *A burden*

	No. answers
A starting point	10
A guide	4
A starting point and a guide	2
A burden	4

Answers from Italian teachers to Question 9

	No. answers
A starting point	5
A guide	12
A starting point and a guide	2
A burden	2

Answers from Italian students to Question 9

	No. answers
A starting point	2
A guide	17
A starting point and a guide	13
A burden	1
All you need	2
All you need and a guide	1
All you need and a burden	2

Answers from Spanish students to Question 9

Figure 75: Answers to question 9

The difference between the answers of the Italian teachers and the students of the same country is striking. Whereas the former in the main consider the book as a starting point, the latter see it more as a guide for math teaching at school. The Spanish participants also consider the textbook as a guide, although a third of the participants see it as both a guide and a starting point.

5. Conclusions: reporting, recommendations and proposals for improvement

When analysing the answers given to the questionnaire, there is a clear difference between the Spanish and the Italian participants. The latter show a greater appreciation of the use of both the History of Mathematics and stories for the transmission of mathematical contents, which is related to the fact that some of them have certainly read some mathematics books and know more about the History of Mathematics and its main protagonists.

In the case of the Spanish participants, the use of the History of Mathematics and stories in teaching is less valued, probably because these resources have not been used in their training. This could also be the cause of Spanish students showing a more rigid view of mathematics, conceived as a complete, well-structured body of knowledge, a vision not generally shared by the Italian participants.

The analysis of the results leads us to recognize the importance of designing a workshop on the History of Mathematics and the teaching thereof which, instead of just providing a list of contents to be learnt, helps future teachers acquire a dynamic view of mathematics, which they will later transmit in schools.

We also propose some changes with regard to the wording of the questionnaire itself. In this respect, although the answer to Question 3 on the *appropriateness of transmitting mathematical contents to students through history and stories* might be obvious to the Italian participants, the fact that almost a quarter of the Spanish students do not consider this approach appropriate has let to us keeping the question as initially drafted.

On the other hand, in Question 4 the participants provided unanimous answers regarding the *systems of numeration different from our decimal and positional* system. The answer is obvious to all the participants given that this topic is included in the syllabi of both Spanish and Italian BA Degrees in Education. Therefore, in the latest version of the questionnaire, it is proposed to replace this with the following question (Appendix Q5):

> *4. Systems of numeration different from ours:*
>
> > *Write an example of a positional system of numeration that is different from the decimal one, if you know it*

Last but not least, Question 6 was worded differently in the two versions of the questionnaire (the Spanish one and the Italian one). The following wording is proposed (Appendix Q5):

> *We all know famous Greek mathematicians, like Pythagoras, Euclid, Archimedes, and Tales. Do you know any other ancient mathematician? If so, write down who s/he is.*

Report of questionnaire Q6: Geometry

Table of contents

1. Abstract

This report describes the design process followed for the questionnaire about Geometry, which has already been delivered to both in-service teachers and prospective teachers of the institutions associated with the ANFoMAN project. The analysis of the data collected shows clear differences between the participants of the Spanish institutions and those of the Italian institutions, mainly in their conception of the nature of geometry, which is much more technical in the Spanish case and more associated with human activity in the Italian case (appendix Q6[12]).

2. Questionnaire design: background and objectives

In many cases, maths at schools has been reduced to arithmetic. Although every curriculum contains different thematic blocks devoted to aspects such as geometry or statistics, the difficulties that many children have with algorithms and arithmetic problems lead to topics such as Geometry becoming of secondary importance in mathematics teaching in the Primary Education stage. Geometry issues are often reduced to topics characterized by superficiality, such as discrimination and classification of figures, or calculation of measurements through formulae. Moreover, the area of geometry is not well-known to teachers and society in general, for whom mathematics is merely a series of operating procedures that enable the performance of several calculations, either with discrete amounts, with money expressed in currency, or with measures of magnitudes.

The ANFoMAM project will design a workshop on Geometry for teachers in both initial and continuous training. We will work on the value of the primordial concepts, which are related to connection, order, congruence, comparison, construction, decomposition. These concepts express an abstract-quantitative view of the world that surrounds us (in contrast to a synthetic-qualitative vision). Geometry allows us to design activities with problems which are similar to those of mathematical research, in contrast to traditional arithmetic activities, which tend to be simulations of everyday life (derived from training for trades). In order to transmit to teachers the life-forming role of mathematics, Geometry is crucial, as it is a discipline which helps us to understand the world and ourselves and to develop conceptual thinking

[12] Questionnaire Q6:
https://docs.google.com/forms/d/1GPnSXgao4RWPhcRgKqGk1ciAKMBEs5tTKucc7HDqFAc/copy

and imagination. Before performing the activities, the participants may be invited to fill out a questionnaire which allows them to reflect on their beliefs and knowledge about geometry, as well as the most appropriate way of working on this area of mathematics at school.

A priori we expect to find different profiles of participants:

- Undergraduate students and in-service teachers who view mathematics as an already established and finished discipline. For these participants, the teaching of geometry consists of the transmission of a series of previously determined concepts. For this reason, we expect that, when they are teaching at school, they will, in the main, carry out activities related to both recognition and classification of figures. Similarly, they will work on the issue of measurement such as length and area in a mechanical way, providing students with methods or formulae for the calculation of areas and the changes of units.

- Participants with a dynamic idea of mathematics, for whom the teaching of geometry will be an opportunity to convey how the discipline has been developed, as a representation of the real space in which we live in response to problems faced by different civilizations throughout human history. This view is expected to lead to a more active way of working on geometry at school, in which students can perform activities of construction and mathematical discovery through experimentation and observation.

3. Data collect

The questionnaire has been given to the following participants:

- Undergraduate students of both Infant and Primary Education at the Public University of Navarre (38).
- Students enrolled in the *Laboratory of Mathematics and Didactics of Mathematics* (Programme B: Geometry, Writing, Expression) in the URT, where they had reflected on elementary geometry and the teaching thereof (32).
- Students enrolled in the *Course of Mathematics and Didactics of Mathematics* who had previously reflected on the teaching of geometry to children (16).
- Undergraduate students of Primary Education at the University of Zaragoza (89)
- In-service teachers participating in activities organized by *ToKalon Matematica* (43)

4. Analysis of data

The collected data have been processed in the following way:

1. For all questions requiring two or three words, the following simplifications have been made:

   - Words in their plural and singular forms: one of the two forms has been chosen (for example, instead of distinguishing between *calcolo* and *calcoli,* they have been unified in the *calcolo* form, or *cálculo* in Spanish).

- Words present as a noun or as a verb: one of the two forms has been chosen (for example, the words *conteggio* and *contare* have been unified in the *contare* form, or *contar* in Spanish).

- Words present as a noun or as an adjective: one of the two forms has been chosen (for example, in the case *of creativo* and creatività, creatività has been chosen, or *creatividad* in Spanish).

  Some groups of words referring to a single recognisable concept have been unified in a single form, preferably the one most cited (e.g. for *trigonometria, seno* and *coseno* and *funzioni goniometriche,* the word *trigonometria* has been chosen)

2. Numbers 1 to 4 are related to the following qualitative expressions:
   1= Nothing at all; 2 = Little; 3 = Quite a lot; 4 = A lot

3. The analysis regroups the data by separating the questionnaires from Italy (distinguishing between pupils and teachers) from those from Spain, which are compared with the previous ones.

4. The data charts related to the Italian participants sometimes bring together students and teachers; at other times, students and teachers are dealt with separately.

5. The data have been prepared graphically. After that, the researchers have carried out a qualitative analysis based on a collective discussion of the different perspectives.

We analyse the answers obtained question by question:

4.1. Question 1:
When you think of Geometry, what are the three words that come to your mind?

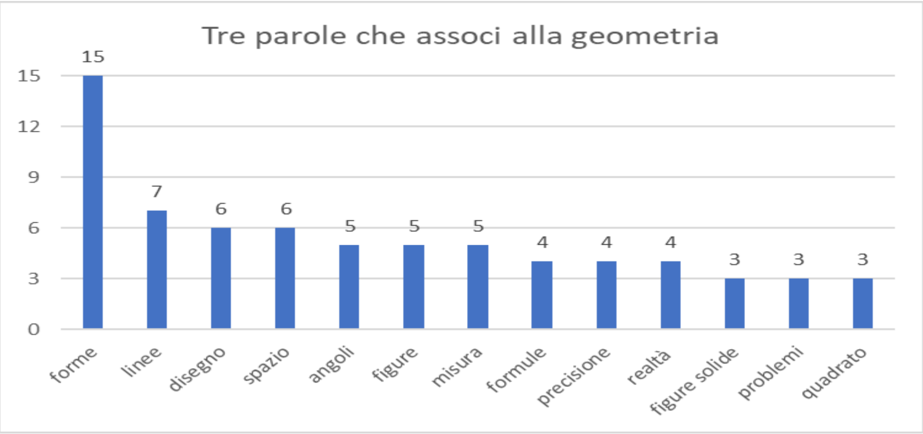
Among Italian teachers, these are the words that appear only once:

> *Harmony, art, circle, colours, concrete, environments, discovery, challenge, Egyptians, geometric features, evolution, fascinating, geometric figures, plane figures, instruments, garden with fountain, game with polygons, skyscrapers, footprints, pencil, straight line, maps, the world in which we live, objects, observation, paving, perception, perfection, map, Cartesian plane, Pythagoras, polygons, reasoning, rules.*

These are the words that appear twice:

> *Abstraction, centimetres, construction, creativity, difficult, buildings, intuition, dot, theorems, triangles*

The words that appear most often are listed in the following graphic, along with the number of times each of them appears:



Three words that you associate with geometry
shapes, lines, design, space, angles, figures, measurement, formulae, precision, reality, solid figures, problems, square

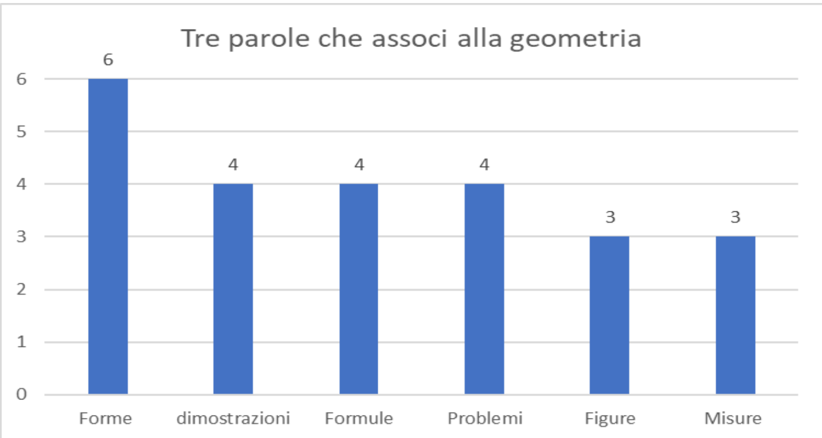
Figure 76: Answers to Question 1 given by the Italian in-service teachers

For the Italian students, the words that appear only once are:

> *Boring, angles, areas, complexity, continuity, corollary, difficult, design, fascinating, imagination, interesting, intuition, mysterious, nature, Cartesian plane, representation, similar, solution, theorem, Pythagorean theorem, day-to-day life.*

There is only one word that appears twice: *Segments*

The words that appear more than twice, together with the number of times each of them appears, are represented in the following graph:



Three words that you associate with geometry
Shapes – proofs – formulae – problems – figures – measurement

Figure 77: Answers to Question 1 given by the Italian students

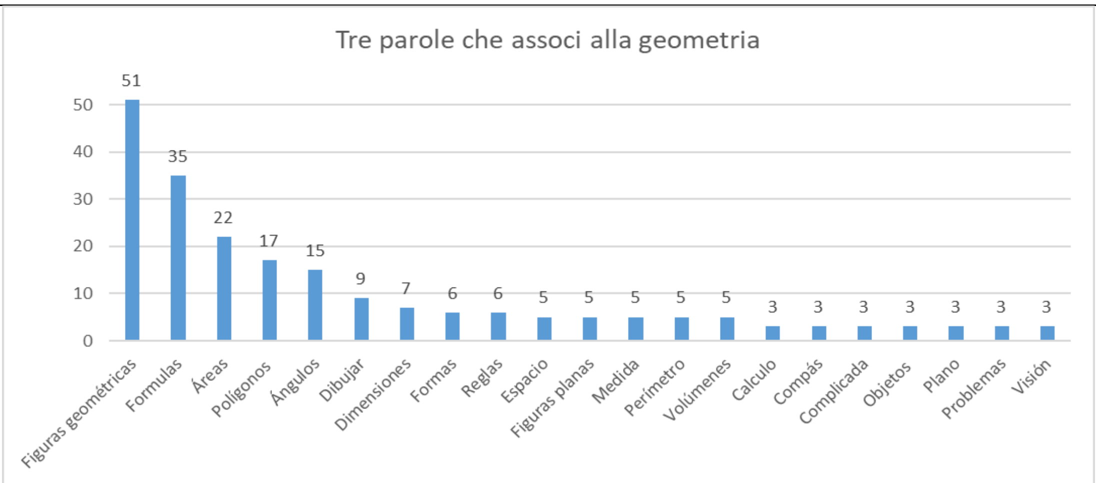
In the sample of the Spanish participants, the words that appear only once in the questionnaires are:

Boring, harmony, architecture, area calculation, circle, concentration, constructions, cubes, equations, more mechanical exercises, three-dimensional figures, Geogebra, skill, mental skill, interesting, length, manipulative, is not as simple as it seems, orientation and spatial perspective, polyedra, precision, depth, properties, projections/perspectives, radius, relationships between figures, representation, segment, symmetry, protractor

Those words that appear twice are:

Abstraction, areas, square, sides, numbers, perspective, Pythagoras, polycubes, prisms, straight lines

The ones that appear more times are listed in the following graphic:



Three words that you associate with geometry

Key

Spanish word	Translation	Spanish word	Translation
Figuras geométricas	Geometric figures	Medidas	Measurements
Fórmulas	Formulae	Perimetro	Perimeter
Áreas	Areas	Volúmenes	Volumes
Polígonos	Polygons	Cálculo	Calculation
Ángulos	Angles	Compás	Compass
Dibujar	Draw	Complicada	Complicated
Dimensiones	Dimensions	Objetos	Objects
Formas	Shapes	Plano	Plane
Reglas	Rulers	Problemas	Problems
Espacio	Space	Visión	Vision
Figuras planas	Plane figures		

Figure 78: Words that appear more than twice in the Spanish answers to Question 1

We make some comparisons between the answers given by the Italian and the Spanish participants by means of a qualitative analysis of these data. It is worth noting that the human dimension linked to mathematics, highlighted, albeit partially, by the Italian participants, is almost entirely absent in the case of the Spanish participants.

