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MARIA ALESSANDRA CONGEDO,
MASSIMILIANO CORRADINI,
ANDREA GHENO, JESSICA RICCIONI,
CHIARA SANSONI

# PORTFOLIO SELECTION USING BEHAVIORAL MODELS

RomaTrE-Press
2023

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Portfolio Selection using behavioral models

*Maria Alessandra Congedo, Massimiliano Corradini, Andrea Gheno, Jessica Riccioni, Chiara Sansoni*

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# Portfolio Selection using behavioral models

Maria Alessandra Congedo, Massimiliano Corradini, Andrea Gheno,
Jessica Riccioni, Chiara Sansoni*

ABSTRACT

Nell'ambito dei problemi di scelta in condizioni di rischio introduciamo un nuovo modello comportamentale e implementiamo un'analisi empirica basata sul confronto con il classico approccio della Teoria del Prospetto. L'applicazione di questi modelli è validata nella selezione di portafoglio attraverso tre modelli di portafoglio tradizionali utilizzati come riferimento (portafoglio a varianza minima, portafoglio a deviazione mediana assoluta e portafoglio equamente ponderato). L'obiettivo di questo lavoro di ricerca è quello di incorporare la percezione del rischio degli investitori nelle scelte dei portafogli ottimali. Proponiamo un'analisi out-of-sample di quattro indici azionari per dimostrare la superiorità dei modelli comportamentali di selezione di portafoglio rispetto a quelli tradizionali.

KEYWORDS: Portfolio Selection; Behavioral Models; Prospect Theory

PAROLE CHIAVE: Selezione di Portafoglio; Modelli Comportamentali; Teoria del Prospetto

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## 1. Introduction and literature review

This research work arises from the need to deal with decision-making under risk problems in the context of portfolio theory. In particular, our aim is to decline the Prospect Theory (PT) (Kahneman and Tversky, 1979) to the portfolio selection problem investigating how people's behavior affects portfolio decisions. We are aware that applying portfolio selection models that do not incorporate the investor's perception of risk can lead to sub-optimal investment decisions (see, e.g., Hens and Bachmann, 2008; Shefrin and Statman, 1985). Motivated by the idea that the rational process of choice is conditioned by cognitive distortions (or biases) and by the research work of Kahneman and Tversky (1979), we propose an empirical analysis based on a behavioral approach using both the Kahneman and Tversky function and a new function devised by Cenci *et al.* (2015) which represents a novelty in the field of portfolio selection. To corroborate the use of behavioral models in portfolio selection, we propose a comparison with traditional portfolio models through some performance indicators from a rolling-windows analysis perspective.
The descriptive models of individuals' behavior in decision-making under risk developed over the years and originated in the mid-1900s. Until then, the choice of an ordering criterion that transforms risky alternatives into certain equivalents and allows for a comparison between the various alternatives was entrusted to the logarithmic utility function proposed by Bernoulli (1738). By the St. Petersburg Paradox, one of the oldest paradoxes, Bernoulli overcomes the criterion of mathematical expectation that neglects the riskiness of the decision maker (DM) and proposed to model the preferences of individuals by introducing their attitude towards risk with the moral expectation.
For two centuries, Bernoulli's criterion was the predominant one until the expected utility theory (EU) proposed by von Neumann *et al.* (1944). According to the EU, if the preferences of individuals in risky decision situations respect some fundamental axioms, then there is rationality in the decision-making process, and the DM maximizes the expected utility function.
According to the EU, individuals always choose rationally. Many criticisms have followed over the years due to the incompatibility between theorized rational behavior and real behavior which can be influenced by multiple factors.
The EU theory is overcome with the Allais paradox (Allais, 1953), which exposes human irrationality through the systematic violation of one of the key axioms of the EU, which is that of independence. In the following years, many researchers have tried to develop models that describe DM preferences

by weakening the assumptions underlying the EU theory (see Weber and Camerer (1987)).
A turning point comes with Kahneman and Tversky's experimental evidence (Kahneman and Tversky, 1979) and the introduction of Prospect Theory (PT) into the literature. With PT, they explained and modeled how individuals make economic choices observed in the laboratory. These experiments correspond to hypothetical choice problems between two random options with known probabilities.
According to PT, behaviors are affected by cognitive biases. The DM exhibits risk-averse behavior over gains and risk-seeking behavior over losses. The property of loss aversion also holds since they are more sensitive to losses than gains on equal terms. In choosing among various alternatives, investors link potential gains and losses to a reference point. They overestimate the probabilities of large gains and losses.
The DM choices are modeled based on the value and probability weighting functions. The value function is concave for gains and convex for losses, so risk aversion is incorporated into the gains and risk seeking into the losses domain. The value function is steeper below the reference point to reflect the characteristic that losses outweigh gains.
The weighting function $\pi(p)$ represents decision weights as a function of probabilities and it is monotone increasing in the probability interval $[0, 1]$. Through the bias provided by the weighting function, which is a non-linear transformation of probabilities, DM transforms the objective probabilities by underestimating the medium-high probabilities and overestimating the low ones linked to extreme events.
Thus, the value function models individuals' risk attitude and loss aversion, and the weighting function models the perception of risk through probability bias.
The PT was further developed by Tversky and Kahneman (1992) with the introduction of the cumulative prospect theory (CPT) to include more than two outcomes and to solve the problem of PT of a potential violation of first-order stochastic dominance. In the CPT, the weights depend on the cumulative distribution function.
According to the CPT, individuals are risk averse to low probability and potentially large losses and risk seeking to small and very probable losses. For gains, it is the opposite; they are risk averse in situations where gains are fairly possible and not too large in amount. Instead, they are risk-seeking when they consider very high and unlikely gains.
Over the years, many researchers have worked on violations involving both the

PT and the CPT (see, e.g., Baltussen *et al.,* 2006; Humphrey, 1995; Marley and Luce, 2005; Wu and Markle, 2008; Wu *et al.,* 2005).

