

Global analysis of charm mixing parameters and determination of the CKM angle γ

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We present an updated global analysis of beauty decays sensitive to the angle γ of the Cabibbo-Kobayashi-Maskawa matrix and of D -meson mixing data in the framework of approximate universality, in which CP violation in $D - \bar{D}$ mixing is described in terms of two universal weak phases corresponding to dispersive and absorptive contributions. We extract the fundamental theoretical parameters determining absorptive and dispersive contributions to D meson mixing and CP violation, together with the angle γ . The results for the charm mixing parameters are $x_{12} \simeq x = (0.401 \pm 0.043)\%$ and $y_{12} \simeq y = (0.610 \pm 0.017)\%$, while the two CP -violating phases are given by $\phi_2^M = (0.13 \pm 0.70)^\circ$ and $\phi_2^{\Gamma} = (2.1 \pm 1.6)^\circ$. The angle γ is found to be $\gamma = (65.7 \pm 2.5)^\circ$, in excellent agreement with the indirect determination from the unitarity triangle analysis.

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I. INTRODUCTION

Charm mixing occurs through flavor changing neutral currents (FCNC) that are absent at the tree level in the Standard Model (SM) and suppressed by the Glashow-Iliopoulos-Maiani (GIM) mechanism [1] and by the hierarchical structure of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [2,3]. Thus, new physics (NP) effects due to heavy particles may appear in the neutral D system [4–23] through observables such as the amount of CP violation. The latter could be used as an interesting benchmark of the SM since it is suppressed by the fourth power of the sine of the Cabibbo angle θ_C .

The experimental study of neutral D mixing has been continuously progressing in the past two decades [24–79].

The most precise estimates of the charm parameters are obtained from the rates of Cabibbo favored (CF) (i.e., proportional to $V_{cs}V_{ud}^*$) or doubly Cabibbo suppressed (DCS) (i.e., proportional to $V_{cd}V_{us}^*$) D decays [52–55], from charm decays to CP eigenstates [56–79], and from Dalitz plot analyses of the three-body modes $K_S^0\pi^+\pi^-$, $K_S^0K^+K^-$, or $\pi^+\pi^-\pi^0$ [30–32,36–38]. In particular, LHCb has achieved impressive results during the LHC Run II, including the observation of direct CP asymmetries in the singly Cabibbo suppressed (SCS) decays of D mesons to K^+K^- and $\pi^+\pi^-$ [72,75] and the very recent measurements of CP -violating observables in $D^0 \rightarrow K^\pm\pi^\mp$ [55].

This remarkable increase in the experimental accuracy calls for an update of our previous works on D meson mixing [80,81], generalizing the so-called “superweak assumption,” in which the weak phase describing CP violation in the interference between D meson decays with and without absorptive mixing is set to zero, to the more suitable “approximate universality” scenario introduced in Ref. [82] to be detailed below.

Charm mixing also plays an important role in the extraction of the CKM angle $\gamma = \arg(-V_{ud}V_{ub}^*V_{cb}V_{cd}^*)$

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from tree-level B decays [83–122], calling for a simultaneous combination of charm and beauty measurements, as pioneered by the LHCb Collaboration [123]. Our results can be compared with the latest LHCb [124] estimates, the Belle/Belle II analysis [125] for γ , and HFLAV [126] for the charm parameters. The consistency of the fit is checked by comparing γ estimates obtained by combining measurements involving different types of B mesons separately: charged B^\pm or neutral B^0 and B_s^0 . The impact of the global fit on the charm parameters is assessed by comparing it with the results of a fit to D observables only.

The paper is organized as follows: The dispersive-absorptive formalism for D meson mixing in the framework of approximate universality is briefly summarized in Sec. II. The expressions of the charm observables are described in Sec. III, while the beauty observables are introduced in Sec. IV and their dependence on the parameters of interest of this study is reported in Appendixes A and B. We present and discuss the most relevant results in Sec. V. The posterior probability intervals of the fit parameters are given in Appendix C. A summary can be found in Sec. VI.

II. CHARM MIXING FORMALISM

In this section, we present a parametrization of the quantities needed to describe charm decays and D mixing underlying all the observables considered in the combination.

We indicate with M (\bar{M}) a generic B (\bar{B}) or D (\bar{D}) meson, while its decay amplitudes to CP conjugate final states f and \bar{f} are denoted as

$$\begin{aligned} \mathcal{A}_M^f &= \langle f | \mathcal{H} | M \rangle, & \mathcal{A}_M^{\bar{f}} &= \langle \bar{f} | \mathcal{H} | M \rangle, \\ \mathcal{A}_{\bar{M}}^f &= \langle f | \mathcal{H} | \bar{M} \rangle, & \mathcal{A}_{\bar{M}}^{\bar{f}} &= \langle \bar{f} | \mathcal{H} | \bar{M} \rangle, \end{aligned} \quad (1)$$

where \mathcal{H} is the relevant effective Hamiltonian involved in the process.

