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Modified Einstein sediment transport method to simulate the local scour evolution downstream of a rigid bed

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ABSTRACT

The present study consists of a new mathematical-numerical modelling formulation to simulate the spatial and temporal scour development downstream of a rigid bed for both a non-cohesive sediment bed and a cohesive sediment mixture with relatively small percentage of cohesive material. Laboratory tests were conducted in a rectangular tilting flume having a recess box filled with the selected bed sediments and placed downstream of a rigid rough bed. The scour pattern was accurately acquired with a 3D Laser Scanner at various time instants. The numerical code was calibrated through the scour profile data obtained under steady-state flow condition and then validated on the basis of scour patterns acquired under both steady and unsteady flow conditions (symmetric and asymmetric hydrographs). On the contrary of most previous studies conducted with an issuing jet, the experiments of the present study were performed under non-strictly uniform flow conditions. The numerical model utilized information concerning the erosive turbulent flow

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characteristics as well as physical and mechanical properties of the movable-bed materials. The mathematical framework of the numerical model was a second order partial differential parabolic equation in which the form of the scoured bed was assumed as an unknown parameter. Instantaneous near-bed velocity data, acquired with an acoustic Doppler velocimeter, was used to modify the Einstein sediment transport method, leading to a significant improvement of the simulated longitudinal scour profiles. Simulated temporal evolution of the scour profiles were in good agreement with the measured ones for both cohesive and non-cohesive bed materials under steady and unsteady flow conditions.

Keywords: Cohesive sediment, Scour, Experimental study, Numerical model, Steady flow, Unsteady flow, Open channel.

INTRODUCTION

The presence of a structure in a river alters the flow field, along with the turbulence level and the local sediment transport. In fact, in most cases the turbulence level and consequently the sediment mobility rate increase downstream of a hydraulic structure. This phenomenon leads to local scouring in the neighborhood of the hydraulic structure, which potentially endangers its stability. A protection layer, e.g. an apron, creating a rigid bed, may be placed downstream of the hydraulic structure to mitigate the turbulence energy before impacting on the natural river bed. Nevertheless, when the flow enters the river, it may still have erosive force and a scour hole may develop downstream of the rigid bed. Owing to the complexity of the flow field around a hydraulic structure and its influence on sediment transport, earlier investigations mainly dealt with experimental studies.

A huge number of experiments on the scour hole evolution downstream of a rigid bed was conducted at Delft Hydraulics, The Netherlands (see Hoffmans and Verheij 1997 for details). The result analysis of the Dutch experiments evidenced two distinguishable phases of scour evolution:

1) the development phase, during which the maximum scour depth is smaller than the initial flow depth; 2) the equilibrium phase, when the maximum scour depth is greater than the initial flow depth and scour develops with a smaller rate than in phase 1). Breusers (1967) and Dietz (1969) made several attempts to obtain the parameters of a power function describing the maximum scour depth time evolution in a two-dimensional scour hole. For the same purpose, Hassan and Narayanan (1985) proposed a semi-empirical theory on the basis of the mean velocity within the scour hole. Chatterjee et al. (1994) conducted an experimental study to derive empirical formulas for the prediction of the equilibrium time, the locations of the maximum scour depth and the peak of the dune, as well as the temporal variation of the maximum scour depth downstream of a rigid apron. The complexity of the scour process is more notable in cohesive bed materials, where soil erodibility is significantly affected by inter-particle forces. In this context, Mazurek et al. (2003) analyzed and scaled longitudinal scour hole profiles in a cohesive soil taking into account the maximum scour depth and the longitudinal distances.

Since the experimental results are generally limited to specific experimental conditions and obviously affected by scale effects, numerical simulations have been widespread accepted as an alternative to empirical formulas. In this context, simulations are mostly based on the Navier-Stokes equations coupled with a bed-load model. Hoffmans and Booij (1993) developed a model on the basis of the Navier-Stokes and convection-diffusion equations. The bed-load transport and the sediment concentration close to the bed were described through a stochastical methodology. Hoffmans and Booij (1993) reported acceptable results in simulations of time-dependent flow velocities, sediment concentrations and scoured bed patterns. García-Martínez et al. (1999) formulated sediment transport by means of a hydrodynamic model and a convection-diffusion equation. The model included source and sink terms to represent bed level changes. García-Martínez et al. (1999) calibrated and validated the model to the predict flow pattern and the channel bed evolution. Brethour (2003) utilized the FLOW-3D® model to simulate a part of an experimental dataset acquired by Chatterjee et al. (1994). The model includes two main components: drifting and lifting. Drifting operates on suspended sediment, whereas lifting occurs at the interface between the bed sediment and the fluid flow. Results of simulated scour depth and deposited dune by Brethour (2003) were fairly in agreement with the laboratory experiments. Adduce and La Rocca (2006) conducted an experimental study on the local scour downstream of a trapezoidal drop caused by three different types of water jets, namely a submerged jet, a surface wave jet, and an oscillating jet. By means of a mathematical model, the authors could reproduce velocity profiles in various locations along the scoured bed. They also concluded that the presence of a scour hole leads to weaken the stability of the surface wave flow. Adduce & Sciortino (2006) developed a mathematical-numerical model, consisting of a second-order partial differential parabolic equation, to simulate the evolution of the local scour due to a horizontal turbulent jet downstream of a rigid apron. The model took into account the continuity equation of local sediment discharge, information concerning the velocity distribution close to the apron edge, as well as the physical and mechanical nature of the mobile bed material. Since the wall-jet like velocity profile at the end of the apron was found to change into a free-jet like velocity distribution in the scour hole, Adduce & Sciortino (2006) modified accordingly the hydrodynamic bed shear stress by using a Gaussian function. The authors simulated the temporal evolution of the scour depth, obtaining a good agreement between computed and measured values. Termini (2011) developed a one-dimensional numerical model to simulate the scour process downstream of a rigid apron. Time and spatial evolution scales were derived from laboratory experiments and included into the numerical model. The author showed that the scour hole length is the most important parameter for the spatial scaling; in addition, the time scale is related to the Shields parameter.