In recent times, Birnbaum (2008) developed an empirical study that proposed 11 new paradoxes to refute PT as a theory capable of modelling the behavior of DM under risk.

In literature, Birnbaum (2019), Birnbaum and Chavez (1997), Marley and Luce (2001), have presented many models that have tried to overcome PT. Here, we want to deepen the model developed by Cenci *et al.* (2015), through which they try to capture the behavior of individuals with a theory that takes the name of "Half-Full/Half-Empty" (HF-HE). These authors intended to propose a less complex model than existing in the literature, with fewer parameters and which could explain many of the paradoxes that have occurred over the years. Motivated by the fact that individuals can choose to pay attention to the negative aspects of a situation (pessimists) or the positive ones (optimists), evaluating the same situation differently, the authors take the average value of a lottery as a reference point and people are divided between optimists and pessimists according to their evaluation of the possible lottery values (above or below the average). It is an intuitive model, easy to apply and models behavior through three parameters calibrated to respect the paradoxes of Kahneman and Tversky: $\lambda+$, $\lambda-$ e $q$.

$\lambda$ considers the DM's degree of optimism/pessimism, while $q$ incorporates the probability distortion to obtain the decision weights.

The classical PT approach and the HF-HE model are tested in this paper to address a portfolio selection problem. In portfolio selection, individuals make a profitable investment in the stock market to minimize risk. The two key theories of this field are those proposed by Markowitz (1952) and Konno and Yamazaki (1991) (Markowitz's portfolio optimization model and Mean-Absolute Deviation portfolio optimization model), which we use as comparison models in our empirical analysis together with an equally weighted portfolio which represents our benchmark.

The merging between portfolio theory and behavioral models takes place to incorporate the risk perceived by investors in the selection choices of the stocks that make up the optimal portfolios. Otherwise, the risk is that of making sub-optimal investment choices (Hens and Bachmann, 2008). There is theoretical evidence in the literature about risk reduction in portfolio models based on the behaviors of DM who are naturally loss averse (Grishina *et al.*, 2017).

There are many behavioral portfolio models that are based on PT or CPT.

Benartzi and Thaler (1995) use a simulation approach to propose a combination of portfolio choice and PT called myopic loss aversion. H. Levy and M. Levy (2003) optimize according to the mean-variance criterion and select the portfolios on the efficient frontier with the highest prospect utility in the case of normal returns. Gomes (2005) related portfolio selection to stock trading volume considering investors loss averse. Kahneman and Tversky (1979) used the PT with equally probable scenarios to solve the portfolio optimization problem using a heuristic approach. Barberis *et al.* (2006), Bernard and Ghossoub (2010), He and Zhou (2011) and Consigli *et al.* (2019) modeled portfolio selection problems focusing the analysis on the CPT and exploiting the certain equivalent associated with the objective function of the CPT. Pirvu and Schulze (2012) generalized results of H. Levy and M. Levy (2003) for elliptic symmetric distributions of risky assets. Barro *et al.* (2020) maintained the probability bias and solved the computational difficulties by exploiting meta-heuristic algorithms. The structure of this research is the following: in Section 2 we present the behavioral models compared in the empirical analysis. In Section 3 we illustrate the classic portfolio models used as benchmarks and necessary for validating the results obtained. Section 4 contains the empirical analysis including the description of the dataset, the performance measures adopted, the optimization algorithm used and the results obtained. In the last section are included the conclusions.

## 2. The behavioral models

We first introduce the notation used. Linear returns were considered in the analysis. If $p_{kt}$ is the price of asset $k$ at time $t$, we have $r_{k,t} = \frac{p_{kt} - p_{k,(t-1)}}{p_{k,(t-1)}}$ which represents the return at time $t$. Assuming an investment universe composed of $n$ assets, the portfolio return $R$ with weights $x$ at time $t$ is $R_t(x) = \sum_{k=1}^{n} x_k r_{k,t}$. $\mu$ is the vector of the expected returns of the $n$ assets and $\sum$ is the covariance matrix, where the generic element $\sigma_{k,j}$ is the covariance of the return of assets $k$ and $j$ with $k, j = 1, \dots n$.

For all models, the eligible portfolios are determined by the budget constraint ($\sum x_k = 1$) and the impossibility of short sales ($x_k \geq 0 \quad k = 1, \dots, n$).

## 2.1 Classical approach of prospect theory

The portfolio model is the one that uses the PT value function (PT model) with equally probable historical scenarios.

$$
\begin{aligned}
\max_{x} \quad & \frac{1}{T}\sum_{t=1}^{T}\Big[\Big(\Big(\sum_{k=1}^{n} x_k r_{kt}\Big)_{+}\Big)^{\alpha} - \beta\Big(\Big(\sum_{k=1}^{n} x_k r_{kt}\Big)_{-}\Big)^{\alpha}\Big] \\
\text{s.t.} \quad & \sum_{k=1}^{n} x_k = 1 \quad k = 1, \dots, n, \\
& \quad x_k \geq 0 \quad k = 1, \dots, n
\end{aligned} \tag{1}
$$

where:

$$
\begin{aligned}
(a_k)_{-} &= \min\{a_k, 0\} \\
(a_k)_{+} &= \max\{a_k, 0\}
\end{aligned}
$$

We implement the theoretical formalization proposed in De Giorgi *et al.* (2007) by making some modifications. We have a problem when applying PT to portfolio theory: the value function is concave in the domain of gain and convex in the domain of losses. As a consequence the first-order condition can only describe local optima. Problem (1) is then solved with a multistart heuristic approach similar to the one proposed by Gilli *et al.* (2019). More precisely, we use the Matlab *fmincon* local solver, and the starting points have been selected by Rubistein's algorithm (interior point). We generate the starting points randomly and uniformly on the simplex.