4.2. Question 2:

Between arithmetic and geometry, what do you prefer?

In this case, we wanted to create three charts: one for the Italian participants in general, another one for the Italian in-service teachers and a third one for the Italian students only. It is interesting to see how the result of the first chart does not give any indication of what appears in the other two. For the Italian students the answer is principally *arithmetic*, in an even greater proportion (75%) than with the Spanish students (60.7%).

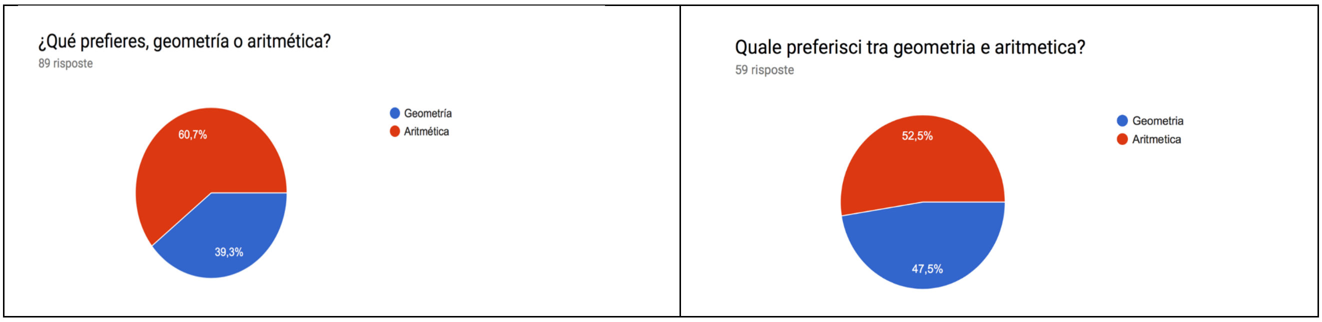


Figure 79: Answers to Question 2 from the Spanish participants versus responses from the Italian ones

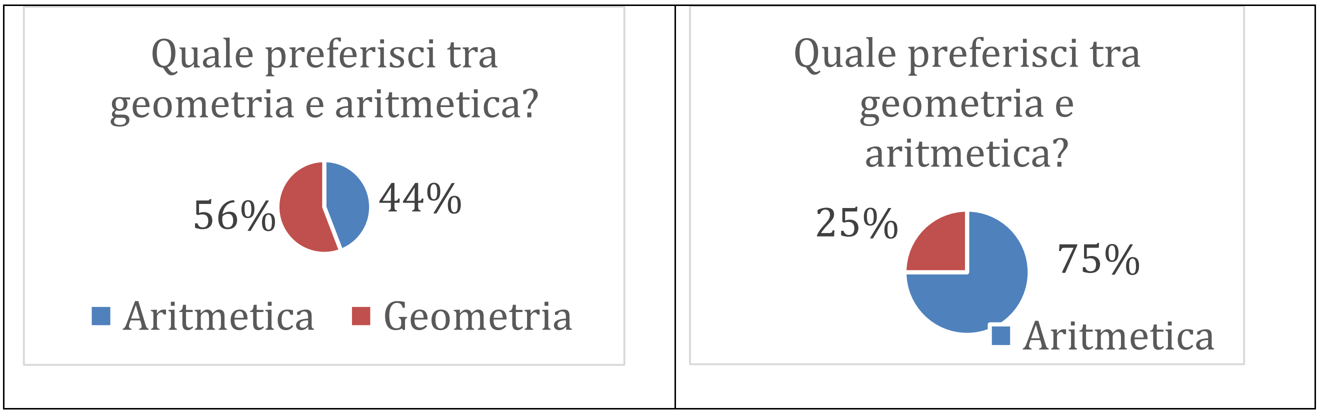


Figure 80: Answers to Question 2 from the Italian teachers versus responses from the Italian students

4.3. Question 3:

What geometry do we start with? Do we first present solid geometry or plane geometry?

In this case, there are no differences between the data analysed separately (teachers on the one hand and students on the other) and the joint data of the Italian participants. Also, as can be seen, almost half the participants opt for one and the other half for the other. There is a marked contrast between these responses and those of the Spanish students, who mostly choose to start with plane geometry.

Figure 81: Responses to Question 3 from the Italian participants vs the Spanish ones

Question 4: *Indicate with a value between 1 and 4 how important you consider the following activities in Geometry*

In the case of the Italian participants, as seen in the following charts, the highest-rated activities, with a value of 4 attributed by more than 80% of the sample, are: *working with the hands, drawing, observing and comparing*, while those that have been lowest valued are *memorizing* and *knowing the formulae*, to which only 23.7% and 27.1% respectively of the participants attributed the value 4.

The following graphs represent the valuation of each term by the Italian and the Spanish participants. The graph for the Italians appears first and that of the Spanish, second. It is surprising that, with the Spanish, the score of 4 has only exceeded 50% for the terms *working with the hands* and *comparing*.

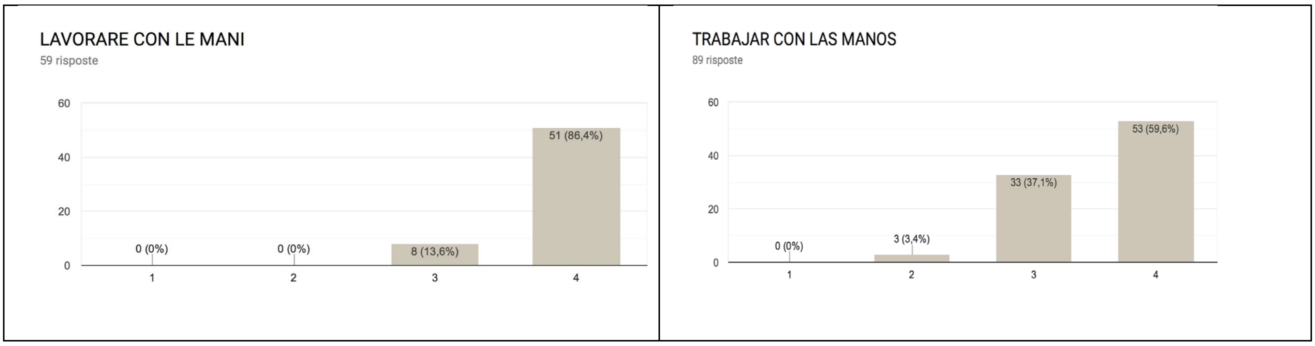


Figure 82: Answers from the Italian participants and the Spanish ones to Question 4a (working with your hands)

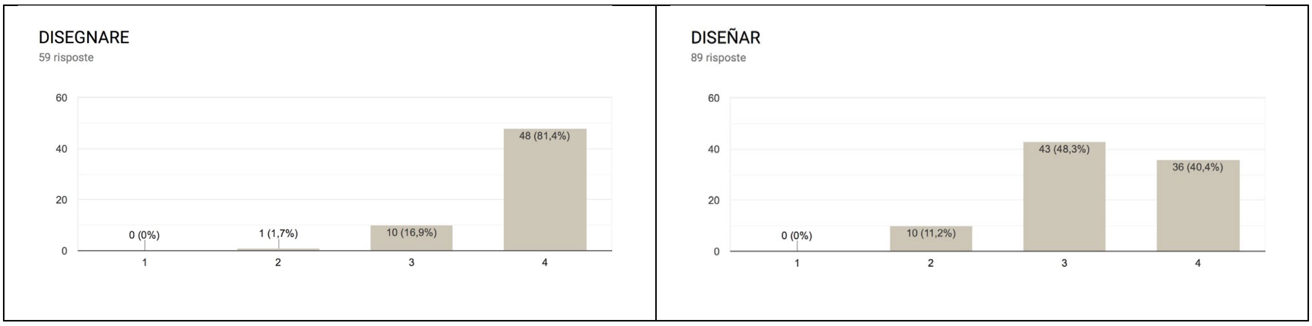


Figure 83: Answers from the Italian participants and the Spanish ones to Question 4b (designing)

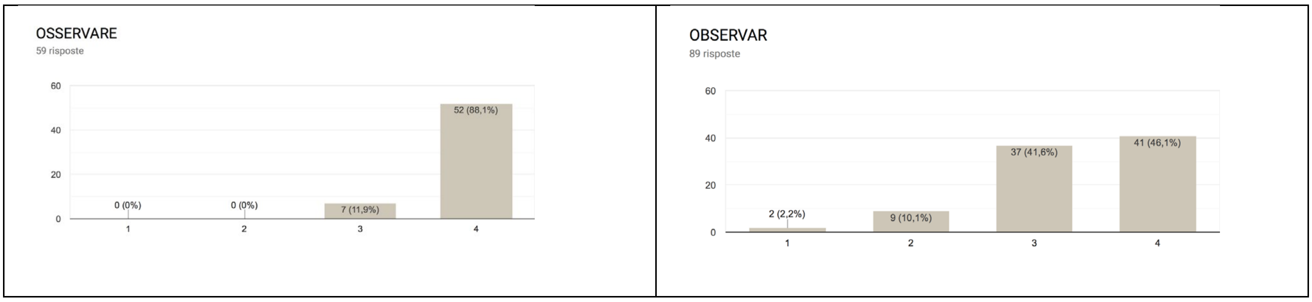


Figure 84: Answers from the Italian participants and the Spanish ones to Question 4c (visual observation)

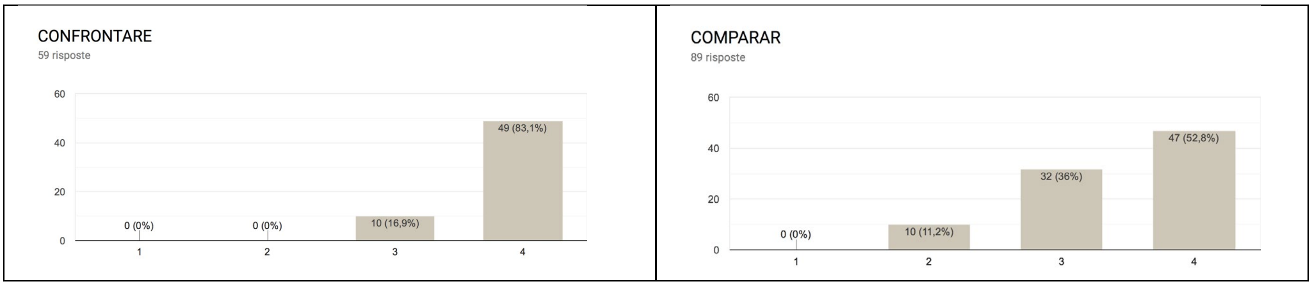


Figure 85: Answers from the Italian participants and the Spanish ones to Question 4d (making comparisons)

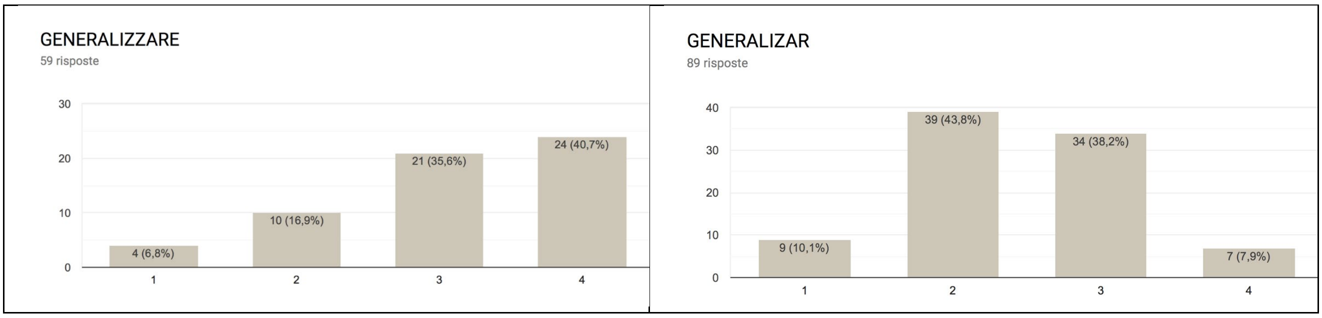


Figure 86: Answers from the Italian participants and the Spanish ones to Question 4e (generalizing)