In the present work, starting from the model developed by Adduce & Sciortino (2006) to simulate the scour process downstream of a rigid bed followed by a movable non-cohesive sand bed, a new mathematical model is developed. In fact, Adduce & Sciortino (2006) formulated a model accounting for a horizontal turbulent jet, whereas the present model adapted the governing equations to open-channel free surface flow conditions. Dodaro et al. (2014) calibrated and verified the proposed new model through a limited experimental dataset acquired under both steady and unsteady flow conditions. In the present study, similar experiments were performed with the same flow conditions of Dodaro et al. (2014), but on a bed made of a cohesive sediment mixture. In addition, new experiments were carried in both steady and unsteady flow conditions with cohesive and non-cohesive sediments, in order to validate the model under different flow intensities. This study, therefore, aims at: 1) considering the role of a small percentage of cohesive sediment in the bed material on the scour development and scour pattern downstream of a rigid bed; 2) developing a mathematical-numerical model to simulate the temporal and spatial evolution of the scour hole in cohesive and non-cohesive beds downstream of a rigid bed; 3) verifying the model under both steady-state and unsteady-state flow conditions.

TESTS

Laboratory installations

Experiments were carried out at the *Laboratorio di Grandi Modelli Idraulici*, *Università della Calabria*, Italy (Dodaro 2013). A 9.66 m-long, 0.485 m-wide rectangular tilting flume was used and set at the constant longitudinal bed slope of 0.52%. The sidewalls were made of Polyvinyl Chloride (PVC) at the left side and transparent glass panels at the right side. The flume was fed with a pipe recirculation system; the discharge was measured at the outlet with of a V-notch Thomson weir.

A false floor was mounted in order to create a 3 m-long, 0.20 m-deep sediment recess box starting at 5.62 m from the flume inlet. A 2 cm-thick, 0.20 m-deep bed-sill made of PVC was placed in the middle of the recess box, dividing it into two parts. Results of the present work relate to the 1.5 m-long first portion, in which the scour process downstream of the rigid bed, upstream to the bed-sill, was observed (Figure 1).

Both a non-cohesive sediment and a mixture of non-cohesive and cohesive sediments (hereafter "mixture") were tested. The non-cohesive sediment had the following characteristics: grain specific weight γ_s=25.82 kN/m^3, natural water content w=0.3%, median diameter d_{50}=0.86

mm and geometric standard deviation of the grain size distribution $\sigma_g=(d_{84}/d_{16})^{0.5}=1.3<1.5$ (uniform sand), d_x being the grain size for which x% by weight of the sediment is finer. The same sand was glued on the upstream fixed bed, in order to keep the bed roughness constant. The cohesive mixture had grain specific weight $\gamma_s=25.73$ kN/m^3, natural water content $w=11.8\%$ and degree of saturation $S=36.8\%$. The X-ray diffraction test was conducted for the identification of clay minerals, which were illite, kaolinite, smectite and vermiculite. After preliminary tests, the composition of the mixture was fixed as 95% sand and 5% clay; in fact, it was observed that with a greater percentage of clay the resistance to erosion increased too much, giving origin to very small scour depths, of the order of millimeters.

A manually-operated tailgate was mounted at the flume outlet and used to adjust the water depth, which was measured with a 0.1 mm-accurate point gauge.

The topographical measurements were carried out with a 3D Laser Scanner (model Vivid 300/VI-300 produced by Minolta®), whereas the velocity components were acquired with a 25 Hz Vectrino acoustic Doppler velocimeter. Post-processing of the Vectrino data was conducted through filtering out the corrupted data detected when the signal-to-nose ratio SNR<15 or correlation between transmitted and received pair of pulses less than 70%. In addition, the phase-space threshold approach (Goring and Nikora 2002, as modified by Wahl 2003) was accepted and utilized for despiking procedure. Details of these instruments can be found in Dodaro et al. (2014).

Experimental procedures

For the tests with the mixture, a soil mixer machine was used to provide a uniformly mixed bed material. The mixed material was then saturated with water for around 24 hours and then placed within the recess box of the flume. For all the tests, an aluminum bar longer than the length of the recess box was employed to level the movable bed materials flush with the upstream and downstream rough false floors. A metal plate was placed on the movable bed to avoid undesirable scouring before the designed experimental conditions were achieved. After switching on the pump, the flow discharge and flow depth were regulated, respectively, with some valves and the tailgate according to the test design. Afterwards, the metal plate was slowly moved towards upstream in order to expose the entire mobile bed to the flow; at this time (t=0) the experiment started. It is also worthwhile to point out that the same selected sand was also glued throughout the metal plate to prevent any change in flow depth due to the variation of bed roughness along the channel.