The parameters are those estimated by Kahneman and Tversky: $\alpha = 0.88$ e $\beta = 2.25$.

## 2.2 Half-Full/Half-Empty approach

### 2.2.1 Preliminary concepts

In the first version of HF-HE (Cenci *et al.*, 2015), only positive lotteries with $Y = (p_1 : y_1, p_2 : y_2, \ldots, p_N : y_N)$ and $y_i \geq 0$ for $i = 1, \ldots N$ are considered.
By the linearity property, we can express the expected value ($EV$) of lottery $Y$ as:

$$H_{EV} = \mathbb{E}[Y] = \mu + \mathbb{E}[Y - \mu] \tag{2}$$

where $\mu = \mathbb{E}[Y]$ is the expected value of $Y$.
Distinguishing the part below the average from the part above the average, we obtain:

$$H_{EV}(Y) = \mu + 2\Big\{\frac{1}{2}\mathbb{E}[(Y-\mu)_+] + \frac{1}{2}\mathbb{E}[(Y-\mu)_-]\Big\} \tag{3}$$

The expected value criterion can be generalized by assuming that the DM assigns a weight different from $\frac{1}{2}$ to the above-average and below-average outcomes. We consider the parameter $\lambda$ and Equation (3) becomes:

$$H_\lambda(Y) = \mu + 2\big\{\lambda\mathbb{E}[(Y-\mu)_+] + (1-\lambda)\mathbb{E}[(Y-\mu)_-]\big\} \quad \text{with} \quad 0 \leq \lambda \leq 1 \tag{4}$$

The optimistic DM assigns a higher weight to the outcomes above the average $\mathbb{E}[(Y-\mu)_+]$ and therefore $\frac{1}{2} < \lambda \leq 1$. If, on the other hand, the DM is pessimistic and $0 \leq \lambda < \frac{1}{2}$, the outcomes below the average have a greater weight $\mathbb{E}[(Y-\mu)_-]$.
Equation (4) leads back to (3) and the DM is neither optimistic nor pessimistic, if $\lambda = \frac{1}{2}$.

To frame the HF-HE functional in portfolio selection, we need to reformulate it in the context of mixed lotteries. If in Model (4), $Y$ admits both positive and negative outcomes, we cannot differentiate the risk attitude of the DM in the domain of gains and losses. If $Y \geq 0$ risk-averse behavior can be described by $\lambda < \frac{1}{2}$ and negative outcomes weigh more. If instead $Y \leq 0$ risk-seeking behavior can be described by $\lambda > \frac{1}{2}$ and positive outcomes weigh more. We repeat Problems (2) and (3) for the random variable $Y$ which admits both positive and negative values.

$$
\begin{aligned}
Y_- &= \big((y_1)_-, (y_2)_-, \ldots, (y_N)_-\big) \\
Y_+ &= \big((y_1)_+, (y_2)_+, \ldots, (y_N)_+\big)
\end{aligned}
$$

where:

$$
\begin{aligned}
(y_i)_- &= \min\{y_i, 0\} \\
(y_i)_+ &= \max\{y_i, 0\}
\end{aligned}
$$

By the linearity property of the expected value, and distinguishing the above-average and below-average parts, we have:

$$
\begin{aligned}
H_1[Y] &= \mathbb{E}[Y - \mu] + \mu = \mathbb{E}[Y_+ + Y_- - \mu_+ - \mu_-] + \mu = \\
&= \mu + \mathbb{E}[(Y_+ - \mu_+)] + \mathbb{E}[(Y_- - \mu_-)]
\end{aligned}
\tag{5}
$$

Given that:

$$
\begin{aligned}
Y_+ - \mu_+ &= (Y_+ - \mu_+)_+ + (Y_+ - \mu_+)_- \\
Y_- - \mu_- &= (Y_- - \mu_-)_+ + (Y_- - \mu_-)_-
\end{aligned}
\tag{6}
$$

Inserting (6) in (5), we obtain:

$$
\begin{aligned}
H_1[Y] &= \mu + \big\{\mathbb{E}\big[(Y_+ - \mu_+)_+ + (Y_+ - \mu_+)_-\big] + \mathbb{E}\big[(Y_- - \mu_-)_+ + (Y_- - \mu_-)_-\big]\big\} = \\
&= \mu + \big\{\mathbb{E}\big[(Y_+ - \mu_+)_+\big] + \mathbb{E}\big[(Y_+ - \mu_+)_-\big]\big\} + \big\{\mathbb{E}\big[(Y_- - \mu_-)_+\big] + \mathbb{E}\big[(Y_- - \mu_-) \\
&= \mu + 2\Big\{\frac{1}{2}\mathbb{E}[(Y_+ - \mu_+)_+] + \frac{1}{2}\mathbb{E}[(Y_+ - \mu_+)_-]\Big\} + \\
&+ 2\Big\{\frac{1}{2}\mathbb{E}[(Y_- - \mu_-)_+] + \frac{1}{2}\mathbb{E}[(Y_- - \mu_-)_-]\Big\}
\end{aligned}
\tag{7}
$$

The expected value criterion expressed in (7) can be generalized as:

$$
\begin{aligned}
H_2[Y] &= \mu + 2\Big\{\lambda_+\mathbb{E}[(Y_+ - \mu_+)_+] + (1 - \lambda_+)\mathbb{E}[(Y_+ - \mu_+)_-]\Big\} + \\
&+ 2\Big\{\lambda_-\mathbb{E}[(Y_- - \mu_-)_+] + (1 - \lambda_-)\mathbb{E}[(Y_- - \mu_-)_-]\Big\}
\end{aligned}
\tag{8}
$$