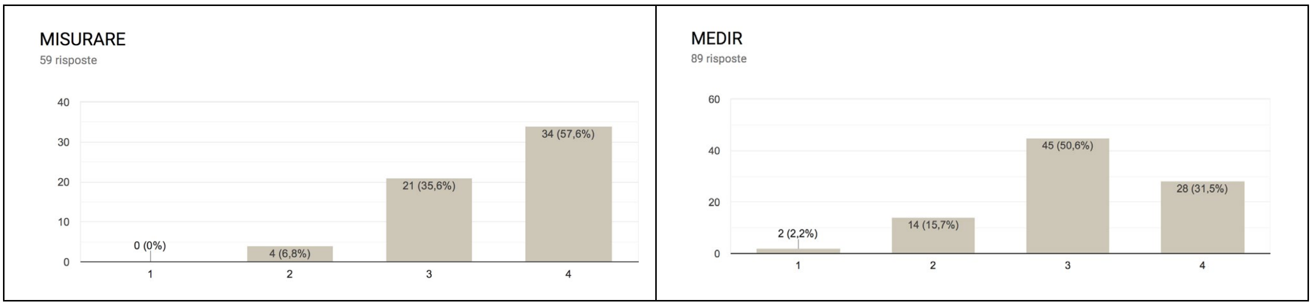


Figure 87: Answers from the Italian participants and the Spanish ones to Question 4f (measurements)

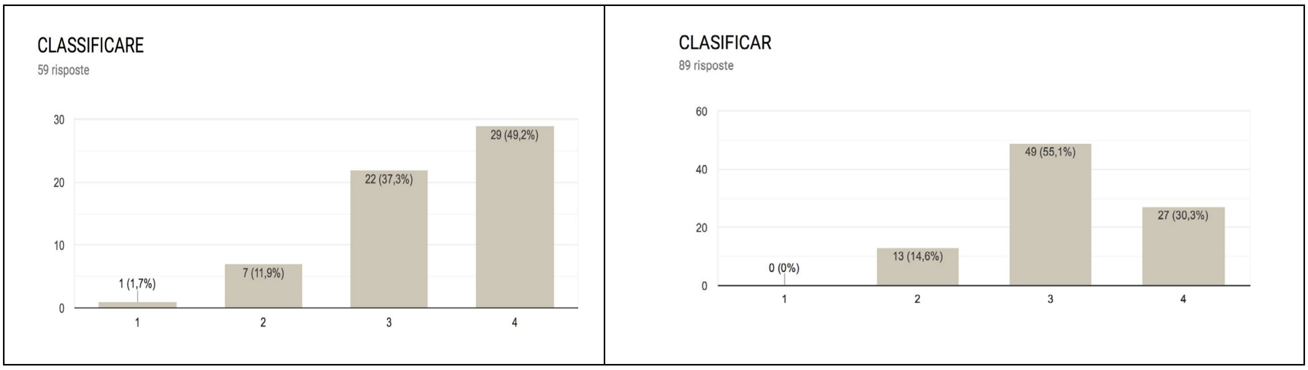


Figure 88: Answers from the Italian participants and the Spanish ones to Question 4g (classifying)

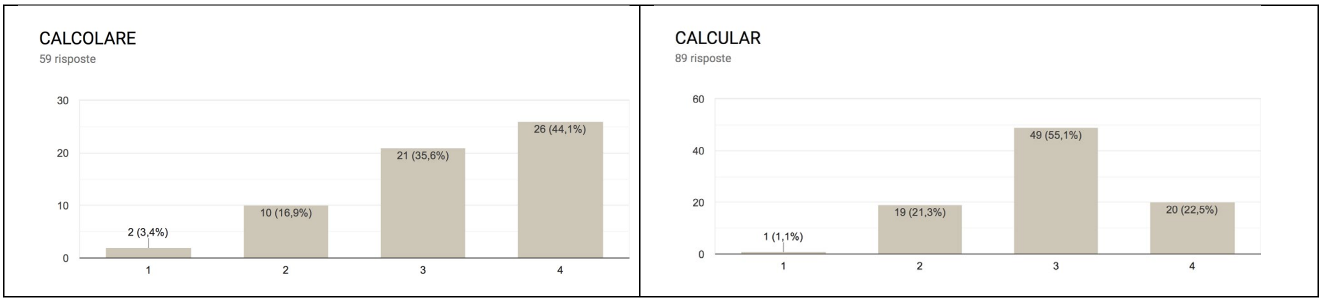


Figure 89: Answers from the Italian participants and the Spanish ones to Question 4h (calculating)

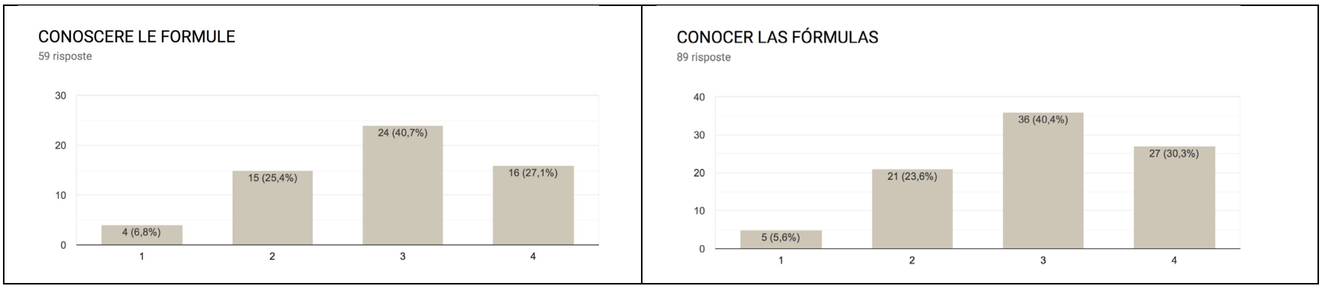


Figure 90: Answers from the Italian participants and the Spanish ones to Question 4i (learning the formulae)

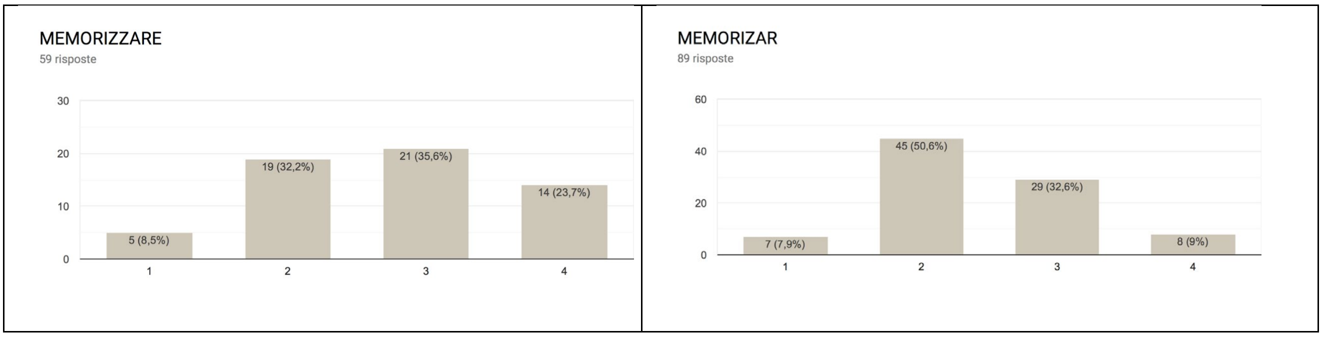


Figure 91: Answers from the Italian participants and the Spanish ones to Question 4j (memorizing)

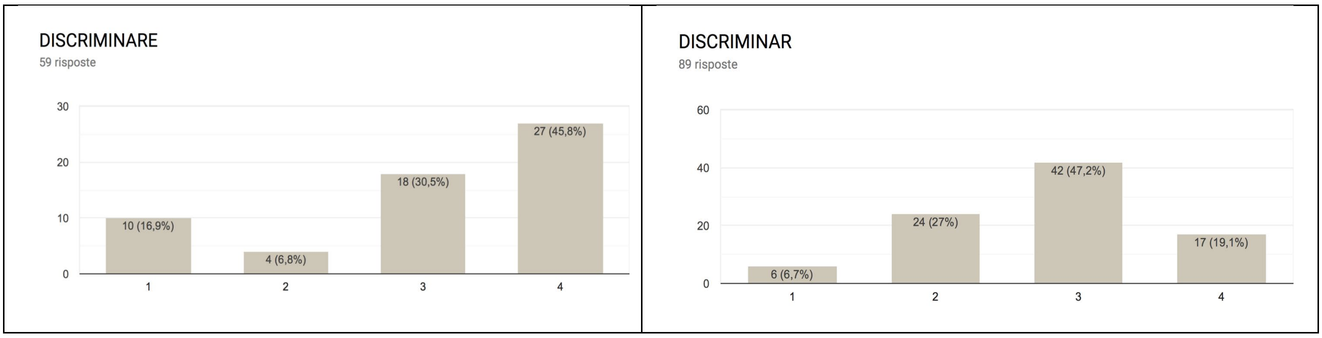


Figure 92: Answers from the Italian participants and the Spanish ones to Question 4k (distinguishing between shapes)

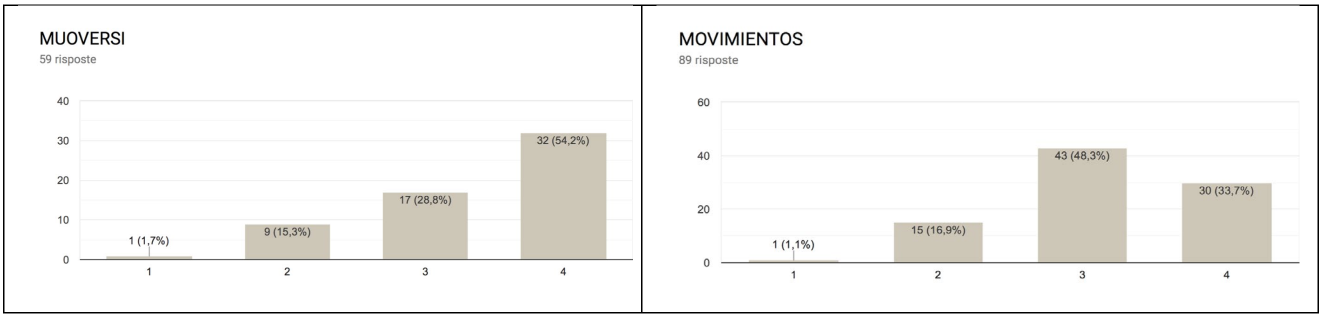


Figure 93: Answers from the Italian participants and the Spanish ones to Question 4l (movements)

Question 5:
Indicate with a number from 1 to 4 how important you consider the following topics in Geometry.

We represent, topic by topic, the opinion of the Italian participants (teachers and students together), first, and then that of the Spanish participants.

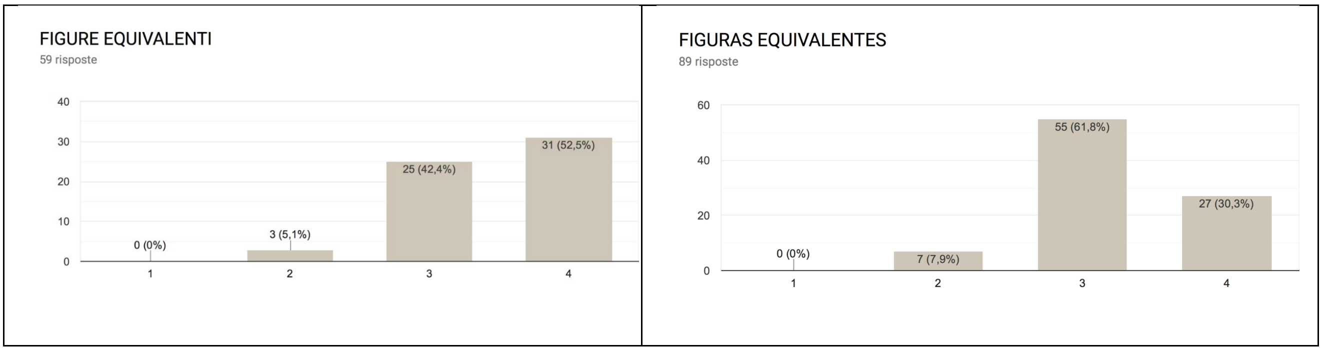


Figure 94: Answers from the Italian participants and the Spanish ones to Question 5a (equivalent figures)

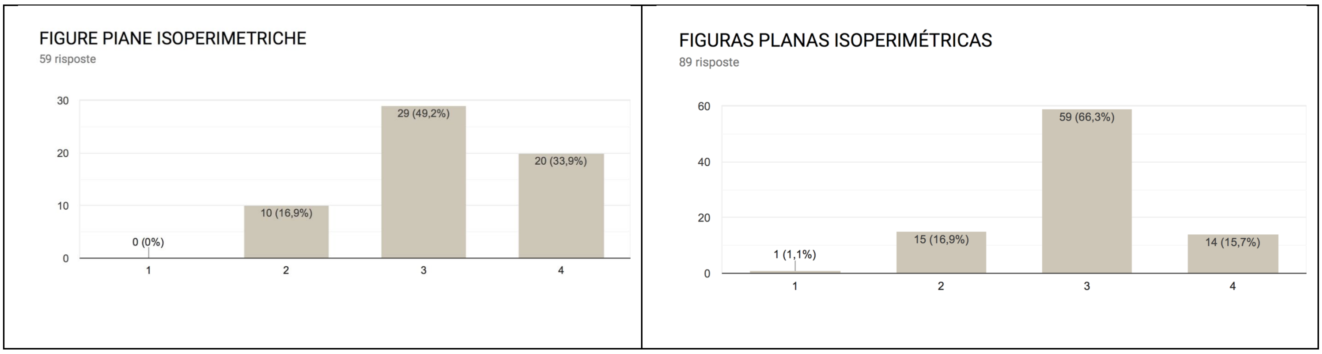


Figure 95: Answers from the Italian participants and the Spanish ones to Question 5b (isoperimetric figures)