Four test series (A, B, C and D) were carried out (see Table 1, where Q is the flow discharge, h the initial mean flow depth along the work section, U the cross-sectional mean velocity, U_c the critical cross-sectional mean velocity for the inception of sediment motion measured in preliminary tests, U/U_c the flow intensity and t the time instant at which the topographical measure of the scoured bed was performed).

Series A was performed with the sand bed in both steady and unsteady flow conditions. The flow was verified to be non-strictly uniform; specifically, it was a decelerating flow. Tests A4 and A5 were presented in Dodaro et al. (2014) and are reconsidered in the present work for comparison with tests C4 and C5, respectively. Analogously for tests B1 and B3 with respect to tests D1 and D3.

For a given discharge, all the tests started from the initial condition of flat bed at the time instant t=0. At the end of most tests, the eroded bed surface was acquired with the 3D Laser Scanner after draining the bed. It is to be noted that the flow velocity near the side walls of an open channel is much smaller than the mean flow velocity in the cross-section. This phenomenon reduces the sediment transport rate near the walls, in particular in cohesive tests in which greater bed shear stress and flow velocity than in sand tests is needed to detach sediment particles from the bed. To overcome this problem and neglect undesirable bed features near the channel walls, for all the tests in which the laser scanner was employed, the bed surface elevation data of only a 20cm-wide strip along the channel center was analyzed.

As an example, Figures 2a,b,c,d show the scour hole digitized with the 3D Laser Scanner after 48 and 96 h in both the sand and the mixture bed, respectively; x, y and z are the streamwise coordinate along the channel centerline starting from the upstream edge of the recess box, the spanwise coordinate and the orthogonal coordinate with respect to the channel bed, respectively (z=0 is the location of the movable bed surface at time t=0). The maximum scour depth increased and its location moved towards downstream over time. This figure also clarifies that the scour shape for non-cohesive sand bed is 2D, whereas it exhibits a 3D shape in the cohesive-mixture test, even though the percentage of cohesive material was only around 5%. In the present study, each longitudinal profile was obtained by averaging the scour depths across the central 20 cm-wide strip. In fact, to evaluate the numerical model, one mean streamwise 2D profile was extracted from each 3D scour surface acquired with the 3D Laser Scanner (see the figures in the 'Results analysis' section).

In order to obtain experimental data in a shorter time, though, scoured bed data of runs A8, A9 and A10 were achieved by suspending the previous run (A7, A8 and A9, respectively; all the operations were conducted with particular care, minimizing the disturbances to the scour hole by rising gradually the tailgate, thus increasing the flow depth and decreasing the flow velocity) and starting the next one without re-flattening the bed. The partial durations were: 2 h for run A8; 21 h for run A9; 24 h for run A10. At the end of each run, the longitudinal profile of the scour hole was measured at the transparent right side-wall with a ruler (accuracy: 1 mm); in order to verify that the side-wall longitudinal profile was very close to the centerline one, both were acquired at the end of run A10, after having drained the flume, using a 0.1 mm-accurate point gauge along the centerline; such profiles looked very close to each other. It is to be noted that such procedure may be applicable only for non-cohesive bed materials, where the scoured bed exhibits a longitudinal 2D pattern. To acquire scoured bed pattern in cohesive bed sediment, it is essential to utilize an instrument which facilitates 3D bed acquisition. In present study, all the scoured cohesive bed data was obtained with the 3D Laser Scanner.

In the steady state experiments, three different discharges were tested using sand material (Q=18.20, 22.37, 30.35 l/s) and two with the mixture (Q=22.37, 27.50 l/s). For each discharge, the scoured bed was measured at different times, to observe the temporal evolution of the scour hole towards equilibrium. The flow depth slightly increased along the work section from 10 cm at x=0 to 10.6 cm at x=150 cm, being h=10.3 cm on average (x is the longitudinal coordinate equal to 0 and 150 cm at the upstream and downstream end of the mobile bed, respectively).

Tests A1-C1, A2-C2 and A3-C3 were carried out with the same duration and with a unique flow intensity (U/U_c=1.11) in order to be compared in a next section to evaluate the differences between sand and mixture in terms of scour depths. Analogously, a comparison will be performed for tests A4-C4 and A5-C5 with a higher flow intensity (U/U_c=1.36). Also test A6 will be compared to test A3, to observe the effect of the flow intensity. For all the above tests, the measured scour depths will be compared with the simulated ones, deriving from the mathematical model described later. Eventually, tests A7 to A10 will be used to check the model validity in case of high flow intensity (U/U_c=1.85).

In unsteady flow conditions, two tests were performed with the sand (B2 with symmetric hydrograph and B4 with an asymmetric hydrograph) and four with the mixture (D1 and D2 with a symmetric hydrograph; D3 and D4 with an asymmetric hydrograph). Other two tests with the sand, reported by Dodaro et al. (2014), are reconsidered here. Hence, the following comparisons will be accomplished in a next section: B1-D1 and B2-D2 with the symmetric hydrograph; B3-D3 and B4-D4 with the asymmetric hydrograph. The hydrographs were generated with an electrical current regulator controlling a motorized valve (Dodaro et al., 2014) and lasted 4.33 to 7 h, with rising limb much shorter than the recession branch in the asymmetric hydrographs. Figures 3a,b show respectively the flow rate and depth variation over time at the undisturbed section of local abscissa x=-95 cm for tests B2 and B4; the same hydrographs were used in tests D2 and D4, respectively, whereas the hydrographs of tests D1 and D3 were the same as those presented in Dodaro et al. (2014) for tests B1 and B3, respectively.