Through (6), Equation (8) can be expressed as:

$$\begin{aligned} H_2[Y] &= \mu + 2\Big\{\lambda_+\mathbb{E}[(Y_+ - \mu_+)_+] + (1-\lambda_+)(-1)\mathbb{E}[(Y_+ - \mu_+)_+]\Big\} + \\ &\quad + 2\Big\{\lambda_-\mathbb{E}[(Y_- - \mu_-)_+] + (1-\lambda_-)(-1)\mathbb{E}[(Y_- - \mu_-)_+]\Big\} = \\ &= \mu + 2\Big\{(2\lambda_+ - 1)\mathbb{E}[(Y_+ - \mu_+)_+]\Big\} + 2\Big\{(2\lambda_- - 1)\mathbb{E}[(Y_- - \mu_-)_+]\Big\} \end{aligned} \tag{9}$$

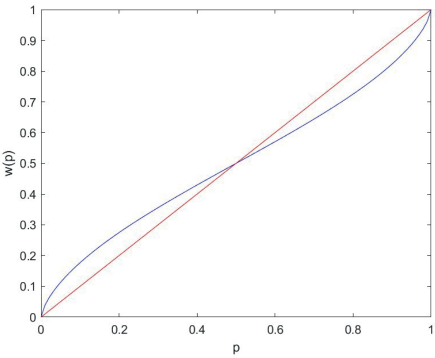
Corradini (2022) proposed an extension of the HF-HE model in the context of negative lotteries (experiments of Kahneman and Tversky (1979)), demonstrating that parameter $\lambda_-$ is compatible with $\lambda_- = 1 - \lambda_+$ (chosen for simplicity). Relation (9) can then be rewritten as:

$$\begin{aligned} H_2[Y] &= \mu + 2(2\lambda_+ - 1)\mathbb{E}[(Y_+ - \mu_+)_+] + 2(1 - 2\lambda_+)\mathbb{E}[(Y_- - \mu_-)_+] = \\ &\mu + 2(2\lambda_+ - 1)\mathbb{E}[(Y_+ - \mu_+)_+ - (Y_- - \mu_-)_+] \end{aligned} \tag{10}$$

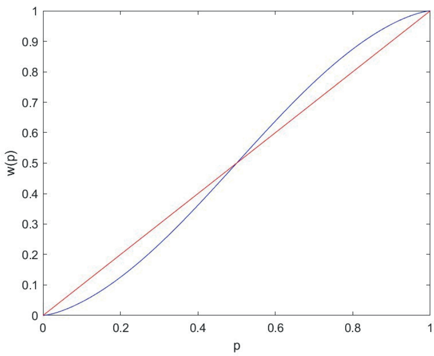
Expression (10) can be expressed in terms of mean absolute deviation (MAD) that applies the same logic to the positive and negative parts.
Recalling that $(Y - \mu)_+ = \frac{|(Y-\mu)| + (Y-\mu)}{2} \Rightarrow \mathbb{E}[(Y-\mu)_+] = \frac{1}{2}\mathbb{E}[|Y-\mu|]$, the HF-HE model turns out to be:

$$H_2[Y] = \mu + (2\lambda_+ - 1)\mathbb{E}\Big[|Y_+ - \mu_+| - |Y_- - \mu_-|\Big] \tag{11}$$

To incorporate the probability distortion into the HF-HE model, we introduce the following normalized weight function $w(p_i, p, q) = \frac{p_i}{\sum_{j=1}^{N} p_j^q}$ with $p = \{p_1, p_2, ..., p_N\}$. In Figure 1 is represented the weight function $0 < q < 1$ and $q > 1$ for a binary lottery $Y = \{(y_1, p); (y_2, 1 - p)\}$.



(a) $q = 0.7$



(b) $q = 1.4$

Figure 1: HF-HE: weight function respectively for $0 < q < 1$ and $q > 1$.

In the case $q < 1$ weight function assumes the typical inverted S $-$ *shape* (the probability of more probable events is underestimated and that of less probable events is overestimated). If instead $q > 1$ weight function assumes the S $-$*shape*. If $q = 1$, there is no probability distortion. We generalize (11):

$$H_2[Y] = \mu_q + (2\lambda_+ - 1)\mathbb{E}_q\big[|Y_+ - \mu_+| - |Y_- - \mu_-|\big]$$

where:

$$\begin{aligned} \mu_q &= \mathbb{E}_q[Y] \\ \mathbb{E}_q[g(Y)] &= \sum_{j=1}^{N} w_q(p_j) g(y_j) \\ w_q(p_i) &= \frac{p_i}{\sum_{j=1}^{N} p_j^q} \end{aligned} \tag{12}$$

### 2.2.2 HF-HE portfolio model

In the context of portfolio selection, we use historical data and all observations are equally probable. Probability distortion, therefore, does not affect optimal portfolio choice.