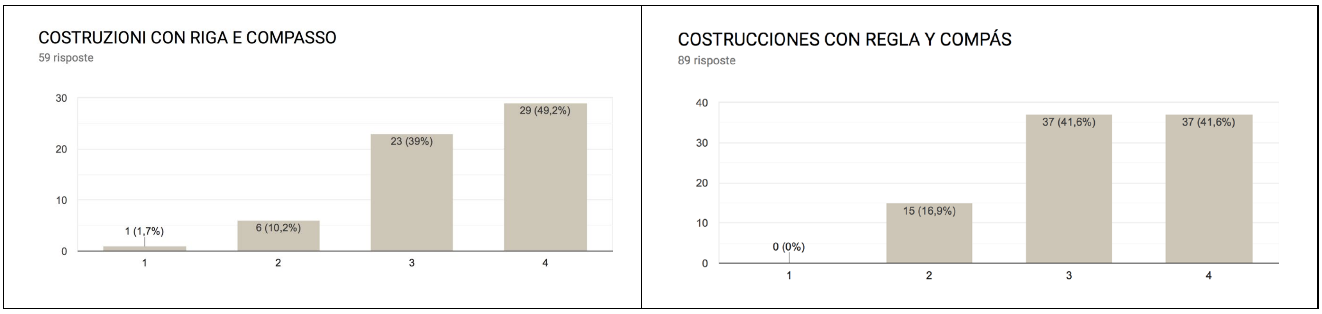


Figure 96: Answers from the Italian participants and the Spanish ones to Question 5c (constructions with ruler and compass)

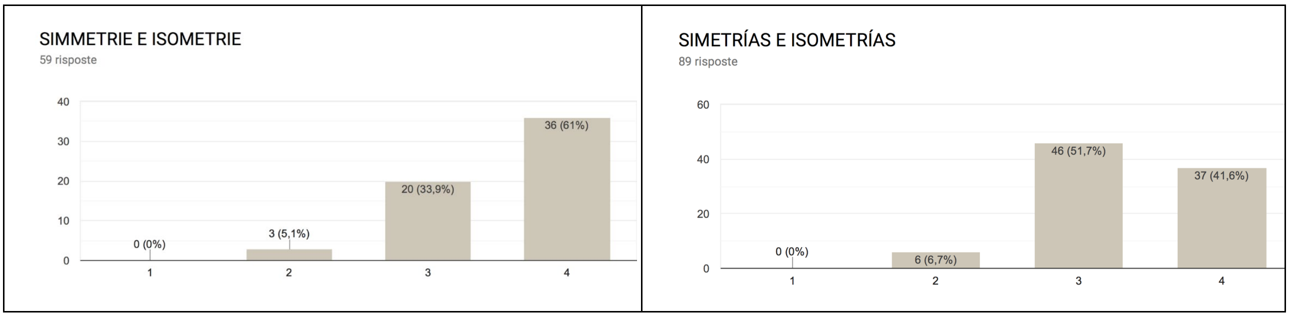


Figure 97: Answers from the Italian participants and the Spanish ones to Question 5d (symmetries and isometries)

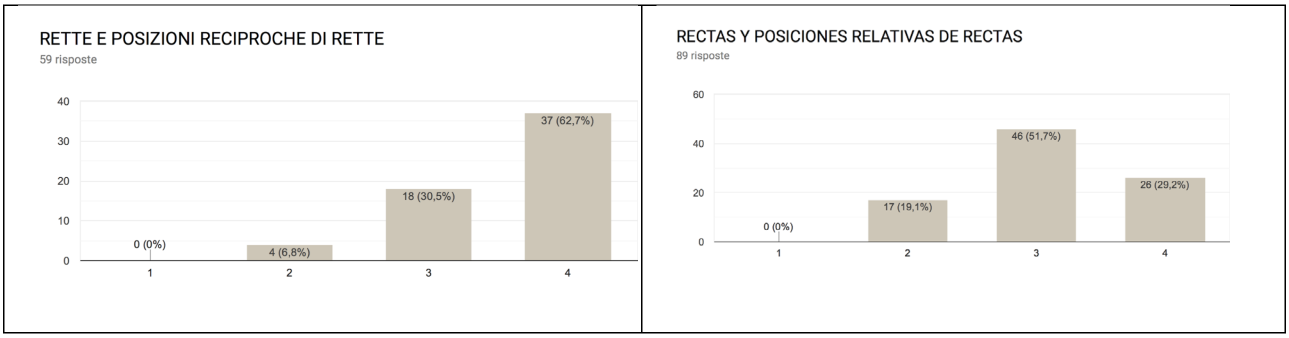


Figure 98: Answers from the Italian participants and the Spanish ones to Question 5e (straight lines and their relative positions)

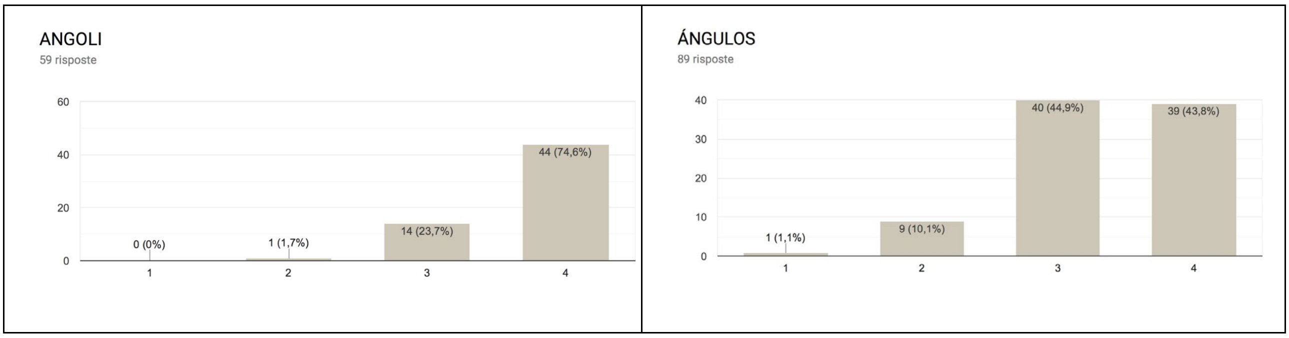


Figure 99: Answers from the Italian participants and the Spanish ones to Question 5f (angles)

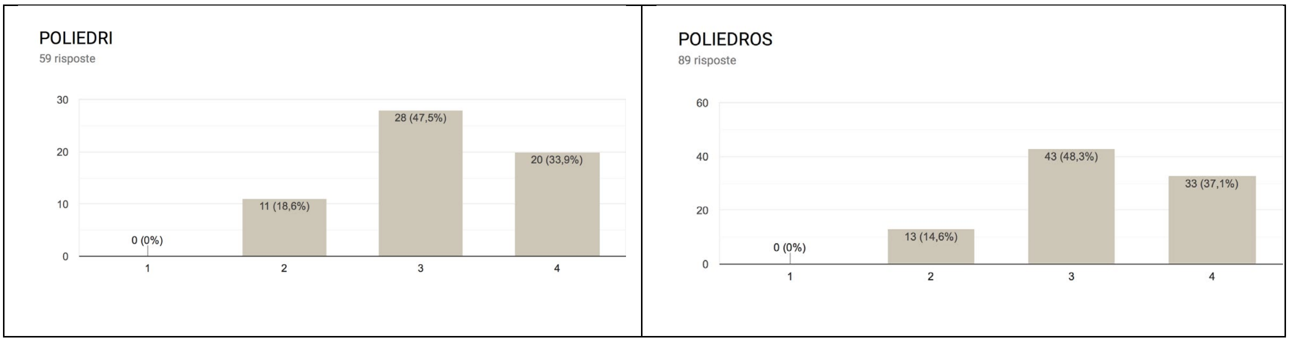


Figure 100: Answers from the Italian participants and the Spanish ones to Question 5g (polyhedra)

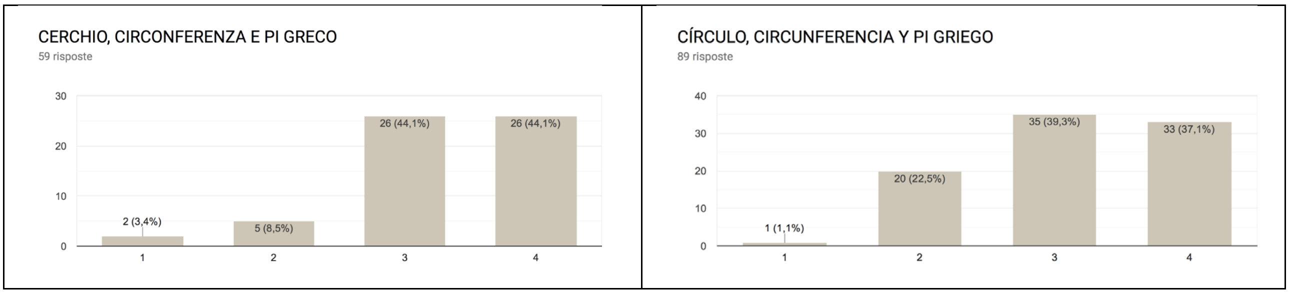


Figure 101: Answers from the Italian participants and the Spanish ones to Question 5h (circle, circumference and Greek pi)

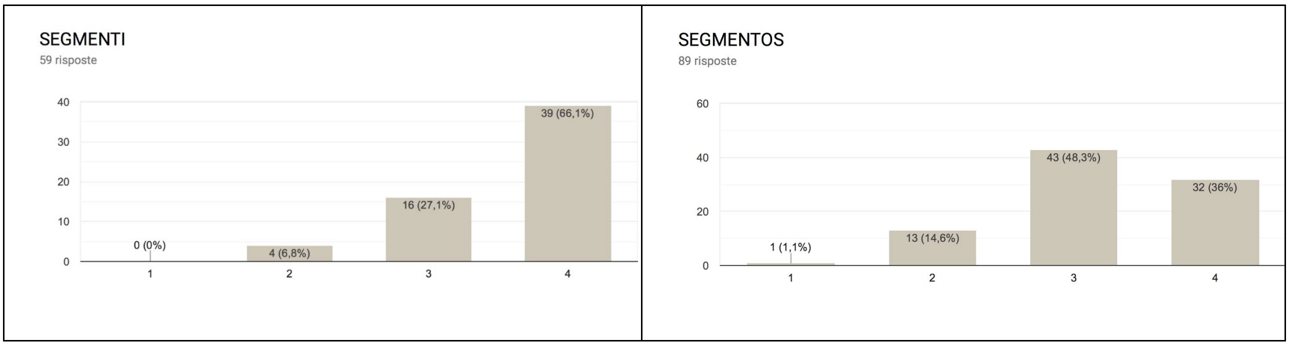


Figure 102: Answers from the Italian participants and the Spanish ones to Question 5i (segments)

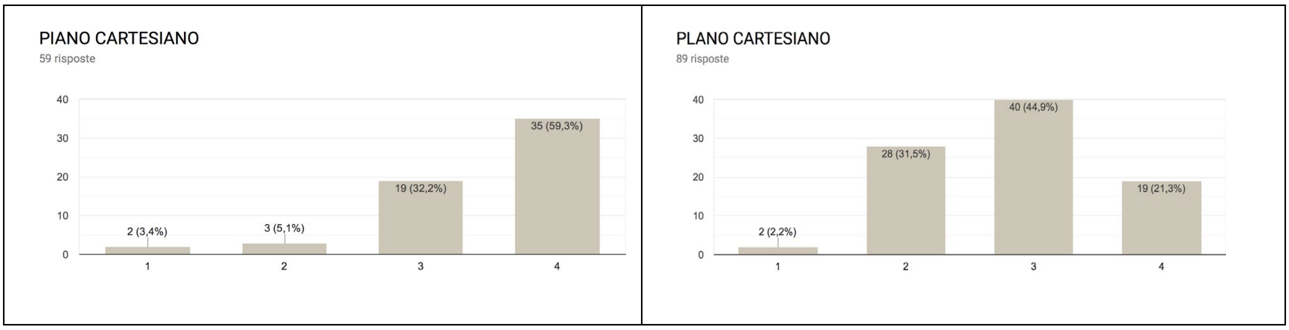


Figure 103: Answers from the Italian participants and the Spanish ones to Question 5j (cartesian plane)

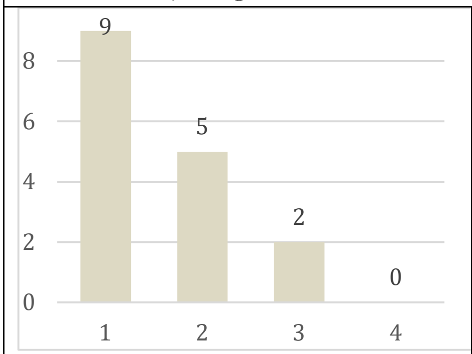
The topics most frequently mentioned, with a value of 4 attributed by more than 60% of the Italian sample, were: *symmetries and isometry, straight lines and their relative positions, angles* and *segments*.