To estimate bed shear stress and its critical value for the inception of sediment motion $\bar{\tau}_{cr}$, the acoustic Doppler velocimeter named Vectrino (manufactured by Nortek) was utilized. Dodaro et al.

(2014) presented the details of a methodology used to calculate the Reynolds stress along the flow depth and estimated the bed shear stress by extrapolating the Reynolds stress at the bed level, on the basis of the Song and Chiew (2001) equation developed for decelerating flow.

Dodaro et al. (2014) carried out tests with only non-cohesive material (uniform sand) and obtained the critical values $\bar{\tau}_{cr}$=0.47 Pa and U_c=0.328 m/s (see Table 1, tests A4, A5, B1 and B3). In the present study, an attempt was made to estimate also the critical values of bed shear stress and mean velocity for the incipient motion of the cohesive mixture, a task much more complex than for non-cohesive material. In fact, depending upon the clay amount in the bed material and energy slope, different types of incipient condition of sediment motion may be observed. Kothyari and Jain (2008) observed three types of erosion in incipient cohesive sediment motion: 1) pothole erosion, 2) line erosion and 3) mass erosion. The first type occurs at low percentages of clay (less than 30%); the detachment of bed material initiates in form of individual particles, leading to the formation of potholes at various locations in the movable bed surface. In the present work, since the percentage of cohesive material was 5%, pothole erosion was observed as expected; potholes were properly distinguished in the cohesive mixture bed, permitting the identification of the critical condition for the inception of sediment motion and the assessment of the critical bed shear stress, $\bar{\tau}_{cr}$=0.7 Pa, and the critical mean flow velocity, U_c=0.405 m/s.

MATHEMATICAL MODEL

Non-local Exner equation

A “non-local” 1D mathematical model, defined by a partial differential equation of parabolic type derived by the Exner equation, is used to simulate the space-time evolution of the local scour downstream of a rigid bed. Non-local models are used to solve several mathematical and physical problems (Betancourt et al. 2011, Du et al. 2012, Stark et al. 2009) and are characterized by the presence of “global” terms in the differential equations or in the boundary conditions. These global terms generally are defined by integral operators or by functionals that cause a time evolution of the unknowns depending on the whole spatial domain and not only on local spatial coordinates.

Following the approach of Adduce and Sciortino (2006), the continuity equation for the solid material (i.e., the Exner equation) can be written as follows:

$$\frac{\partial z_b}{\partial t}+\left(\frac{1}{1-p}\right)\frac{\partial}{\partial x}\left(\frac{Q_s}{\sqrt{1+(\partial z_b/\partial x)^2}}\right)=0 \qquad (1)$$

where $z_b(x,t)$ is the instantaneous scoured hole profile, p is the sediment porosity and Q_s is the volumetric solid discharge per unit width. In order to model a bed load transport formula for a sloping bed, a functional relation is supposed as follows:

$$Q_s = Q_s(Y) \qquad (2)$$

where Y is the mobility Shields parameter. For a flat bed (Yalin, 1977):

$$Y \equiv \overline{Y} \equiv \frac{\tau_{hydr}}{(\gamma_s-\gamma)d} \qquad (3)$$

where $\overline{Y}$ denotes shields parameter for a flat bed, γ the water specific weight, γ_s is the grain specific weight, d a characteristic particle size and τ_{hydr} is the hydrodynamic bed shear stress. For a flat bed, the critical bed shear stress is:

$$\overline{\tau}_{cr} = \overline{Y}_{cr}(\gamma_s-\gamma)d \qquad (4)$$

where:

$$\overline{Y}_{cr} = \overline{Y}_{cr}(D_s) \tag{5}$$

is the critical Shields parameter and $D_s=d[(\gamma_s-\gamma)/(\rho v^2)]^{1/3}$ is the sedimentological diameter, ρ and v are the fluid density and the kinematic viscosity, respectively.

As a consequence, the mobility Shields parameter for a flat bed can be written as follows:

$$\overline{Y} = \overline{Y}_{cr} \frac{\tau_{hydr}}{\overline{\tau}_{cr}} \tag{6}$$

Following the approach of Adduce & Sciortino (2006), the condition of incipient motion for a sloping bed can be written as follows:

$$\frac{\tau_{hydr} + \tau_g}{\tau_{cr}} > 1 \tag{7}$$

where, taking into account equation (4):

$$\tau_{cr} = \frac{\overline{\tau}_{cr}}{\sqrt{1+(\partial z_b/\partial x)^2}} = \overline{Y}_{cr} \frac{(\gamma_s - \gamma)d}{\sqrt{1+(\partial z_b/\partial x)^2}} \tag{8}$$

is the critical bed shear stress for sloping bed and:

$$\tau_g = \frac{\overline{\tau}_{cr}\,\partial z_b/\partial x}{\tan\varphi\sqrt{1+(\partial z_b/\partial x)^2}} \tag{9}$$

is the gravity shear stress due to the local slope of the bed and φ is is the angle of internal friction of the sediment.