In Equation (12) replacing $p_i = \frac{1}{T}$, the model leads back to (11):

$$H(Y) = \mu + (2\lambda_+ - 1)\mathbb{E}\big[|Y_+ - \mu_+| - |Y_- - \mu_-|\big] \tag{13}$$

where $Y$ is the random variable, i.e. portfolio weighted returns $\sum_{k=1}^{n} x_k r_{kt}$ and it is possible to distinguish the above-average and below average parts as follows:

$$Y_+ = R_P^+(Y) = \max(Y,0) = \max\Big(\sum_{k=1}^{n} x_k r_{kt}, 0\Big)$$

$$\mu_+ = \mathbb{E}[Y_+] = \frac{1}{T}\sum_{t=1}^{T}\max\Big(\sum_{k=1}^{n} x_k r_{kt}, 0\Big)$$

$$Y_- = R_P^-(Y) = \max(-Y,0) = \max\Big(-\sum_{k=1}^{n} x_k r_{kt}, 0\Big)$$

$$\mu_- = \mathbb{E}[Y_-] = \frac{1}{T}\sum_{t=1}^{T}\max\Big(-\sum_{k=1}^{n} x_k r_{kt}, 0\Big)$$

$$Y = Y_+ - Y_-$$

For the random variables $Y_-$ (negative portfolio weighted returns), we preferred to use the notation $\max\left(-\sum_{k=1}^{n} x_k r_{kt}, 0\right)$ which allows us to work with positive quantities.

Model (11) represents the objective function that the individual wants to maximize subject to the constraints of the inhibition of short selling and full investment. The optimization problem can be reduced to:

$$\begin{aligned}
\max_x \quad & \mu + (2\lambda_+ - 1)\frac{1}{T}\sum_{t=1}^{T}\Big|\max\Big(\sum_{k=1}^{n} x_k r_{kt}, 0\Big) - \frac{1}{T}\sum_{t=1}^{T}\max\Big(\sum_{k=1}^{n} x_k r_{kt}, 0\Big)\Big| + \\
& - (2\lambda_+ - 1)\frac{1}{T}\sum_{t=1}^{T}\Big|\max\Big(-\sum_{k=1}^{n} x_k r_{kt}, 0\Big) - \frac{1}{T}\sum_{t=1}^{T}\max\Big(-\sum_{k=1}^{n} x_k r_{kt}, 0\Big)\Big| \\
\text{s.t.} \quad & \\
& \sum_{k=1}^{n} x_k = 1 \\
& x_k \geq 0 \qquad k = 1,\ldots,n
\end{aligned} \tag{14}$$

The final problem appears similar to MAD, although MAD considers equally the positive and negative outcomes. Instead, the HF-HE model differentiates the two components through the parameter $\lambda$ ($\lambda_+$ and $\lambda_-$).

It is possible to observe that, for risk-averse investor, therefore with $\lambda_+ < 0.5$, the optimization problem implies that he is simultaneously minimizing the 'positive MAD' and maximizing the 'negative MAD'; the reasoning is mirrored in the case of $\lambda_+ > 0.5$. To determine the values of $\lambda$, Cenci *et al.* (2015) repeated the experiments carried out by Kahneman and Tversky and Birnbaum fixing the probability distortion parameter $q$ and taking all those eligible $\lambda$ values.
The value of $\lambda$ that allows all paradoxes to be satisfied simultaneously is approximately $\lambda = 0.31$ (on average). Since in this paper we consider equiprobable scenarios, we do not take into account the distortion of the probabilities $q$. We have decided to take a wide range of values of $\lambda$. Problem (14) has been solved for $\lambda = 0.29$, $\lambda = 0.30$, $\lambda = 0.31$ and $\lambda = 0.32$.
It is classified as a nonlinear programming problem since the objective function is piecewise linear.
The proposed method for solving the optimization problem involves the use of *fmincon* solver with multistart algorithm.

# 3 Other portfolio models analysed

In this section, we describe some classical portfolio selection models that are used to compare with the performance of the behavioral models of the previous section: minimum-variance portfolio, mean absolute deviation portfolio and portfolio equally weighted. The portfolio equally weighted (EW) is used as a benchmark. It is a portfolio in which the capital is equally distributed among the assets. Each asset $k = 1, \ldots, n$ has weight $x_k = \frac{1}{n}$ (De Miguel *et al.*, 2009).

## 3.1 Minimum-variance portfolio

The minimum-variance portfolio (MinV) of Markowitz (Markowitz, 1952) is obtained by minimizing the variance of the portfolio return without any constraint on the expected return:

$$\begin{aligned} \min_x \quad & \sum_{k=1}^{n}\sum_{j=1}^{n} \sigma_{k,j} x_k x_j \\ \text{s.t.} \quad & \sum_{k=1}^{n} x_k = 1 \\ & x_k \geq 0 \quad k = 1, \ldots, n \end{aligned} \tag{15}$$

Differently from PT, variance is a risk measure which weighs the negative and positive deviations equally. It is a model that is not sensitive to the highest moments of the distribution (Hens and Bachmann, 2008).

## 3.2 Mean absolute deviation portfolio

The mean absolute deviation portfolio (MAD) (Konno and Yamazaki, 1991) is the expected value of the absolute deviations of portfolio returns $R_P(x)$ from their mean $\mu_P(x)$:

$$MAD(x) = E[|R_P(x) - \mu_P(x)|] = E\left[\left|\sum_{k=1}^{n} R_k x_k - \sum_{k=1}^{n} \mu_k x_k\right|\right]$$

We minimize this risk measure and assuming equally probable historical scenarios, the optimization problem becomes:

$$\begin{aligned} \min_{x} \quad & \frac{1}{T}\sum_{t=1}^{T}\left|\sum_{k=1}^{n} r_{k,t} x_k - \sum_{k=1}^{n} \mu_k x_k\right| \\ \text{s.t.} \quad & \sum_{k=1}^{n} x_k = 1 \\ & x_k \geq 0 \quad k = 1, \ldots, n \end{aligned} \tag{16}$$