The only issues with more than 40% in a same valuation (but less than 50%) for the Spanish participants are: *constructions with ruler and compass, angles, symmetries and isometry.* The valuation of *Segments (36%)* contrasts markedly with the Italian valuations *(66%).* A similar contrast occurs in the valuation of *straight lines and their relative positions* (29% vs. 63% in Italy).

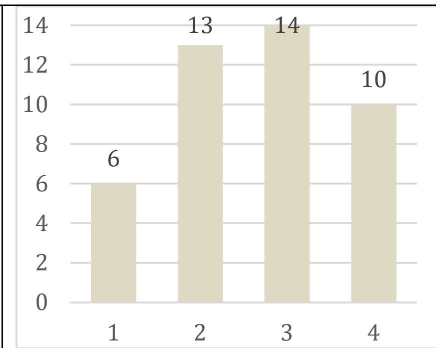
4.6. Question 6: *Express with a number from 1 to 4 your degree of agreement with the following statements.*
a) For young children, the distinction between square, triangle and circle should be emphasized.
b) It is possible to teach in an elementary class a concept such as the straight line which is tangential to a curve.

In the case of the Italian participants, it is interesting to look separately at the teachers and the students. It appears, for example, that students have a clear understanding of the didactic irrelevance of the distinction between square, triangle, and circle, while teachers seem much more confused about this issue. In the case of the Spanish participants, almost 90% agree (grade 3 or 4) with the first statement. Regard the second statement, more than 56% of the Spanish agree (grade 3 or 4), similar to the Italian case (there being no differences between the responses given by teachers and students).

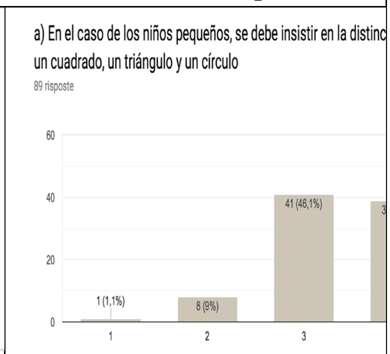
For young children, the distinction between square, triangle and circle should be emphasized.



Italian students

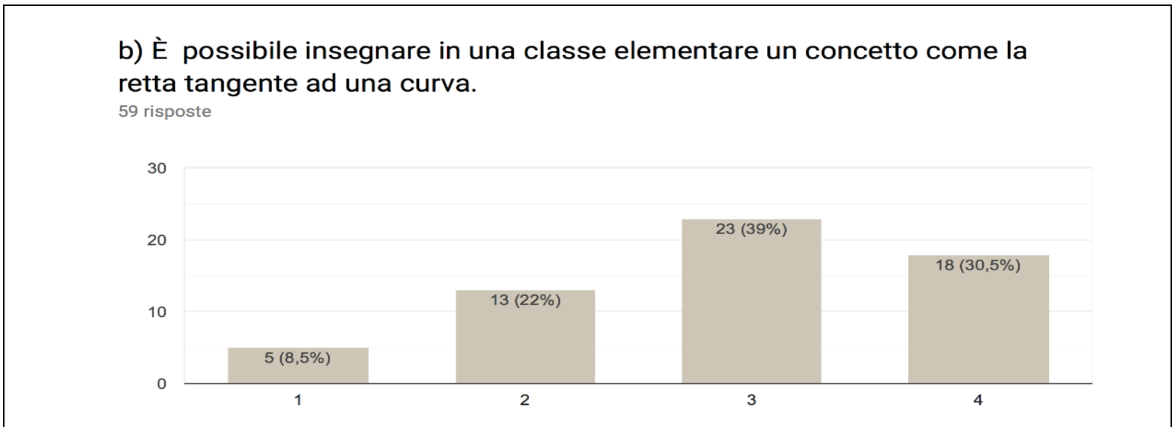


Italian teachers



Spanish participants

Figure 104: Answers from the participants to Question 6a



It is possible to teach in an elementary class a concept such as the straight line which is tangential to a curve

Figure 105: Answers from the Italian participants to Question 6b

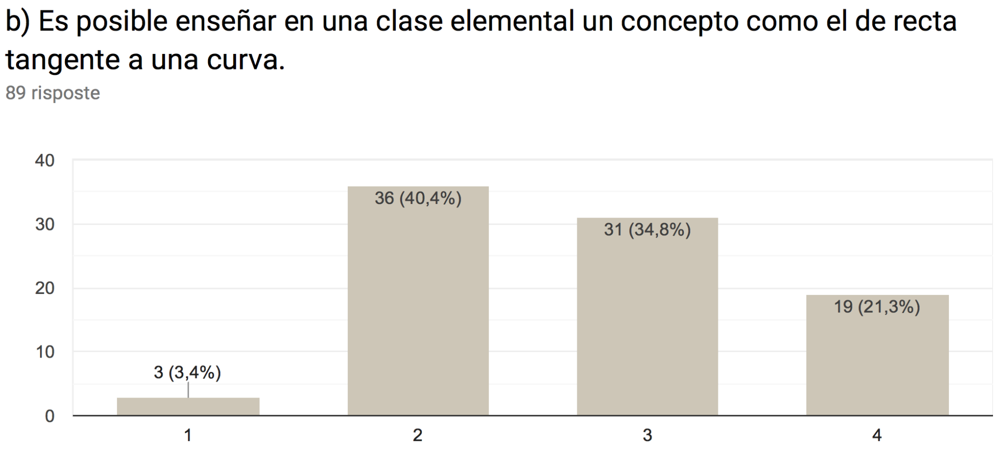
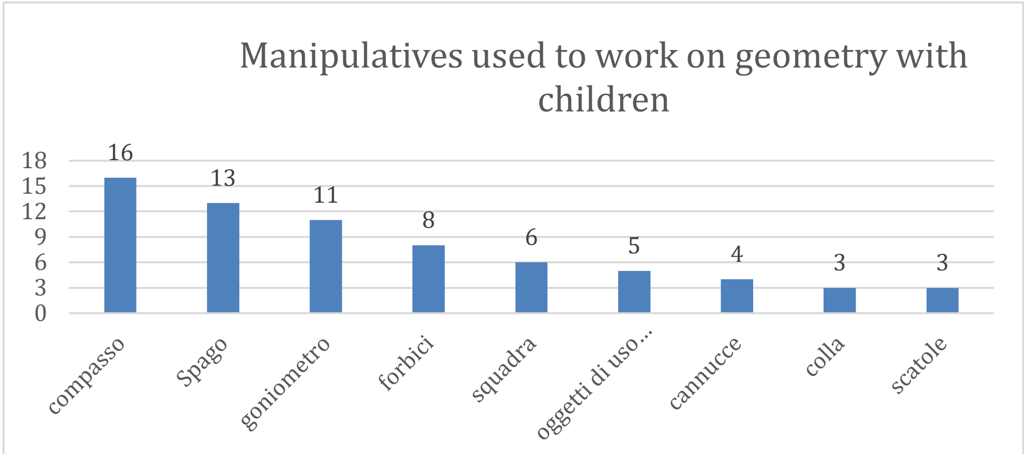


Figure 106: Answers from the Spanish participants to Question 6b

4.7 Question 7: *Paper, pen, pencil, ruler, are essential for doing geometry with children. Indicate at least two other useful things that could also be used.*

In this case, as in that of the words related to geometry, we prefer to consider separately the answers given by the teachers and those given by the students. 42% of the Italian teachers name only two objects, whilst 58% mention three. The following graph lists the objects that appear more than twice, along with the number of times they are named by the Italian teachers:



compass - rope – protractor– scissors – set square – commonly used objects – straws – cola – boxes

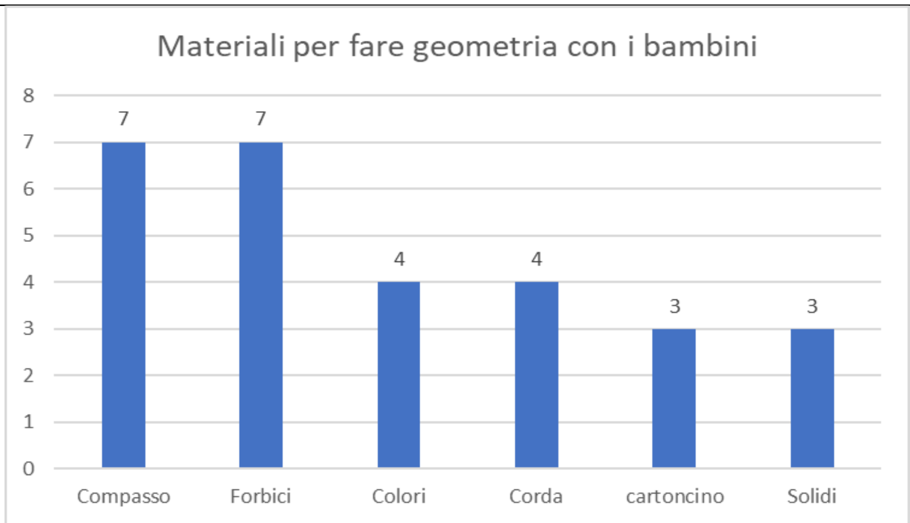
Figure 107: Answers given by the Italian teachers to Question 8

The words that are named only once are:

> *logical blocks, nails, lego pieces, clay, wooden cubes, metal fasteners, wires, sheet of paper, geometric shapes, ropes and other gym objects, geometric board, chalk, tessellation materials, duct tape, floor, solids, toothpicks.*

The words that have been named twice are: *cardboard, body, rubber band, book, hands, origami, notebook, Tangram.*

In the case of the Italian students, 63% of the Italian students name three objects, while 37% name only two. The objects that are named only once are: *bars, silk paper, city map, everyday objects, metal blinders, any geometric object, rulers, stories, or stories about geometry*. The words that appear twice are*: geometric shapes and protractor*. And those that appear most often among Italian students are represented in the following table*: compass, scissors, colours, rope, card, solids.*

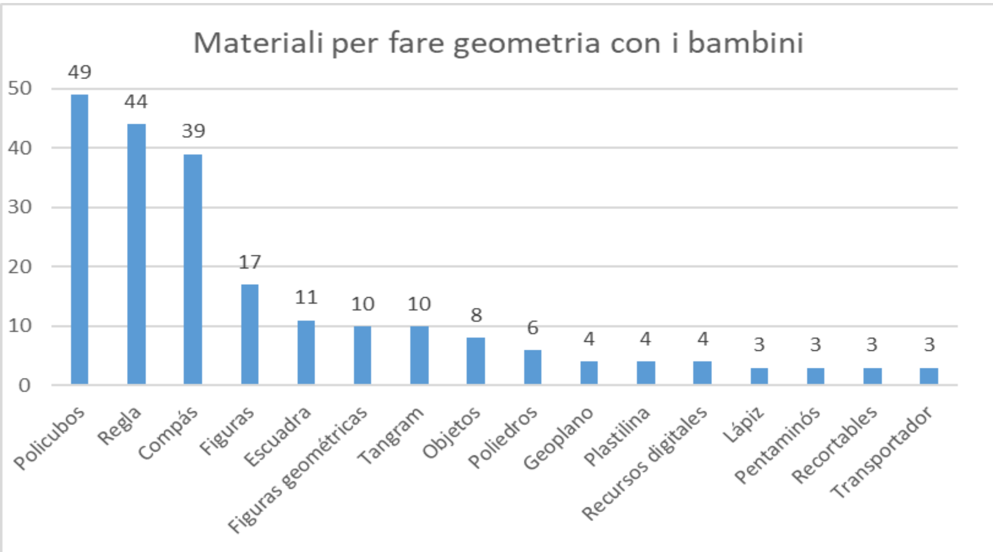


Materials for working on geometry with children

Compass - scissors - colours - string - cardboard - solids

Figure 108: Answers given to Question 8 by the Italian students

In the case of the Spanish students, the objects that were named more than twice are shown in the following table:



Manipulatives used to work on geometry with children

Spanish word	Translation	Spanish word	Translation
Policubos	Polycubes	Poliedros	Polyhedra
Regla	Ruler	Geoplano	Geoplane
Compás	Compass	Plastilina	Plasticine
Figuras	Figures	Recursos digitales	Digital resources
Escuadras	Set squares	Lápiz	Pencils
Figuras geométricas	Geometric figures	Pentaminos	Pentaminos
Tangram	Tangram	Recortables	Cotouts
Objetos	Objects	Transportador	Protractor

Figure 109: Answers given to Question 8 by the Spanish participants

The objects that are only named once by the Spanish participants are:

> *Clay, plastic blocks (referring to logical blocks), boxes, calculator, polycubes, visual environment, hand-outs to reinforce the contents, gridded sheets, sheets of different moving and modifiable materials, pencil, lines and angles, manipulative material, shaped materials, drawing materials, molds and clay, organicbies, chopsticks, pentominoes, parts, paintings to differentiate the parts, paper templates with drawings , wooden prisms, puzzles, strips, calculus board, different sizes, Tetris, different textures, isometric weft, folding rods, interactive videos.*

And those that have been named twice are:

> *Cubes, rope, geometric bodies, games, manipulative materials, slate and protractor*

5. Conclusions: observations, reflections, and proposals for improvement

After a qualitative analysis of the answers given in response to the questionnaire, we can say that mathematics in general, and geometry in particular, for some teachers and students, bears a relation to their own life experiences, evoking a "positive" memory of the past. For others, on the contrary, their experience of maths constitutes a missed opportunity in this respect.