Condition (7) can also be written as follows:

$$\overline{Y}_{cr} \frac{\tau_{hydr} + \tau_g}{\tau_{cr}} > \overline{Y}_{cr} \tag{10}$$

This last condition suggests to define the mobility Shields parameter Y for a sloping bed as follows:

$$Y \equiv \overline{Y}_{cr} \frac{\tau_{hydr} + \tau_g}{\tau_{cr}} \tag{11}$$

The definition (11) reduces to (6) for a flat bed, i.e. $\partial z_b/\partial x \rightarrow 0$. Adopting the Mohr-Coulomb failure criterion (Labuz & Zang 2012) for a cohesive bed with a cohesion parameter c, a generalization of Eq. (11) can be obtained by adding c to the the critical bed shear stress for a sloping bed , τ_{cr}:

$$Y \equiv \overline{Y}_{cr} \frac{\tau_{hydr} + \tau_g}{\tau_{cr} + c} \tag{12}$$

The parameter c, as well-known in soil mechanics, has the dimensions of a stress. The value of c adopted for the numerical simulations was experimentally obtained by the evaluations of $\bar{\tau}_{cr}$ and U_c (as described in section 2 for the mixture) in order to be in the condition of incipient motion, i.e. $Y = \overline{Y}_{cr}$.

In order to complete the definition of the mathematical model, first the way to calculate the hydrodynamic bed shear stress τ_{hydr} in terms of the unknown $z_b(x,t)$ has to be defined, then a suitable choice of the bed load transport formula is necessary.

Analogously to Adduce & Sciortino (2006), the hydrodynamic bed shear stress τ_{hydr} is modelled by:

$$\tau_{hydr} = G(x,t)\frac{\gamma Q^2}{\chi^2 A^2} \tag{13}$$

where $G(x,t)$ is a dimensionless Gaussian-like function defined below, $\gamma Q^2/(\chi^2 A^2)$ is the bed shear stress of a uniform free surface flow given by the Chézy relation with $\chi=(1/n)R_h^{1/6}$, in which $n=0.13(2d)^{1/6}/g^{1/2}$ is the Manning's roughness coefficient, $R_h=b[h-z_b(x,t)]/[b+2(h-z_b(x,t)]$ is the hydraulic radius, with b and h the width of the channel and the initial flow depth averaged along the mobile bed, respectively, Q is the flow discharge, A is the cross-sectional area and g the gravity acceleration. The Gaussian-like function is used to modify the hydrodynamic bed shear stress for a uniform free surface flow into the bed shear stress occurring during a scour process (Hogg et al. 1997, Adduce and Sciortino 2006). During the scouring process the uniform free surface flow velocity profile over the mobile bed changes and the associated bed shear stress is modified as well (analogously to Adduce & Sciortino, 2006). At the beginning of the scouring process, the mobile bed at the downstream edge of the rigid apron starts being eroded. Then the uniform free surface flow velocity profile changes from the rigid apron to the eroded bed into a velocity profile with a weaker transport action. This latter resembles a uniform free surface flow velocity profile after a length scale of the order of one half of the maximum scour length, as in Adduce & Sciortino (2006). This length scale is very short during the first period of the scouring process and becomes longer and longer as time progresses. Then a phenomenological approach, like in Hogg et al. (1997), is followed and the bed shear stress is supposed to evolve with an approximate law given by the product of the shear stress due to a uniform free surface flow and a Gaussian function defined below. Equation (14) has to simulate the transition from a non-strictly uniform free surface flow to a uniform free surface flow condition in a region between the downstream edge of the apron and the maximum scour abscissa. The Gaussian function varies the transport action, which is weak downstream of the rigid apron and becomes stronger and stronger as the maximum scour abscissa is approached. Then the downslope scour profile with $x \leq X_c(t)$ has to be distinguished by to the rest of the scour profile, i.e. $x > X_c(t)$, and the Gaussian law is defined by the following nonlinear piecewise function:

$$G(x,t) = \begin{cases} \varsigma_1 + (1-\varsigma_1)\mathrm{Exp}\left[-\left(\dfrac{x - X_c(t)}{\lambda X_c(t)}\right)^2\right] & \text{if } x \leq X_c \\ 1 & \text{if } x > X_c \end{cases} \tag{14}$$

where ς_1 and λ are calibration parameters of the model; $X_c(t)$ is the instantaneous abscissa of the maximum scour depth, which can be calculated when the instantaneous scour profile $z_b(x,t)$ is known for all the values of the abscissa x. As a consequence, the function $G(x,t)$ defined by Eq. (14) makes the model non-local owing to the fact that, for each time t, $X_c(t)$ is a functional of $z_b(x,t)$: $X_c=F[z_b]$.

A modified Einstein's formula

The last step is the definition of a suitable bed load transport formula that is crucial when a local scour occurs in the condition of incipient motion. In particular, a modified Einstein-like formula has been adopted in this work to obtain the best agreement between experimental data and numerical simulations.