The time evolution of a linear combination of D^0 and \bar{D}^0 mesons follows the Schrödinger equation, with a 2×2 non-Hermitian Hamiltonian \mathbf{H} (see, e.g., [127–130]) that in terms of its dispersive \mathbf{M} and absorptive $\mathbf{\Gamma}$ components is given by

$$\mathbf{H} = \mathbf{M} - i/2\mathbf{\Gamma}, \quad (2)$$

so that its matrix element between charm states can be written as

$$\langle D^0 | \mathbf{H} | \bar{D}^0 \rangle = H_{12} = M_{12} - i/2\Gamma_{12}. \quad (3)$$

Adopting the long-lived (L) and short-lived (S) meson notation, the Hamiltonian eigenstates read

$$|D_{L,S}\rangle = p|D^0\rangle \pm q|\bar{D}^0\rangle, \quad (4)$$

with p and q complex coefficients satisfying $|p|^2 + |q|^2 = 1$. Then, the differences between masses and decay widths of

the Hamiltonian eigenstates are parametrized through the usual quantities

$$x = \frac{m_S - m_L}{\Gamma}, \quad y = \frac{\Gamma_S - \Gamma_L}{2\Gamma}, \quad (5)$$

with Γ being the averaged D^0 lifetime.

D meson mixing is described in terms of the matrix elements of \mathbf{H} by the parameters

$$\phi_{12} = \arg\left(\frac{M_{12}}{\Gamma_{12}}\right), \quad x_{12} = 2\frac{|M_{12}|}{\Gamma}, \quad \text{and} \quad y_{12} = \frac{|\Gamma_{12}|}{\Gamma}. \quad (6)$$

The phase ϕ_{12} governs CP violation in pure mixing, while x_{12} and y_{12} are CP conserving.

The CP -violating quantity $|q/p| - 1$ and the observables in Eq. (5) are related to the mixing parameters in Eq. (6) as

$$|x| = x_{12}, \quad \left|\frac{q}{p}\right| - 1 = \frac{x_{12}y_{12}}{x_{12}^2 + y_{12}^2} \sin\phi_{12}, \quad |y| = y_{12}, \quad (7)$$

up to negligible corrections quadratic in $\sin\phi_{12}$.

In the SM, it is possible to decompose the dispersive and absorptive parts of H_{12} in terms of U-spin amplitudes as follows [82]:

$$\begin{aligned} \xi_{12}^{\text{SM}} &= \frac{(\lambda_{uc}^s - \lambda_{uc}^d)^2}{4} \xi_2 + \frac{(\lambda_{uc}^s - \lambda_{uc}^d)\lambda_{uc}^b}{2} \xi_1 + \frac{(\lambda_{uc}^b)^2}{4} \xi_0, \\ \xi &= M, \Gamma, \end{aligned} \quad (8)$$

where $\lambda_{kl}^j = V_{kj}^* V_{lj}$, while $M_{0,1,2}$ and $\Gamma_{0,1,2}$ are the $\Delta U_3 = 0$ components of the $\Delta U = 0, 1, 2$ multiplets, respectively. Thus, the order of magnitude of $\xi_{0,1,2}$ is related to the U-spin breaking parameter ε , such that $\xi_n \approx \mathcal{O}(\varepsilon^n)$.

In the approximate universality scenario, CP violation in the interference between decays with and without dispersive and absorptive mixing can be described through two final state-independent weak phases $\phi_2^{M,\Gamma}$, defined as

$$\phi_2^\xi = \arg\frac{\xi_{12}}{\frac{1}{4}(\lambda_{uc}^s - \lambda_{uc}^d)^2 \xi_2}, \quad \xi = M, \Gamma. \quad (9)$$

Furthermore, the difference between ϕ_2^M and ϕ_2^Γ is exactly ϕ_{12} , meaning that all types of indirect CP violation concerning D meson mixing, namely CP violation (CPV) in mixing and in the interference between mixing and decay, are entirely determined by knowledge of these two universal weak phases.