MAD is similar to variance but uses a piecewise linear objective function. Model (16) can be traced back to a linear programming problem:

$$\begin{aligned} \min_{x,d} \quad & \frac{1}{T}\sum_{t=1}^{T} d_t \\ \text{s.t.} \quad & d_t \geq \sum_{k=1}^{n} (r_{k,t} - \mu_k) x_k \quad t = 1, \ldots, T, \\ & d_t \geq -\sum_{k=1}^{n} (r_{k,t} - \mu_k) x_k \quad t = 1, \ldots, T, \\ & \sum_{k=1}^{n} x_k = 1 \\ & d_t \geq 0 \quad t = 1, \ldots, T, \\ & x_k \geq 0 \quad k = 1, \ldots, n \end{aligned} \tag{17}$$

Problem (17) seems to be very similar to Problem (14). Model (11) with $q = 1$ and $\lambda < 0.5$ is equivalent to the MAD in the case of positive lotteries $Y$. In the case of mixed lotteries, MAD and HF-HE are different; the HF-HE model weighs positive and negative outcomes differently, while the MAD is a symmetric measure of risk and weighs positive and negative parts likewise.

# 4 The empirical analysis

In this section we compare the performance of the models presented above. We use a rolling time window analysis considering 3715 observations ($T$) and an in-sample time window of one year ($M = 250$). The out-of-sample portfolio returns are evaluated in the following month (20 days). To simulate portfolio rebalancing, the in-sample window is shifted by one month and this process is iterated until the time horizon is complete. The aforementioned approach generates $T - M$ daily out-of-sample returns for each portfolio strategy on which performance analysis is performed.
The performance measures used to compare the models are as follows:

- Sharpe ratio (Sharpe, 1966, 1998)
- Sortino ratio (Rollinger and Hoffman, 2013)
- Rachev ratio (Biglova *et al.*, 2004)
- ROI (Phillips and Phillips, 2005)
- Herfindahl index (Rhoades, 1993)
- Ulcer index (Martin and McCann, 1989)
- Turnover (Han, 2017)

## 4.1 The Dataset

The empirical analysis was performed on different investment universes. Each dataset[1] consists of daily stocks prices, adjusted for splits and dividends. The time window of the historical data is from 06/10/2006 to 31/12/2020, and also includes years of market crisis, such as 2008 and 2020.
The choice of such a broad historical series is motivated by the intention of analysing the behavior of the models in all phases of the market. We expect a

[1] For further information, consult the website https://host.uniroma3.it/docenti/cesarone/DataSets.htm.

robust model to be effective even in times of market crisis (Mba *et al.*, 2022).
The datasets are illustrated in the following table:

Table 1: Dataset of the empirical analysis

| Index | #Stocks | Country | Time interval |
|---|---|---|---|
| EuroStoxx 50 | 45 | EU | October 2006 - December 2020 |
| FTSE 100 | 80 | UK | October 2006 - December 2020 |
| Dow Jones Industrial | 28 | USA | October 2006 - December 2020 |
| NASDAQ100 | 54 | USA | October 2006 - December 2020 |

After some preliminary tests, the final setting chosen for the *fmincon* algorithm is reported in the Table 2. This tolerance calibration for the solver was also maintained to solve the nonlinear problem PT (1).

Table 2: Setting *fmincon*

| NSim. | Step tol. | Max fun eval. | Opt. tol. | Fun Tol. |
|---|---|---|---|---|
| 50 | $10^{-10}$ | 15000 | $10^{-6}$ | $10^{-6}$ |

## 4.2 Computational results

To simplify the reading of the results, in the following tables for each row (universe of assets), the performance ranking of the proposed models is shown in different colors (in columns). More specifically, for each dataset, the performance of the models takes on colors ranging from dark green to dark red, where the first represents the best performance and the second the worst. In this way we can more easily identify possible patterns of recurring behavior of a portfolio model across the various investment universes.
Tables 3 and 4 examine two measures of return, respectively the expected daily return and the ROI.
In particular, the HF-HE model shows the best performance in terms of expected daily return, across all investment universes. In particular, the model to which $\lambda = 0.32$ belongs (the least pessimistic) is the best in all rankings. The reason is that the higher the value of the $\lambda$ parameter, the lower the degree of pessimism/risk aversion. Therefore, the model reflects this DM's attitude, choosing stocks with higher returns, and consequently greater volatility.
The PT model is not among the best, except for the European equity market.
The result obtained is confirmed by the ROI, in Table 4. Assuming an investment

horizon of 3 years, the ROI is largely positive for all behavioral models. In the UK market, the HF-HE models have the best performance, while in the European one the PT model performs better.

Table 3: Expected daily return

| | EW | MinV | Min MAD | PT | HFHE 0.29 | HFHE 0.30 | HFHE 0.31 | HFHE 0.32 |
|---|---|---|---|---|---|---|---|---|
| ***Eurostoxx 50*** | 0,0367% | 0,0354% | 0,0341% | 0,0429% | 0,0351% | 0,0343% | 0,0429% | 0,0471% |
| ***FTSE 100*** | 0,0393% | 0,0347% | 0,0258% | 0,0380% | 0,0508% | 0,0618% | 0,0580% | 0,0632% |
| ***Dow Jones*** | 0,0481% | 0,0257% | 0,0317% | 0,0349% | 0,0709% | 0,0683% | 0,0684% | 0,0707% |
| ***NASDAQ 100*** | 0,0683% | 0,0400% | 0,0392% | 0,0535% | 0,1319% | 0,1437% | 0,1411% | 0,1541% |

Table 4: Average ROI

| | EW | MinV | Min MAD | PT | HFHE 0.29 | HFHE 0.30 | HFHE 0.31 | HFHE 0.32 |
|---|---|---|---|---|---|---|---|---|
| ***Eurostoxx 50*** | 135,39% | 142,38% | 142,56% | 153,06% | 128,62% | 130,45% | 134,16% | 143,25% |
| ***FTSE 100*** | 139,21% | 137,44% | 129,38% | 148,39% | 176,59% | 194,65% | 182,70% | 189,27% |
| ***Dow Jones*** | 144,96% | 126,45% | 131,56% | 135,00% | 160,85% | 160,06% | 157,45% | 163,77% |
| ***NASDAQ 100*** | 168,09% | 141,44% | 140,74% | 165,00% | 296,19% | 306,83% | 329,97% | 338,46% |

From the results on daily volatility in Table 5, as expected, the minimum risk portfolio strategies (MinV and Min MAD) have the best performance in terms of volatility. However, excellent results are also highlighted for the PT model, in all dataset. The HF-HE models show the greatest volatility, which however improves as the parameter $\lambda$ decreases.