In the Einstein model (Yalin 1977) a relation between the dimensionless transport rate:

$$\phi \equiv \frac{Q_s \rho^{1/2}}{((\gamma_s - \gamma)d)^{3/2}} \tag{15}$$

and the probability P_1 that one grain of the bed will be detached at least one time is constructed as follows:

$$\phi = \frac{P_1}{A_*(1-P_1)} \tag{16}$$

where A_* is a parameter of the Einstein formula, which was assumed equal to 43.50 in the present model, as in Yalin (1977). P_1 is the probability that the random variable with a unit expected value of:

$$R \equiv F_y / < F_y > \tag{17}$$

exceeds the value of:

$$a \equiv G / < F_y > \tag{18}$$

where F_y is the lift force acting on a grain, $<F_y>$ its mean value and G the submerged weight of a grain. As a consequence:

$$P_1 = \mathrm{Prob}\left[R > a\right] = \int_a^{+\infty} \mathrm{Pdf}(r)dr \tag{19}$$

where Pdf(r) is the probability density function of the dimensionless random variable R and r is a dummy variable. At this stage Einstein made two strong assumptions: the first is that Pdf(r) is a normal distribution, the second that the condition $R > a$ can be replaced by $|R| > a$. Both these assumptions are unrealistic, in particular for the last condition as observed by (Yalin, 1977).

To avoid these strong assumptions, a more realistic Pdf(r) function is proposed, starting from the experimental data and assuming valid the expression for the lift force F_y near the bed (Yalin, 1977):

$$F_y = \rho d^2 u^2 \phi_y \tag{20}$$

where u is the instantaneous streamwise velocity component near the bed and ϕ_y is, near the bed, a function depending on the grain shape and the particle Reynolds number, $\mathrm{Re}_* = u_* d / \nu$, where u_* is the friction velocity. In rough turbulent flow, ϕ_y can be assumed as a constant. As a consequence from Eq. (20) it results:

$$F_y / < F_y >= R = u^2 / < u^2 >= \frac{1}{(\frac{< u^2 >}{< u >^2})} \frac{u^2}{< u >^2} = \frac{1}{(\frac{< u^2 >}{< u >^2})} \tilde{U}^2 \tag{21}$$

where:

$$\tilde{U} \equiv u / < u > \tag{22}$$

is the dimensionless streamwise velocity with a unit expected value.

From the Reynolds decomposition:

$$u =< u > + u' \Rightarrow u^2 =< u >^2 + u'^2 + 2 < u > u' \Rightarrow < u^2 >=< u >^2 + < u'^2 > \tag{23}$$

it follows that:

$$\frac{<u^2>}{<u>^2} = 1 + \frac{<u'^2>}{<u>^2} = 1 + \frac{<(u-<u>)^2>}{<u>^2} = 1 + <(\frac{u}{<u>} - 1)^2> = 1 + \sigma^2 \tag{24}$$

where σ^2 is the variance of the dimensionless random variable $\tilde{U}$. Then, the Eq. (21) becomes:

$$R = \frac{\tilde{U}^2}{1+\sigma^2} \tag{25}$$

Figure 4a shows the comparison between the measured probability density hystogram of the random variable $\tilde{U}$ and the Gaussian probability density function (pdf) defined by a unit expected value and the measured standard deviation. The velocity measurements were performed at a distance of 0.5 cm from the bed at *x*=0.2 m. Then, as experimentally detected (see Figure 4a), the random variable $\tilde{U}$ follows with a good approximation a normal distribution:

$$f_{\tilde{U}}(\tilde{u}) = \frac{\mathrm{Exp}\left(-\frac{(\tilde{u}-1)^2}{2\sigma^2}\right)}{\sigma\sqrt{2\pi}} \tag{26}$$

where $\tilde{u}$ is a dummy variable. Hence, it is possible to obtain the probability density function $f_R(r)$ of *R* with simple calculations. Owing to the quadratic relation in Eq. (25), it may be written:

$$\begin{aligned} &\mathrm{Prob}[r < R < r + dr] = f_R(r)dr = \\ &\mathrm{Prob}[\tilde{u} < \tilde{U} < \tilde{u} + d\tilde{u}] + \mathrm{Prob}[-\tilde{u} < \tilde{U} < -\tilde{u} + d\tilde{u}] = (f_{\tilde{U}}(\tilde{u}) + f_{\tilde{U}}(-\tilde{u}))d\tilde{u} \end{aligned} \tag{27}$$

then:

$$f_R(r) = \left. \frac{(f_{\tilde{U}}(\tilde{u}) + f_{\tilde{U}}(-\tilde{u}))}{dr / d\tilde{u}} \right|_{\tilde{u}=\sqrt{r(1+\sigma^2)}} = \frac{(f_{\tilde{U}}(\sqrt{r(1+\sigma^2)}) + f_{\tilde{U}}(-\sqrt{r(1+\sigma^2)}))}{2\sqrt{r(1+\sigma^2)}/((1+\sigma^2))} \tag{28}$$

where the inverse relation of Eq. (25):

$$\tilde{u} = \sqrt{r(1+\sigma^2)} \tag{29}$$

was used.

This is the adopted probability density function for the random variable R and then formula (16) for the transport rate ϕ becomes:

$$\phi = \frac{\int_a^{+\infty} f_R(r)dr}{A_*(1 - \int_a^{+\infty} f_R(r)dr)} \tag{30}$$

In Figure 4b, a comparison between experimental data of random variable R and the adopted probability density function $f_R(r)$ is shown and highlights a very good agreement.