Employing in the definition of Eq. (9) the expressions of M_{12}^{SM} and Γ_{12}^{SM} given in Eq. (8) and neglecting the third (smallest) contribution, the orders of magnitude of the two CP -violating phases in the SM can be roughly estimated to be [82]

$$\begin{aligned}
(\phi_2^M) &\sim (\phi_2^\Gamma) \simeq \left| \frac{\lambda_{uc}^b}{\theta_C} \right| \sin \gamma \times \frac{\Gamma_1}{\Gamma_2} \sim \left| \frac{\lambda_{uc}^b}{\theta_C} \right| \sin \gamma \times \frac{1}{\varepsilon} \\
&\sim (2.2 \times 10^{-3}) \times \left(\frac{0.3}{\varepsilon} \right). \quad (10)
\end{aligned}$$

From Eqs. (6) and (8), we get the relation $|\Gamma_2| \simeq y_{12}\Gamma/|\lambda_{uc}^s|^2$, which can be employed in Eq. (10) to provide an upper bound on $|\phi_2^\Gamma|$ as [82]

$$|\phi_2^\Gamma| \simeq \frac{|\lambda_{uc}^b \lambda_{uc}^s \sin \gamma|}{y_{12}} \times \frac{|\Gamma_1|}{\Gamma} < (5 \times 10^{-3}) \times \left(\frac{0.61\%}{y_{12}} \right), \quad (11)$$

where we used the conservative assumption $|4\Gamma_1/(\Gamma_0 - \Gamma_2)| < 1$, a ratio which is nominally of $\mathcal{O}(\varepsilon)$.

For a given final state f , CP violation in the interference between decays with and without mixing can be equivalently described by the weak phase of the observable

$$\lambda_D^f \equiv \frac{q}{p} \frac{\mathcal{A}_D^f}{\mathcal{A}_D^f}, \quad (12)$$

which becomes universal in the framework of approximate universality and we denote by ϕ_2 . The posterior of ϕ_2 is determined from the charm parameters as

$$\tan 2\phi_2 = - \left(\frac{x_{12}^2 \sin 2\phi_2^M + y_{12}^2 \sin 2\phi_2^\Gamma}{x_{12}^2 \cos 2\phi_2^M + y_{12}^2 \cos 2\phi_2^\Gamma} \right). \quad (13)$$

Notice that ϕ_2 corresponds to the phase ϕ extracted in global analyses under the assumption of final state independence (see, e.g., Refs. [123,124]).

In the superweak assumption, dispersive mixing is supposed to be the dominant source of CPV in the charm system, corresponding to the approximation where $\phi_2^\Gamma = 0$ and $\phi_{12} = \phi_2^M$ or, equivalently, $\phi = \phi_2 = \arctan \frac{1-|q/p|^2 x}{1+|q/p|^2 y}$. Then, going from the superweak assumption, adopted in our previous works [80,81], to approximate universality, used in this work, involves moving from a description in terms of a single independent CPV parameter (ϕ_{12} or ϕ_2) to a scenario with two free CPV parameters (ϕ_2^M and ϕ_2^Γ or ϕ_2 and $|q/p| - 1$).

TABLE I. Charm observables used in the combination. We use XX to indicate both $\pi^+\pi^-$ and K^+K^- states, while $K\pi$ stands for $K^\mp\pi^\pm$. We refer to the multibody final states $K^\mp\pi^\pm\pi^+\pi^-$ and $\pi^+\pi^-\pi^+\pi^-$ as $K3\pi$ and 4π , respectively. The state f^+ stands for a CP -even final state. A description of the observables can be found in Secs. III A–III C.