Table 5: Daily volatility

| | EW | MinV | Min MAD | PT | HFHE 0.29 | HFHE 0.30 | HFHE 0.31 | HFHE 0.32 |
|---|---|---|---|---|---|---|---|---|
| ***Eurostoxx 50*** | 0,0144 | 0,0102 | 0,0103 | 0,0116 | 0,0179 | 0,0183 | 0,0184 | 0,0186 |
| ***FTSE 100*** | 0,0126 | 0,0091 | 0,0094 | 0,0111 | 0,0205 | 0,0211 | 0,0211 | 0,0216 |
| ***Dow Jones*** | 0,0127 | 0,0096 | 0,0096 | 0,0108 | 0,0178 | 0,0179 | 0,0182 | 0,0183 |
| ***NASDAQ 100*** | 0,0138 | 0,0101 | 0,0100 | 0,0124 | 0,0242 | 0,0247 | 0,0251 | 0,0257 |

We now pass to the risk-adjusted performance. The Sharpe ratio and the Sortino ratio, respectively in Tables 6 and 7, show the same classification. This result suggests that the differences between the risk measures adopted are negligible in the historical scenarios considered. The results are different depending on the

universe considered. In the European and UK stock market, the PT model is among the best, alongside the traditional models. While, in the American market, the HF-HE models excel, in particular, the one with $\lambda = 0.29$ is in first place for the Dow Jones, while $\lambda = 0.32$ is for the NASDAQ 100.
The Rachev ratio calculated in Table 8, which compares the positive and negative tails of the distribution, shows excellent results for all HF-HE portfolios, in all investment universes. The traditional models are placed in the middle and the PT model in last place.

Table 6: Sharpe ratio

| | **EW** | **MinV** | **Min MAD** | **PT** | **HFHE 0.29** | **HFHE 0.30** | **HFHE 0.31** | **HFHE 0.32** |
|---|---|---|---|---|---|---|---|---|
| ***Eurostoxx 50*** | 0,0255 | 0,0347 | 0,0333 | 0,0371 | 0,0196 | 0,0187 | 0,0233 | 0.0253 |
| ***FTSE 100*** | 0,0311 | 0,0380 | 0,0274 | 0,0343 | 0,0248 | 0,0294 | 0,0274 | 0,0293 |
| ***Dow Jones*** | 0,0378 | 0,0269 | 0,0331 | 0,0323 | 0,0399 | 0,0382 | 0,0376 | 0,0386 |
| ***NASDAQ 100*** | 0,0495 | 0,0396 | 0,0391 | 0,0433 | 0,0546 | 0,0582 | 0,0562 | 0,0599 |

Table 7: Sortino ratio

| | **EW** | **MinV** | **Min MAD** | **PT** | **HFHE 0.29** | **HFHE 0.30** | **HFHE 0.31** | **HFHE 0.32** |
|---|---|---|---|---|---|---|---|---|
| ***Eurostoxx 50*** | 0,0363 | 0,048 | 0,046 | 0,0523 | 0,0274 | 0,0263 | 0,0328 | 0.0357 |
| ***FTSE 100*** | 0,0440 | 0,053 | 0,038 | 0,0472 | 0,0347 | 0,0419 | 0,0390 | 0,0418 |
| ***Dow Jones*** | 0,0541 | 0,038 | 0,047 | 0,0457 | 0,0588 | 0,0562 | 0,0553 | 0,0569 |
| ***NASDAQ 100*** | 0,0704 | 0,056 | 0,056 | 0,0616 | 0,0801 | 0,0860 | 0,0830 | 0,0892 |

Table 8: Rachev ratio

| | **EW** | **MinV** | **Min MAD** | **PT** | **HFHE 0.29** | **HFHE 0.30** | **HFHE 0.31** | **HFHE 0.32** |
|---|---|---|---|---|---|---|---|---|
| ***Eurostoxx 50*** | 0,9577 | 0,9030 | 0,9091 | 0,9363 | 0,9276 | 0,9396 | 0,9480 | 0,9505 |
| ***FTSE 100*** | 0,9723 | 0,9382 | 0,9218 | 0,9093 | 0,9631 | 0,9966 | 0,9920 | 1,0008 |
| ***Dow Jones*** | 0,9676 | 0,9214 | 0,9459 | 0,9342 | 1,0104 | 1,0233 | 1,0246 | 1,0311 |
| ***NASDAQ 100*** | 0,9487 | 0,9368 | 0,9373 | 0,9311 | 1,0480 | 1,0528 | 1,0533 | 1,0679 |

In Table 9 we can see the Ulcer index. Empirical evidence shows that in the event of deep losses, HF-HE portfolios are the ones that have suffered the most impact. In all the investment universes considered, the best performance is from traditional models, with the PT model (yellow-green zone) at the center of the ranking. From a psychological point of view, the investor prefers portfolios with a lower index value, as they are able to absorb drawdowns and quickly return to previous levels of returns.