Following Einstein (Yalin, 1977) the parameter a can be modelled as follows:

$$a = \frac{B_* \eta_0}{Y} \tag{31}$$

where B_* and η_0 are 0.143 and 0.5 respectively, then $B_*\eta_0$=0.0715. Finally from the definition of Eq. (15) of ϕ and Eq. (30), Eq. (31) it follows:

$$Q_s(Y)=\frac{\left((\gamma_s-\gamma)d\right)^{3/2}}{\rho^{1/2}}\frac{\int_{\frac{B_*\eta_0}{Y}}^{+\infty} f_R(r)dr}{A_*(1-\int_{\frac{B_*\eta_0}{Y}}^{+\infty} f_R(r)dr)} \tag{32}$$

which is the transport bed load formula adopted in this study.

The previous mathematical model was numerically integrated by means of a Mac-Cormack like scheme (see Adduce & Sciortino, 2006). Suitable boundary conditions were applied and the flat bed was used as initial condition as in Dodaro et al. (2014).

RESULT ANALYSIS

Selection of the sediment transport formula

In Figure 5 a comparison is shown between the measured scour hole for tests A4 and A5 and the predicted ones by the use of three different sediment transport formulas, Meyer-Peter and Müller (1948), denoted by MPM, Einstein (1950) and Modified Einstein. The deterministic Meyer-Peter and Müller formula is not able to predict the scour hole evolution, as soon as the mobility Shields parameter is lower than the critical Shields parameter, i.e. $Y<\overline{Y}_{cr}$ because in this condition the MPM transport rate is zero. Then the scour hole predicted by the MPM formula stops its evolution at about $t\approx$3h and underestimates the measured scour hole for runs A4 and A5, as displayed in Figure 5. The scour profiles predicted by the two probabilistic formulas of Einstein and modified Einstein show a better agreement with the measured scour holes, if compared to the scour profiles predicted by the MPM formula. In fact the use of probabilistic transport formulas let to have a transport rate that is not zero when $Y<\overline{Y}_{cr}$. In addition, the modified Einstein formula shows a better agreement with the measurements if compared to the Einstein formula. Then, it was used for all the following numerical simulations of this paper.

Steady state flow tests

Measured and simulated scour profiles of the steady state runs A1, A2, A3, A6 (i.e., sediment bed without cohesion), C1, C2, C3 (i.e., sediment bed with cohesive bed) are presented in Figure 6. For tests C1, C2 and C3 performed with U/U_c=1.11, no cohesion effect on the scour hole shape is observed. When U/U_c=1.36, as for run A6, the maximum scour hole, as expected, is deeper than that one with a lower U/U_c (i.e., tests A3 and C3). For the tests with U/U_c=1.11 the scour profiles show an evident flex point positioned downstream the abscissa of the maximum scour depth, while no flex point is observable for test A6 with U/U_c=1.36. The simulated scour profiles are in agreement with the measured ones for all the performed tests; however, for test A1 the numerical prediction slightly overestimates the maximum scour depth. No flex point is shown in the simulated scour profiles.

Figure 7 illustrates comparisons between the measured scour profiles for steady states tests A4, A5 (with a non-cohesive bed), C4, C5 (with a cohesive bed) and the simulated results. Fig. 7 was prepared on the basis of the experimental results with a flow intensity of U/U_c=1.36. Slight differences between the measured scour profiles of run C4 and A4 can be observed in Figure 7a, while no appreciable changes can be observed for the most part of the scour profile in Figure 7b. No flex point was observable for both the measured scour holes of runs A4 and A5, in agreement with what happened for run A6 with the same U/U_c. Test A6 was carried out with the same water depth and velocity of run C3. The deeper scour in run A6 with respect to the run C3 is due to the higher value of U/U_c in run A6. Agreement between the measured and simulated scour profiles is shown in Figure 7. The simulated scour holes highlight a small difference between the cohesive and non-cohesive tests that is less evident in the measured data.

The comparison between the measured scour profiles for the steady runs A7, A8, A9 and A10 and the simulated ones is shown in Figure 8. For all the runs of Figure 8 the value of U/U_c is 1.85 and these runs are used to validate the numerical model, which was calibrated by runs A1, A2, A3, A4, A5, A6, C1, C2, C3, C4, C5, C6. A sensitivity analysis of the calibration parameters was performed and a variation of ±20% of the calibrated parameters was found to affect the dimensionless difference between the predicted and measured scour profiles less than 16%. This difference was evaluated as a mean squared error. The maximum scour depth of A9 and A10 with U/U_c=1.85 is, as expected, deeper than the ones of the runs performed with a lower U/U_c, i.e. U/U_c=1.11 and 1.36.

Unsteady flow tests

Figure 9 shows the comparison between the measured and simulated scour profiles under unsteady flow conditions. Tests B1 and B2 and tests D1 and D2 were performed, respectively, with non-cohesive and cohesive bed materials. The unsteady runs B1, B2, D1 and D2 were carried out under a symmetric hydrograph. The cohesion affects the measured scour profiles, resulting in a deeper scour hole for run B1 with a non-cohesive bed comparing to run D1 with a cohesive bed (see fig. 9a). A similar behavior is observed in Figure 9b illustrating profiles of runs B2 and D2. In addition, as expected, the scour hole deepens as the duration of the experiments increases. The simulated scour profiles are in agreement with the measured ones and both the cohesion and the time dependence effects are captured by the predicted scour profiles.