In particular, the answers given to the first question show that the terms that imply a greater personal relationship with geometry (fascinating, challenging, discovery, difficult and boring) are named only once, suggesting that the experience of mathematics is unique for each person.

The human aspects of mathematical activity, highlighted, albeit partially, by the Italian participants, is almost entirely absent in the case of the Spanish participants (see in particular the answers to questions no. 1 and 4). Activities such as working with the hands, drawing, and moving, to which the Italian students and teachers give special relevance, are not valued by the Spanish students. It is surprising, however, that the importance placed on aspects such as memorization and discrimination is greater among Italian participants than among the Spanish ones.

The analysis of the results reflects a need on the part of the in-service and future to offer a more human view of mathematics, beyond knowledge of definitions and procedures. For instance, in Question 4, the participants emphasize formulae and memorization as important aspects in geometry-related activities, which shows a static and instructive view of geometry. However, the importance placed on movement in question 4, very highly valued by the Italian participants, represents an openness to another way of addressing mathematics.

The results of the analysis impel us to design a math workshop for teachers in initial and continuous training that provides them with a dynamic and human vision of geometry, with activities that leave room for visual observation, movement, designing and making constructions.

Concluding remarks

As we have shown in the analysis of the data which have been collected by means of the questionnaires, the participants' experience with mathematics during school is strongly correlated to their beliefs on both the nature of maths and the way this subject should be worked on at school.

What is clear from the analysis is that, when the future and current teachers have been taught mathematics as a completely finished and rigid body of concepts and procedures, they do not regard themselves as capable of adapting the activities to their pupils in the classroom or even helping them to understand the subject. This vision of maths is present in a high percentage of the Spanish university students, who typically associate maths with arithmetic tasks and problems. In spite of the high value they give to the teaching of geometry in primary school, they do not feel comfortable when working with geometrical procedures. This is probably due to the way in which they have been taught geometry, with methods focused on both the classification of the plane shapes and the use of formulae to calculate areas and volumes.

Given this experience, it seems logical to these participants not to value either the use of the history of maths or the use of narration in teaching. The mathematics they knew at school is completely separated from the human context in which it arises, as well as completely unlinked to its evolution through the history of humanity. This might also be the reason why the Spanish students show a more rigid view of maths than the other participants, a vision which regards its utility as the main aim of teaching maths to children, i. e. how it may serve for their future academic and professional success.

The Italian participants present a profile which is different to that of the Spanish ones. The Italian participants include a higher percentage of in-service teachers than in the other nationalities. In addition, the Italian in-service teachers who participated in the study usually take part in the professional development courses which are offered by the ToKalon Association, in which a dynamic vision of maths is presented. In this vision, Geometry plays a leading role, a geometry in which activities such as working with hands, drawing and moving are particularly valued. Furthermore, the Italian participants have certain knowledge about the history of maths, which enables them to relate mathematical concepts to their historical inception and evolution, including the way in which the development of the discipline is related to human activities in different civilizations. They favour activities which enable children to inquire and make their own discoveries – even when it comes to using the calculator – despite the fact that they also give value to certain mechanical aspects of learning such as memorization or the early introduction of the arithmetical algorithms at school.

The profile of the French participants is a mixture of a dynamic vision of maths and an emphasis on tasks related to perseverance and memorization. When these participants reflect on geometry, they include issues such as deductive logic and, when they refer to the algorithms of the elementary arithmetic operations, they assign value to effort and repetition in the learning thereof, as well as mental computation. When the participants refer to carrying out computation and estimation tasks, they assign value to the formative aspects of the procedure per se, rather than to speed in calculation.

It is reasonable to think that these differences between the profiles of the participants are due to the different ways they have been taught. However, in spite of having lived different experiences as pupils, the analysis of the participants' answers to the questionnaires reflects a shared desire to find new ways of working on maths at school. They express, for example, the desirability of children understanding the dynamics underlying the algorithms, as well as acquiring familiarity with numbers and their properties. Moreover, the participants understand the arithmetic problems as a chance for the pupils to acquire confidence in their own capacities and learn to dialogue using mathematical language and argumentation. They lean towards ways of teaching which give sense to the subject, connect different areas of maths and also link the more technical aspects of maths with the human and personal dimensions.

The analysis of the answers to the questionnaires reflects, on the part of most of the participants, a lack of knowledge and resources to carry out this kind of teaching in the classroom. Consequently, we think that the aims of the project, together with the topics we have chosen for the design of workshops in the ANFoMAM project, are both relevant and necessary:

- To promote a vision of the arithmetical algorithms focused on the understanding of the operations and the numerical structures.
- To work on techniques of arithmetical problem solving focused on the understanding and representation of the situations which have generated them.
- To promote activities which show the interconnections between arithmetic and geometry, both in terms of their nature and how they are taught at school.
- To design activities to work on mental computation in combination with a suitable use of the calculator.
- To foster a dynamic vision of maths by means of knowledge of its historical evolution.
- To work on the aspects of geometry which are closest to human activity, such as observation, construction or movement.

We have also shown the necessity of defining a specific format for the workshops in which the activities will be designed and implemented. This format seeks to make it possible for the participants to live new experiences related to maths. The goal is not only for them to adopt a new vision of the nature of maths, gaining insight into different aspects of the discipline and acquiring resources for the teaching thereof. A further aim, somewhat more ambitious, is for the participants to achieve a new relationship with the discipline, which gives them confidence to teach maths in the future and enables them to use maths to facilitate the personal growth and development of the children. These are our challenges when designing and implementing the ANFoMAM "talleres" of mathematics.

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Esta conferencia tiene como objetivo vincular las preocupaciones y cuestiones de los profesionales y el público en general con las producciones científicas sobre el aprendizaje y los números y cálculos de enseñanza, por un lado. Las recomendaciones fueron redactadas por un jurado al final de la conferencia.

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Appendix P0: Text of the general questionnaire (pilot phase)

According to your own experience
1.- Associate three words to mathematics
1)
2)
3)
2.- Value from 1 to 4 the weight you give to the following aspects in the mathematical activity:

- Experimentation
- Fantasy
- Creativity
- Reasoning
- Dialogue
- Intuition
- Memory
- Abstraction
- Attention
- Speed
- Perseverance
- Construction
- Subjectivity
- Formulae

3.- Enunciate the three most difficult themes of mathematics that you have studied
1)
2)
3)
4.- Help to learn mathematics
Who/what has helped you learn mathematics?

- A teacher of primary education
- A teacher of compulsory secondary education
- A teacher of high school
- A university teacher
- A private teacher
- A member of your family
- A book
- A game
- A classmate
- A video/a film
- Others

5.- Hindrance to learn mathematics
Who/what has hindered your mathematics learning?

- A teacher of primary education

- A teacher of compulsory secondary education
- A teacher of high school
- A university teacher
- A private teacher
- A member of your family
- A book
- A game
- A classmate
- A video/a film
- Others

6.- Assign a value between 1 and 4 to your degree of agreement with the following statements: (1= disagree, 2= slightly disagree, 3= slightly agree, 4= strongly agree)

a) To perform well at maths, it is necessary to be 'gifted'
b) One's ability at mathematics can be improved over time
c) Mathematics is useful in day-to-day life
d) Mathematics is essential to find a job
e) Mathematics is beautiful
f) In mathematics, the result is what matters
g) Mathematics is difficult
h) Mathematics is a fun
i) Mathematics is 'always the same thing'
j) Mathematics contributes to personal growth
k) In Mathematics, making mistakes must be avoided
l) Mathematics structures your mind

7.- Mark the studies in which you last studied mathematics

- Compulsory secondary education
- Science high school
- Professional studies
- University studies different from those addressed to primary teaching
- Others

8.- Which of the following sentences describes best your current situation

- I am working as a primary teacher
- I worked as a teacher, although not at this moment
- I have never worked as a teacher, although I plan to do so
- I am not planning to work as a teacher

Appendix Q0: Text of the general questionnaire (final version)

https://docs.google.com/forms/d/1J15LUMwkk45JHBpA6iUUCVw5ZPnRhc4e-yfPWahirnk/copy

According to your own experience
1.- Associate three words to mathematics
1)
2)
3)
2.- Value from 1 to 4 the weight you give to the following aspects in the mathematical activity:

- Experimentation
- Fantasy
- Creativity
- Reasoning
- Dialogue
- Intuition
- Memory
- Abstraction
- Attention
- Speed
- Perseverance
- Construction
- Subjectivity
- Formulae

3.- According to your experience, assign a value between 1 and 4 to the difficulty you find in the following kinds of mathematical activities:
(1= very easy, 2= affordable, 3= difficult, 4= very difficult)

a) Performing the algorithms of the basic operations
b) Solving arithmetic problems
c) Mental computation
d) Measurement and change of units
e) Solving geometrical problems
f) Making deductive reasoning

4.- Help to learn mathematics
Who/what has helped you learn mathematics?

- A teacher of primary education
- A teacher of compulsory secondary education
- A teacher of high school
- A university teacher
- A private teacher
- A member of your family
- A book
- A game

- A classmate
- A video/a film
- Others

5.- Hindrance to learn mathematics

Who/what has hindered your mathematics learning?

- A teacher of primary education
- A teacher of compulsory secondary education
- A teacher of high school
- A university teacher
- A private teacher
- A member of your family
- A book
- A game
- A classmate
- A video/a film
- Others

6.- Assign a value between 1 and 4 to your degree of agreement with the following statements: (1= disagree, 2= slightly disagree, 3= slightly agree, 4= strongly agree)

a) To perform well at maths, it is necessary to be 'gifted'
b) One's ability at mathematics can be improved over time
c) Mathematics is useful in day-to-day life
d) Mathematics is essential to find a job
e) Mathematics is beautiful
f) In mathematics, the result is what matters
g) Mathematics is difficult
h) Mathematics is a fun
i) Mathematics is 'always the same thing'
j) Mathematics contributes to personal growth
k) In Mathematics, making mistakes must be avoided
l) Mathematics structures your mind

7.- Mark the studies in which you last studied mathematics

- Compulsory secondary education
- Science high school
- Humanities high school
- Professional studies
- University studies different from those addressed to primary teaching
- Others

8.- Which of the following sentences describes best your current situation

- I am working as a primary teacher
- I worked as a teacher, although not at this moment
- I have never worked as a teacher, although I plan to do so
- I am not planning to work as a teacher

Appendix P1: Pilot text of the questionnaire about understanding arithmetic algorithms

1- Assign a value between 1 and 4 to indicate the degree the following statements reflect your experience with the arithmetic algorithms:
(1= not identified at all, 4= totally identified)

a) They have always been boring and repetitive for me
b) I have never been interested in knowing why they are designed the way they are
c) I understand fairly well the reason for each step that is given when an algorithm is applied
d) I found them easy to learn
e) As soon as I was allowed to, I used the calculator to avoid having to apply them

2- Assign a value between 1 and 4 to the following reasons for teaching arithmetic algorithms to children in primary education
(1= disagree, 4= strongly agree)

a) It is useful for both the academic and professional future of the pupils
b) It helps children better understand the properties of numbers and arithmetic operations
c) It offers the pupil the security that their calculations are right
d) It helps the pupil understand the iterative processes of computer programming
e) Faced with an arithmetic problem, the student can focus on the resolution strategy, having mastered the calculations

3- What importance do you give to the following aspects of the teaching of the arithmetic algorithms in order to facilitate students' understanding of the steps involved?

(1= not important at all, 4= very important)

a) Combining their teaching with that of the properties of numbers and arithmetic operations
b) Encouraging the use of a precise vocabulary to name the different units involved
c) Accompanying their teaching with appropriate graphs, diagrams or schemes
d) Accompanying their teaching with suitable manipulatives

4- Assign a value from 1 to 4 regarding with the degree of appropriateness of using traditional arithmetic algorithms to perform the following types of operations:
(1= not suitable at all, 4= very suitable)

- 23+15 operations
- 24x25 operations
- 234 x 346 operations
- 876 - 582 operations

5- Point out the reasons (they may be more than one) why you think children encounter difficulties when applying arithmetic algorithms

- Each specific example differs in some way from the others
- They do not practice enough at home
- They have learnt to apply rules without understanding their meaning
- They get distracted while they are applying the algorithms
- They are not able to make a mental image that helps them in the calculation
- They are not used to breaking down numbers
- They do not know how to use a diagram, a graph or a manipulative to support them

- They find no sense in applying them to perform operations without a context situation
- They feel blocked by a fear of making mistakes
- None of the above

6- Assign a value from 1 to 4 to indicate your degree of agreement with the following statements about the teaching and practice of arithmetic algorithms in primary education
(1= strongly disagree, 4= strongly agree)

a) Nowadays it is useless to know the arithmetic algorithms since the calculator can be used
b) They should be taught at the earliest possible age
c) They have to be taught after the meaning of the arithmetic operations has been understood
d) It is not appropriate that children devote time to fully understanding the arithmetic algorithms, as this delays their acquisition of mechanical skills
e) Practising arithmetic algorithms is a closed task that leaves no room for the pupils' initiative
f) In primary education, the learning of arithmetic algorithms must be prioritized over problem solving, so that pupils acquire a feeling of security with calculations
g) In the teaching of algorithms, the emphasis should be placed on children acquiring speed when applying them
h) If it is performed separately from problem solving, the practising of arithmetic algorithms can result in students losing interest in mathematics

Appendix Q1: Text of the questionnaire about understanding arithmetic algorithms (final version)

https://docs.google.com/forms/d/1Qa_IMwpyG9DIZkxynqi88OuvPJ93RQ0Sxo7lJQ4ICLA/copy

1- Assign a value between 1 and 4 to indicate the degree the following statements reflect your experience with the arithmetic algorithms:
(1= not identified at all, 4= totally identified)

a) They have always been boring and repetitive for me
b) I have never been interested in knowing why they are designed the way they are
c) I understand fairly well the reason for each step that is given when an algorithm is applied
d) I found them easy to learn
e) As soon as I was allowed to, I used the calculator to avoid having to apply them

2- Assign a value between 1 and 4 to the following reasons for teaching arithmetic algorithms to children in primary education
(1= disagree, 4= strongly agree)

a) It is useful for both the academic and professional future of the pupils
b) It helps children better understand the properties of numbers and arithmetic operations
c) It offers the pupil the security that their calculations are right
d) It helps the pupil understand the iterative processes of computer programming
e) Faced with an arithmetic problem, the student can focus on the resolution strategy, having mastered the calculations

3- What importance do you give to the following aspects of the teaching of the arithmetic algorithms in order to facilitate students' understanding of the steps involved?