Table 9: Ulcer index

| | EW | MinV | Min MAD | PT | HFHE 0.29 | HFHE 0.30 | HFHE 0.31 | HFHE 0.32 |
|---|---|---|---|---|---|---|---|---|
| ***Eurostoxx 50*** | 0,1544 | 0,1327 | 0,1317 | 0,2007 | 0,3161 | 0,3208 | 0,2907 | 0,2952 |
| ***FTSE 100*** | 0,1244 | 0,0865 | 0,0966 | 0,1428 | 0,2648 | 0,2449 | 0,2477 | 0,2561 |
| ***Dow Jones*** | 0,0977 | 0,0915 | 0,0943 | 0,1157 | 0,1874 | 0,2028 | 0,2050 | 0,2155 |
| ***NASDAQ 100*** | 0,1139 | 0,0922 | 0,0844 | 0,1321 | 0,2303 | 0,1914 | 0,2679 | 0,1991 |

We focus now on risk concentration. From the calculation of the average of the normalized Herfindahl index in Table 10, it appears that the HF-HE portfolios are those with the greatest concentration of risk. The PT model appears clearly more diversified.

Table 10: Herfindahl index

| | EW | MinV | Min MAD | PT | HFHE 0.29 | HFHE 0.30 | HFHE 0.31 | HFHE 0.32 |
|---|---|---|---|---|---|---|---|---|
| ***Eurostoxx 50*** | 0,9950 | 0,8178 | 0,8269 | 0,7557 | 0,4712 | 0,4290 | 0,3977 | 0,3668 |
| ***FTSE 100*** | 0,9958 | 0,9028 | 0,9014 | 0,8404 | 0,5353 | 0,5036 | 0,4684 | 0,4476 |
| ***Dow Jones*** | 0,9896 | 0,7815 | 0,8008 | 0,7736 | 0,4085 | 0,3843 | 0,3559 | 0,3205 |
| ***NASDAQ 100*** | 0,9940 | 0,8286 | 0,8317 | 0,8176 | 0,4122 | 0,3698 | 0,3458 | 0,3032 |

Finally, the turnover is reported in Table 11. The turnover considered is that generated by the models, and not by price adjustments. So, by construction, the EW portfolio has zero turnover. Among the portfolio models examined, HF-HE portfolios have higher turnover than traditional models. The PT model is in the middle. In general, it is observed that higher turnover can reduce the performance of the models, due to the transaction costs necessary to apply the strategy.

Table 11: Turnover

| | EW | MinV | Min MAD | PT | HFHE 0.29 | HFHE 0.30 | HFHE 0.31 | HFHE 0.32 |
|---|---|---|---|---|---|---|---|---|
| ***Eurostoxx 50*** | 0,0000 | 0,2296 | 0,2958 | 0,5163 | 0,7984 | 0,7906 | 0,7797 | 0,7465 |
| ***FTSE 100*** | 0,0000 | 0,3272 | 0,4625 | 0,6369 | 0,8352 | 0,8103 | 0,8498 | 0,8450 |
| ***Dow Jones*** | 0,0000 | 0,2052 | 0,2759 | 0,5470 | 0,7176 | 0,7263 | 0,7349 | 0,7460 |
| ***NASDAQ 100*** | 0,0000 | 0,2348 | 0,3137 | 0,6269 | 0,6607 | 0,6503 | 0,6521 | 0,6398 |

From the analysis carried out it is clear that the HF-HE model applied in portfolio selection is very sensitive to the variation of the $\lambda$ parameter. In particular, in the present preliminary study, it was decided to adopt $\lambda_- = 1 - \lambda_+$ for simplicity. However, from the results obtained, this choice seems to have favoured risk-seeking investors.
It is possible to weigh the attitude towards losses differently by modifying the parameter's value.

## 5 Conclusions

This paper compares two portfolio models through two behavioral approaches: the classic one of PT and the HF-HE model. As a benchmark, we have therefore used traditional portfolio models, such as the minimum-variance portfolio, the minimum MAD portfolio and the equally weighted portfolio. The out-of-sample analysis was carried out on four equity markets (two from the US, one from the UK and one from Europe). The HF-HE model appears to be the most profitable in terms of expected daily return and average ROI over three years, except for Eurostoxx50, where the PT model excels. Even considering the extreme tails of the distribution of returns, the HF-HE model reaches the highest Rachev ratio, with the exception of Eurostoxx 50. The Sharpe and Sortino ratios show that the HF-HE model performs well in two out of four datasets. The reason is that HF-HE models are characterized by a high level of risk in terms of volatility and the Ulcer index. However, the degree of risk of the HF-HE model, as demonstrated by the sensitivity analysis performed, can be easily modified. In fact, the HF-HE model has proved to be very sensitive to the value of $\lambda$. To improve the volatility of the model, we propose to use a different value of $\lambda$ for future work, however compatible with the experimental tests. However, there is a trade-off between the goal of improving performance and the inclusion of an additional parameter.
We also refer to future research, the application of alternative weight functions to equiprobable scenarios, which may introduce the distortion of probabilities; and solving the HF-HE problem with partial or full linearization for more accurate results.

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We introduce a new behavioural model, the Half-Full/Half-Empty portfolio selection model, in decision-making under risk problems. We implement an empirical analysis based on the comparison with the classic behavioral model of Prospect Theory and we validate the use of these approaches in portfolio selection by proposing three traditional portfolio models as benchmarks (minimum-variance portfolio, mean absolute deviation portfolio and portfolio equally weighted). The aim of this paper is to incorporate investors' perception of risk into the choices of optimal portfolios. Out-of-sample analysis of four stock indexes is proposed to demonstrate the superiority of behavioral portfolio selection models over traditional ones.

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