The comparison between the measured scour profiles under unsteady flow runs B3 and B4 conducted with a non-cohesive bed and D3 and D4 performed with a cohesive bed along with the simulation results are shown in Figure 10. The unsteady flow runs B3, B4, D3 and D4 were performed with an asymmetric hydrograph. As for the runs performed with a symmetric hydrograph, the cohesion affects the measured scour profiles, developing a deeper scour hole for both runs B3 and B4 with a non-cohesive bed, if compared to runs D3 and D4 with a cohesive bed, respectively. Figure 10 shows that the simulated scour profiles are in good agreement with the measured ones and both the cohesion and the time dependence effects are captured by the predicted scour profiles.

CONCLUSIONS

The main aim of this study was to develop a mathematical and numerical model to simulate scour evolution downstream of a rigid bed with both non-cohesive and cohesive mixture. This goal was achieved, thereby a good agreement between experimental data and simulated results. In addition, following results are also drawn from the study:

1) scour forms a 2D shape in non-cohesive sediments, whereas exhibits a 3D shape even with a relatively small percentage of cohesive material. Nevertheless, a cross-sectional averaged scour profile on the cohesive mixture during time may behave as the 2D scour profiles of the non-cohesive material. It is to be noted that this finding may be valid only for a cohesive mixture with a small percentage of clay;
2) the scour profiles of non-cohesive and cohesive mixture tests are fairly superimposed for a certain flow intensity value. In fact, the main role of a small percentage of cohesive material, as used in this study, is to increase the critical bed shear stress or mean flow velocity for the incipient sediment motion. In this condition, sediment particles detach individually and are transported as non-cohesive materials. Although the difference between non-cohesive and cohesive bed profiles may increase with time, the dimensionless parameter U/U_c is an important parameter for scaling the scour profiles;
3) by the definition of a suitable non-local mathematical model, it is possible to simulate the local scour holes observed experimentally. The numerical integration of a partial differential equation of parabolic type derived by the Exner equation furnishes results in reasonable agreement with the experimental measurements. This fact confirms the validity of the mathematical approach followed in the present work. The main advantage of the proposed non-local model is the capability of simulating the complex scour phenomenon with a single differential equation having the instantaneous bed profile as unknown.
4) the general sediment transport model for non-cohesive bed material can be assumed to be a function of the Shields mobility parameter, which can be formulated as a function of gravity,

hydrodynamic and critical bed shear stresses caused by the sloping bed. For a cohesive mixture, cohesion parameter should be also taken into account;

5) a correct selection of the bed load transport formula is a key issue in the accurate estimation of scour profiles. An attempt was made by using the well-known Meyer-Peter and Müller formula, but the result was not acceptable; however, a better result was obtained by using the Einstein model. Since some original assumptions considered by Einstein are not realistic, new modifications of his method were taken into account.

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Nomenclature

A = Cross-sectional area;

A_* = A calibration Einstein parameter

a = Einstein parameter

B_* = Einstein parameter

b = Channel width;

c = Cohesion parameter;

D_s = Sedimentological diameter;

d = Characteristic particle size;

d_x = Grain size for which x% by weight of the sediment is finer;

F = A function;

F_Y = Lift force acting on a grain;

G = Submerged weight of a grain;

$G(x,t)$ = A dimensionless Gaussian like function;

g = Acceleration due to gravity;

h = Initial mean flow depth along the work section;

n = Manning's roughness coefficient;

P_1 = Probability that one grain of the bed is detached at least one time;

Pdf(r) = Probability density function of the dimensionless random variable R;

p = Sediment porosity;

Q = Flow discharge;

Q_s = Volumetric solid discharge per unit width;

R = Random variable defined as the ratio between the lift force and the mean lift force

R_h = Hydraulic radius;

Re_* = Shear Reynolds number;

r = Dummy variable;

S = Degree of saturation;

t = Time;

U = Cross-sectional mean velocity;

U_c = Critical cross-sectional mean velocity for the incipient of sediment motion;

$\tilde{U}$ = Dimensionless streamwise velocity;

u = Instantaneous streamwise velocity component near the bed;

$\tilde{u}$	=	Dummy variable;
u'	=	Velocity fluctuation in streamwise direction;
u_*	=	Friction velocity;
w	=	Natural water content;
$X_c(t)$	=	Instantaneous abscissa of the maximum scour depth;
x	=	Streamwise coordinate;
Y	=	Mobility Shields parameter;
$\bar{Y}$	=	Shields parameter for a flat bed;
$\bar{Y}_{cr}$	=	Critical Shields parameter;
y	=	Spanwise coordinate;
z	=	Orthogonal to the bed coordinate;
$z_b(x,t)$	=	The equation of instantaneous scoured bed;
γ	=	Water specific weight;
γ_s	=	Grain specific weight;
ς_1	=	Calibration parameter;
η_0	=	Einstein parameter
λ	=	Calibration parameter;
ν	=	Fluid kinematic viscosity;
ρ	=	Fluid density;
τ_{cr}	=	Critical bed shear stress for sloping bed;
τ_g	=	Gravity shear stress due to the local slope of the bed;
$\bar{\tau}_{cr}$	=	Critical bed shear stress for a flat bed;
τ_{hydr}	=	Hydrodynamic bed shear stress;
ϕ	=	Dimensionless transport rate;
ϕ_y	=	Function of the particle Reynolds number

φ	=	Angle of internal friction of the sediment
χ	=	Chézy coefficient.