(1= not important at all, 4= very important)

a) Combining their teaching with that of the properties of numbers and arithmetic operations
b) Encouraging the use of a precise vocabulary to name the different units involved
c) Accompanying their teaching with appropriate graphs, diagrams or schemes
d) Accompanying their teaching with suitable manipulatives

4- Choose the most suitable way of performing each of the following types of operations:

a) 23+15 operations: mental computation- classical algorithms – calculator
b) 24x25 operations: mental computation - classical algorithms – calculator
c) 234 x 346 operations: mental computation - classical algorithms – calculator
d) 876 - 582 operations: mental computation - classical algorithms – calculator

5- Point out the reasons (they may be more than one) why you think children encounter difficulties when applying arithmetic algorithms

- Each specific example differs in some way from the others
- They do not practice enough at home
- They have learnt to apply rules without understanding their meaning
- They get distracted while they are applying the algorithms
- They are not able to make a mental image that helps them in the calculation
- They are not used to breaking down numbers
- They do not know how to use a diagram, a graph or a manipulative to support them

- They find no sense in applying them to perform operations without a context situation
- They feel blocked by a fear of making mistakes
- None of the above

6- Assign a value from 1 to 4 to indicate your degree of agreement with the following statements about the teaching and practice of arithmetic algorithms in primary education
(1= strongly disagree, 4= strongly agree)

a) Nowadays it is useless to know the arithmetic algorithms since the calculator can be used
b) They should be taught at the earliest possible age
c) They have to be taught after the meaning of arithmetic operations has been understood
d) It is not appropriate that children devote time to fully understanding the arithmetic algorithms, as this delays their acquisition of mechanical skills.
e) Practising arithmetic algorithms is a closed task that leaves no room for the pupils' initiative
f) In primary education, the learning of arithmetic algorithms must be prioritized over problem solving
g) In the teaching of algorithms, the emphasis should be placed on children acquiring speed when applying them
h) If it is performed separately from problem solving, the practising of arithmetic algorithms can result in students losing interest in mathematics

7- Mark the studies in which you last studied mathematics

- Compulsory secondary education
- Science high school
- Arts high school
- Professional studies
- University studies different from those addressed to primary teaching
- Others

8- Which of the following sentences describes best your current situation

- I am working as a primary teacher
- I worked as a teacher, although not at this moment
- I have never worked as a teacher, although I plan to do so
- I am not planning to work as a teacher

Appendix P2: Pilot text of the questionnaire about solving arithmetic problems

1- Try to solve the following problem and write down either your answer or a comment before continuing:

David went from home to school and, after class he went to his grandparents' house. He walked 525 meters in total. If the distance from home to school is four times longer than the distance from the school to his grandparents' house, how many metres did he walk from home to school?

When you finish, answer the following question

2- Choose an assertion (more than one may be selected) that best identify your response to the above question:

a) I felt comfortable solving the problem
b) I felt blocked, not knowing how to start
c) I tried to remember a similar problem that I had previously solved
d) I tried to get a comprehensive view of the situation implicit in the problem
e) I tried to find out which operations were necessary in order to find the solution from the data
f) I did a drawing, a diagram or a graph
g) None of the above

3- Try to solve the following problem and write down either your answer or a comment before continuing:

If the price of a product is decreased by 50% and, later, this second price is incremented by 50%, then is the third price of the product less than, equal to or greater than the first one?

When you finish, answer the following question

4- Choose an assertion (more than one may be selected) that best identify your response to the above question:

a) It was not possible to solve the problem due to insufficient data
b) This is not an arithmetical problem
c) I am sure that I found the right answer
d) I think that I know the right answer, although I do not know how to demonstrate it
e) I solved the problem by thinking of a particular case
f) I solved the problem by means of doing a drawing, a diagram or a graph
g) I found it too difficult to solve
h) I do not find this problem suitable for Primary Education
i) I answered by mere intuition
j) None of the above

5- Assign a value between 1 and 4 to indicate your degree of agreement with the following statements about your preferences regarding the different kinds of problems:
(1= disagree, 2= slightly disagree, 3= slightly agree, 4= strongly agree)

a) I like problems which can be solved with only one operation
b) I like problems which require the use of several operations
c) I like problems about direct and inverse proportionality
d) I like problems about percentages
e) I like problems about great common divisor and about least common multiple
f) I like problems about combinatorics
g) I like problems that cannot be classified as any standard type
h) I do not like solving problems

6- Assign a value between 1 and 4 to the following reasons for giving arithmetic problems to children in primary education
(1= disagree, 2= slightly disagree, 3= slightly agree, 4= strongly agree)

a) To practise the operation that they have just learnt at class
b) To prepare children for situations that they will find in their academic or professional future
c) To show children that arithmetic knowledge can help them in their day-to-day life
d) To show the relationships between different mathematical concepts that have been worked on in class
e) To help the child develop confidence in their own capacities
f) To get the child to face a challenge that encourages them to put their mathematical knowledge and abilities into practice
g) To present specific situations in which children can converse using mathematical language and argumentation

7- Assign a value between 1 and 4 to your degree of agreement with the following statements about the resolution of arithmetic problems in Primary Education:
(1= disagree, 2= slightly disagree, 3= slightly agree, 4= strongly agree)

a) Giving children arithmetic problems which are difficult for them can lead to children losing interest in mathematics
b) We should avoid presenting arithmetical problems to children with learning difficulties so that they do not get frustrated
c) It is almost impossible for a primary school child to design their own strategy to solve a problem which they have never seen
d) If we want children to learn to solve problems, it is desirable to do as many problems as possible in class
e) In primary education, the learning of operating techniques must be preferred to problem solving, to get the children to gain confidence in doing calculus
f) In order to evaluate the resolution of an arithmetic problem, the most important thing is that the right answer has been obtained
g) Frequently, children do not try to get a comprehensive view of the implicit situation of the problem

h) Pupils believe that they are only allowed to do operations and so they do not use other kinds of strategies (imaging the scene, doing a diagram or drawing, etc.)
i) It is possible that pupils' attitudes towards arithmetic problems can change positively by means of a suitable teaching approach

8- Mark the studies in which you last studied mathematics

- Compulsory secondary education
- Science high school
- Arts high school
- Professional studies
- University studies different from those addressed to primary teaching
- Others

9- Which of the following sentences describes best your current circumstances

- I am working as a primary teacher
- I have worked as a teacher, although I am not currently doing so
- I have never worked as a teacher, although I plan to do so
- I am not planning to work as a teacher

Appendix Q2: Text of the questionnaire about solving problems (latest version)

https://docs.google.com/forms/d/1LUEu8vL9nz-zFHwJS4AjAzpaM7wA3Rf8fXOpnIRSk5E/copy

1- Children's strategies:
What do children typically do when they are facing an arithmetic problem?

- They wait for a sudden inspiration
- They use manipulatives
- They do a drawing, a diagram or a graph
- They discover from the data which operations are necessary in order to find the solution
- They experiment with several options
- They feel blocked and do not know how to start
- They encounter the solution, but they do not understand how they have done so
- They look for a similar problem which has been previously solved in their notebook or book
- They ask the teacher or a peer how to start
- Others

2- Children's difficulties
Why do children find difficulties solving problems?

- They are used to solving exercises in a mechanical and repetitive way
- Each new problem is seen as different from the others
- They do not practise enough at home
- They feel blocked by a fear of making a mistake
- They try to guess which operations are necessary in order to find the solution from the data
- They do not understand the text
- They get confused because there is either too much information or too many conditions to be considered
- It is the teachers' fault
- The use of digital devices is to blame
- Others

3- Your favourite strategies
How did you typically face arithmetic problems as a child at school?
(Please use your personal memories as a pupil)

- I waited for a sudden inspiration
- I used manipulatives
- I did a drawing, a diagram or a graph
- I discovered from the data which operations were necessary to find the solution
- I experimented with possible options
- I felt blocked and did not know how to start
- I encountered the solution, but I did not understand how I had done so

- I looked for a similar problem which had been previously solved in my notebook or book
- I asked the teacher or a peer how to start

4- Your main difficulties
(As in the previous question, use your memories as a pupil to answer)
When you found difficulties in solving problems, which was the cause?

- I was used to solving exercises in a mechanical and repetitive way
- Each new problem was different from the others
- I did not practise enough at home
- I felt blocked by a fear of approaching the task in an erroneous way
- I tried to guess which operations were necessary in order to find the solution from the data
- I did not understand the text
- I got confused because there was either too much information or too many conditions to be considered
- It was the teachers' fault
- Some of my classmates were better at mathematics than me and I was demoralized
- I never had difficulties solving problems
- Others

5.- Assign a value between 1 and 4 to indicate how much you like or dislike the following types of problems:
1.- dislike 2.- slightly dislike 3.- slightly like 4.- like

i) Problems which can be solved with only one operation
j) Problems which require the use of several operations
k) Problems about direct and inverse proportionality
l) Problems about percentages
m) Problems about greatest common divisor and about least common multiple
n) Problems about fractions
o) Problems about combinatorics
p) Problems that do not belong to any standard type
q) All arithmetical problems

6- Assign a value between 1 and 4 to the following reasons for giving arithmetic problems to children in primary education
(1= disagree, 2= slightly disagree, 3= slightly agree, 4= strongly agree)
h) To practise the operation that they have just learnt at class
i) To prepare children for situations that they will find in their academic or professional future
j) To show children that arithmetic knowledge can help them in their day-to-day life
k) To show the relationships between different mathematical concepts that have been worked on in class
l) To help the child develop confidence in their own capacities
m) To get the child to face a challenge that encourages them to put their mathematical knowledge and abilities into practice

n) To present specific situations in which children can converse using mathematical language and argumentation

7- Assign a value between 1 and 4 to your degree of agreement with the following statements about the resolution of arithmetic problems in Primary Education:
(1= disagree, 2= slightly disagree, 3= slightly agree, 4= strongly agree)

a) Giving children arithmetic problems which are difficult for them can lead to children losing interest in mathematics
b) We should avoid presenting arithmetical problems to children with learning difficulties so that they do not get frustrated
c) It is almost impossible for a primary school child to design their own strategy to solve a problem which they have never seen
d) If we want children to learn to solve problems, it is desirable to do as many problems as possible in class
e) In primary education, the learning of operating techniques must be preferred to problem solving, to get the children to gain confidence in doing calculus
f) In order to evaluate the resolution of an arithmetic problem, the most important thing is that the right answer has been obtained
g) Frequently, children do not try to get a comprehensive view of the implicit situation of the problem
h) Pupils believe that they are only allowed to do operations and so they do not use other kinds of strategies (imaging the scene, doing a diagram or drawing, etc.)
i) It is possible that pupils' attitudes towards arithmetic problems can change positively by means of a suitable teaching approach

8- Mark the studies in which you last studied mathematics

- Compulsory secondary education
- Sciences high school
- Humanities high school
- Professional studies
- University studies different from those addressed to primary teaching
- Others

9- Which of the following sentences describes best your current circumstances

- I am working as a primary teacher
- I have worked as a teacher, although I am not currently doing so
- I have never worked as a teacher, although I plan to do so
- I am not planning to work as a teacher

Appendix Q3: Text of the questionnaire on Integrated Arithmetic and Geometry

https://docs.google.com/forms/d/1WavTkg-_TC_wVIqgUaahITLnF2mfWxXEXiIfFHN3tMM/copy

1. Problem.

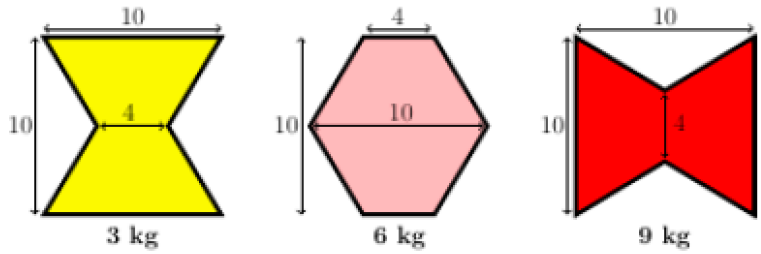
1.1. State a math problem that you would like an eight-year-old to be able to solve at the end of the course when you are a teacher.

Regarding their responses, the respondent is asked to analyze:

1.2. The problem you have chosen belongs to the field of ...
... arithmetic
… geometry
... both
... neither

2. Properties.

2.1. Write four properties that these three colored plates have in common.



Regarding their responses, the respondent is asked to analyze:

2.2. How many properties have you written?
None, one, two, three or four.
2.3. The properties you have found have to do with ...
... only with numeric properties
... only with geometric properties
... with numerical and geometric properties
... with other types of properties

3. Concepts: longitudes and rational numbers.

(How to visualize rational numbers with the help of lengths)

3.1. On your answer sheet, draw two lengths such that one is twice as long as the other.
3.2. On the answer sheet, draw two lengths such that one is one-third the other.
3.3. Draw on the answer sheet two lengths such that one is 4/3 the other.

Regarding their responses, the respondent is asked to analyze:

3.4. Describe how you have tried to solve the problems above:
... using ruler and compass, without using numbers
... visually estimating lengths without using numbers
... using numbers that indicate exact measurements

... using numbers that indicate approximate measurements
... I didn't know

4. Concepts: surface areas and rational numbers.
(How to visualize rational numbers with the help of surfaces)
4.1. On the answer sheet, draw two surfaces such that one is twice as long as the other.
4.2. On the answer sheet, draw two surfaces such that one is one-third the other.
4.3. On the answer sheet, draw two surfaces such that one is 5/4 than the other.

Regarding their responses, the respondent is asked to analyze:
4.4. Describe how you have tried to solve the problems above:
... estimating visually (no numbers)
... using numbers that indicate exact measurements
... using numbers that indicate approximate measurements
... I didn't know

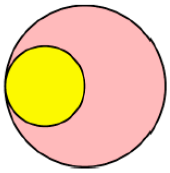
5. Concepts: longitudes and irrational numbers.
(How to visualize irrational numbers with the help of lengths)
5.1. On the answer sheet, draw two lengths such that one is the root of the other twice.

Regarding their responses, the respondent is asked to analyze:
5.2. Have you solved the problem above?
Yes, no, or I'm not sure.
5.3. Describe how you have tried to solve the problems above:
... using the compass (no numbers)
... estimating visually (no numbers)
... using numbers that indicate exact measurements
... using numbers that indicate approximate measurements

6. Concepts: surface areas and longitudes.
6.1. If all the sides of a square are tripled, its area ...
... multiplied by 3
... multiplied by 6
... multiplied by 9
… I'm not sure

Regarding their responses, the respondent is asked to analyze:
6.2. Describe how you have tried to solve the problem above:
... drawing the two squares and using the area formula with the side measurements.
... by drawing the two squares and using the area formula without the side measurements.
... drawing the two squares and reasoning about the figure.
... in any of the above ways.

7. Surface areas and longitudes.
7.1. What is the relationship between the area of the yellow circle and the large one?



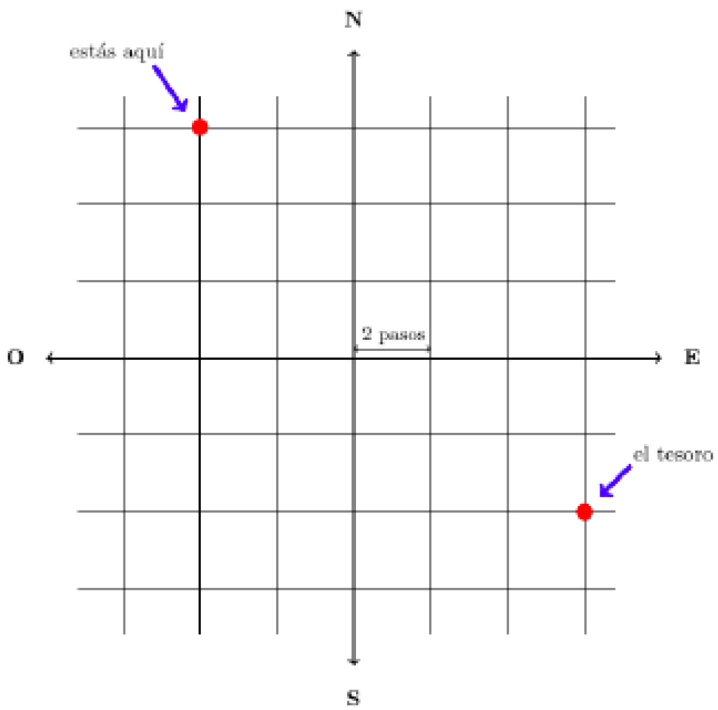
... it is half
... it is one third
… it is one fourth
… I'm not sure

Regarding their responses, the respondent is asked to analyze:

7.2. Describe how you have tried to solve the problem above:
... reasoning only on the figure.
... reasoning about the figure and using the formula for the area with the radius measure.
... reasoning on the figure and using the area formula without the radius measure.
... in none of the above ways.

8. Treasure Map. A practical challenge.

8.1. Write precise instructions to find the hidden treasure for someone who cannot see the map.



Regarding their responses, the respondent is asked to analyze:

8.2. Describe how you have tried to solve the problem above:
... using the references N, S, E, W.
... using Cartesian coordinates.
... using body references (right, left).

... in none of the above ways.

9. Teaching of mathematics

(From each pair of statements, choose the one you agree with the most)

9.1.a. Elementary mathematics should ensure that the child knows how to handle numbers. Visual representations can be used for this.

9.1.b. Mathematics should ensure that the child understands well the geometric relationships of his environment. For this you can use the numbers and their relationships.

9.2.a. To teach elementary mathematics, it is good to teach both arithmetic and geometric concepts, even if it takes longer to become fluent in calculation.

9.2.b. To teach elementary mathematics, it is good to establish an understanding of numbers and their operations, before moving on to studying geometric relationships.

9.3.a. Children understand mathematics in different ways: using numbers, using geometric representations ... We must provide each one with this approximation in all ages.

9.3.b. The use of visual and geometric representations makes it difficult to move towards abstraction and therefore it is better to dispense with them progressively.

9.4.a. I feel more comfortable working with numbers.

9.4.b. I feel more comfortable working with geometric shapes.

10. Question about personal profile.

10.1. Indicate the level of studies in which you took a mathematics class for the last time:

... Compulsory secondary

… Bachelor of Science

... Bachelor of Humanities and Social Sciences

… Vocational training

... University studies other than teacher training

… None of the above

10.2. Which of the following phrases best describes your current situation:

... I am working as a teacher

... I have worked as a teacher, although now I am not

... I have never worked as a teacher, although I would like to do so in the future

... I don't plan to work as a teacher

… None of the above

Appendix Q4: Text of the questionnaire on mental computation and the use of the calculator

https://docs.google.com/forms/d/1bXDuJcWb3zPvXaxBflW-O5HKk4XD7F7BFoGK9dejbIU/copy

1. Regarding mental computation, assign a value between 1 and 4 to the following concepts.
(1. strongly disagree - 4. strongly agree)
Dislike
Initiative
Reasoning
Speed
Play
Training
Strategy
Automatism
Usefulness
Attention

2. In your opinion, why learn mental computation in elementary school? (It is possible to tick several boxes)
   a. To develop memory skills
   b. To develop reasoning and argumentation skills
   c. To build and strengthen knowledge of the properties of numbers and operations
   d. To identify different ways of carrying out the same computation
   e. To check the result displayed by a calculator
   f. To evaluate the order of magnitude of a result
   g. To assist in problem solving

3. Assign a value between 1 and 4 to your degree of agreement with the following sentences.
(1. strongly disagree - 4. strongly agree)
a) To do mental computations, no written support should be provided.
b) Pupils tend to make mental computations by putting numbers in columns in their head.
c) When providing pupils with a complex mental arithmetic activity, the focus should be on procedure rather than speed.
d) In elementary school, mental computation should occupy a privileged place in relation to computation in columns.
e) From the beginning of elementary school, to provide the result of an additive computation, the use of fingers should be prohibited.
f) The lack of mastery of mental computation makes it difficult to learn the arithmetic algorithms.
g) The construction and memorization of the addition and multiplication tables are facilitated by their ritual verbal repetition, in ascending order.

ABOUT THE CALCULATOR (The calculator has now become an object of everyday consumption. In elementary school, can it be used in combination with mental computation and computation in columns?)

4. In your opinion, what is the use of the calculator in elementary school?
(It is possible to tick several boxes)
   - It is a tool for computing

- It is a tool to check the result of a computation
- It is a support to explore numbers
- It is a source of problems and exercises
- It is a tool to reduce memorization
- It is a tool for performing computations with big numbers or with a lot of numbers, otherwise difficult to perform by mental or posed computations
- It is a tool for solving problems that require a lot of testing
- It is a tool for students to solve a problem without worrying about possible errors in the process of computation

5. It is assumed that students are allowed to use the calculator. We should allow them to use it (it is possible to check only one box)
   - after the pupils mastered the algorithms of the basic computations.
   - after the algorithms of the basic computations have been introduced to the pupils.
   - simultaneously while the pupils are learning the algorithms of the basic computations.
   - without the pupils necessarily knowing the algorithms of the basic computations.

6. Assign a value between 1 and 4 to the following sentences.
(1. strongly disagree - 4. strongly agree):

a) The calculator is more often used at home than at school.
b) Introducing the calculator too early hinders the development of students' mathematical skills.
c) Basic mathematical skills deteriorate if the calculator is used indiscriminately.
d) The calculator prevents the pupil from thinking.
e) Over-reliance on the calculator is a sign of deficient mathematical knowledge.
f) If the pupil does not know how to compute, the calculator will not teach them how to do so.
g) The calculator can be used to provide the pupils with interesting learning situations.
h) A variety of numerical skills is required to use the calculator efficiently.
i) The use of the calculator is a significant barrier to mental computation.

7. How would you do the following computations?
(It is possible to tick only one box for each computation)

Mental computation In columns (algorithm) Calculator

- 657 + 95 + 48
- 3456 - 897
- 12 x 19
- 10008 : 9

Appendix P5: Pilot text of the questionnaire about the History of Mathematics and its teaching

1. Books about the history of mathematics:
   Have you ever read or watched anything about the history of mathematics? If so, which one?

2. Assign a value from 1 to 4 to indicate your degree of agreement with the following sentence (1: strongly disagree 4: strongly agree)
   Mathematics has become a complete, well-structured body of knowledge

3. With regard to history:
   Do you find it appropriate to use history and stories as a starting point for transmitting mathematical contents to pupils?

4. Systems of numeration different from ours:
   Is there any system of numeration that is different from our decimal positional system?

5. Do you know any additive system of numeration apart from the Roman one? If so, which one?

6. Names of ancient mathematicians:

   We all know famous Greek mathematicians: Pythagoras, Euclid, Archimedes, and Tales. Do you know any other ancient mathematician? If so, write down who s/he is.

7. Names of more recent mathematicians:

   Do you know non-Italian mathematicians who lived after the Middle Ages? If so, write them down below.

8. Famous books of mathematics:

   If you know any famous the title of a famous book of mathematics, write it here.

9. The text book:

   In mathematics at school, the text book is: (maximum two answers)
   - A starting point
   - All you need
   - A guide
   - A burden

Appendix Q5: Text of the questionnaire about the History of Mathematics and its teaching (final version)

https://docs.google.com/forms/d/1YGUvdI1SDH51RMJPk9PIBdflGl6kEjhiLCqHN54KWlQ/copy

1. Books about the history of mathematics:

Have you ever read or watched anything about the history of mathematics? If so, which one?

2. Assign a value from 1 to 4 to indicate your degree of agreement with the following sentence (1: strongly disagree 4: strongly agree):

Mathematics has become a complete, well-structured body of knowledge

3. With regard to history…

Do you find it appropriate to use history and stories as a starting point for transmitting mathematical contents to pupils?

4. Systems of numeration different from ours:

Write an example of a positional numbering system different from the decimal one, if you know it

5. Mention an example of an additive numbering system different from the Roman one, in case you know another one.

6. We all know famous Greek mathematicians: Pythagoras, Euclid, Archimedes, and Tales. Do you know any other ancient mathematician? If so, write down who s/he is.

7. Do you know non-Italian mathematicians who lived after the Middle Ages? If yes, write them down below.

8. If you know any famous mathematics book, write it here.

9. In the mathematics subject at school, the textbook is: (maximum two answers)
   - A starting point
   - Everything you need
   - A support
   - A charge

Appendix Q6: Text of the questionnaire on Geometry

https://docs.google.com/forms/d/1GPnSXgao4RWPhcRgKqGk1ciAKMBEs5tTKucc7HDqFAc/copy

1. When you think about Geometry, which three words come to mind?

2. What do you prefer, Geometry or Arithmetic?

3. Where should we start, with solid geometry or plane geometry?

4. What importance do you give to the following activities in Geometry?

- Working with your hands
- Designing
- Visual observation
- Making comparisons
- Generalizations
- Measurements
- Classifying
- Calculating
- Learning the formulae
- Memorizing
- Distinguishing between shapes
- Movement

5. What importance do you give to the following topics in Geometry?
- Equivalent figures
- Isoperimetric plane figures
- Constructions with ruler and compass
- Symmetries and isometries
- Straights lines and their relative positions
- Angles
- Polyhedra
- Circle, circumference, and Pi
- Segments
- Cartesian plane

6. Assign a value from 1 to 4 to indicate your degree of agreement with the following statements:
a) With young children, the distinction between a square, a triangle and a circle should be emphasized
b) It is possible to teach a concept such as the line tangent to a curve at primary education

7. Indicate three manipulatives which are useful to work on geometry with children.