Observables	D^0 decays	References	Observables	D^0 decays	References
$(x^2 + y^2)/2$	$D \rightarrow Kl\bar{\nu}_l$	[24–28]	$x_{K\pi^0}^{\prime\pm}, y_{K\pi^0}^{\prime\pm}$	$D \rightarrow K\pi\pi^0$	[29]
x, y	$D \rightarrow K_S^0 XX$	[30]	x, y	$D \rightarrow K_S^0 \pi\pi$	[31]
x, y	$D \rightarrow \pi^+\pi^-\pi^0$	[32]	$x, y, q/p , \phi_2$	$D \rightarrow K_S^0 \pi\pi$	[36]
$(r_D^{K\pi})^2, x^2, y,$ $\cos \delta_D^{K\pi},$ $\sin \delta_D^{K\pi}$	$D \rightarrow K\pi$	[33]	$\frac{B(D^0 \rightarrow K_S^0 K^+ \pi^-)}{B(D^0 \rightarrow K_S^0 K^- \pi^+)},$ $\delta_D^{K_S^0 K\pi},$ $\kappa_D^{K_S^0 K\pi}$	$D \rightarrow K_S^0 K\pi$	[34]
$F_+^{4\pi}$	$D \rightarrow 4\pi$	[39,41]	$F_+^{XX\pi^0}$	$D \rightarrow XX\pi^0$	[39,40]
$\frac{B(D \rightarrow K_S^0 K^+ \pi^-)}{B(D^0 \rightarrow K_S^0 K^- \pi^+)}$	$D \rightarrow K_S^0 K\pi$	[42]	$F_+^{KK\pi\pi}$	$D \rightarrow K^+K^-\pi^+\pi^-$	[44]
$x_{CP}, y_{CP}, \Delta x, \Delta y$	$D \rightarrow K_S^0 \pi\pi$	[37,38]	$(x^2 + y^2)/4$	$D \rightarrow K3\pi$	[45]
$r_D^{K3\pi}, \delta_D^{K3\pi}, \kappa_D^{K3\pi},$ $r_D^{K\pi\pi^0}, \delta_D^{K\pi\pi^0}, \kappa_D^{K\pi\pi^0}$	$D \rightarrow K3\pi$ $D \rightarrow K\pi\pi^0$	[45,47–49]	$A_D(f^+), A_D(\pi\pi\pi^0),$ $r_D^{K\pi} \cos \delta_D^{K\pi}, r_D^{K\pi} \sin \delta_D^{K\pi}$	$D \rightarrow f^+, D \rightarrow K\pi$ $D \rightarrow XX\pi^0$	[50]
\tilde{y}_{CP}	$D \rightarrow XX$	[56–65]	$\Delta Y^{KK} - \Delta Y^{\pi\pi}, \Delta Y$	$D \rightarrow XX$	[67–70]
$(r_D^{K\pi})^2, c_{K\pi}^{(\prime)}, \Delta c_{K\pi}^{(\prime)}, \Delta \tilde{c}_{K\pi}^{(\prime)}$	$D \rightarrow K\pi$	[55]	$(r_D^{K\pi})^2, (x_{K\pi}^{\prime\pm})^2, y_{K\pi}^{\prime\pm}$	$D \rightarrow K\pi$	[52–54]
$A_D^{CP}(KK), \langle \tau \rangle_E^{KK}$	$D \rightarrow K^+K^-$	[71–73]	$\Delta A_D^{CP}, \langle \tau \rangle_E^{KK}, \langle \tau \rangle_E^{\pi\pi}$	$D \rightarrow XX$	[73–75]
$\Delta Y^{KK}, \Delta Y^{\pi\pi}$	$D \rightarrow XX$	[66]	$A_D^{CP}(XX), \langle \tau \rangle_E^{KK}, \langle \tau \rangle_E^{\pi\pi}$	$D \rightarrow XX$	[76–79]
$\frac{B(D^0 \rightarrow K^+ \pi^-)}{B(D^0 \rightarrow K^- \pi^+)}, \frac{B(D^0 \rightarrow K^+ \pi^- \pi^0)}{B(D^0 \rightarrow K^- \pi^+ \pi^0)}$	$D \rightarrow K\pi, D \rightarrow K\pi\pi^0$	[43]			
$\frac{B(D^0 \rightarrow K^+ \pi^- \pi^+ \pi^-)}{B(D^0 \rightarrow K^- \pi^+ \pi^+ \pi^-)}$	$D \rightarrow K3\pi$				

III. CHARM OBSERVABLES

The charm observables used in our combination rely on time-integrated and time-dependent studies of D meson decays, reconstructed from two-body and multibody final states. The final states can be either CP eigenstates or non CP eigenstates originated from CF or DCS decays. We parametrize the decay amplitudes through the ratios of their magnitudes and strong phases and provide expressions for the observables in Table I in terms of charm decay, mixing, and CP -violating parameters in the framework of approximate universality for the neutral D system, following Ref. [82].

A. CF/DCS decays to $K^\pm X$

In this section, we consider CF/DCS decays to $K^\pm X$, such as $D^0 \rightarrow K^\pm \pi^\mp$. We denote with f the final state of the CF decay of the D^0 meson (e.g., $K^- \pi^+$) and by “wrong-sign” (WS) and “right-sign” (RS) the time-dependent DCS (i.e., $D^0 \rightarrow \bar{f}$ and $\bar{D}^0 \rightarrow f$) and CF (i.e., $D^0 \rightarrow f$ and $\bar{D}^0 \rightarrow \bar{f}$) decays respectively. We parametrize the ratios of decay amplitudes in Eq. (1) as

$$r_D^f e^{-i\delta_D^f} = \frac{\mathcal{A}_D^{\bar{f}}}{\mathcal{A}_D^f} = \frac{\mathcal{A}_D^f}{\mathcal{A}_D^{\bar{f}}}, \quad (14)$$

where we have neglected possible new weak phases in CF/DCS decays (weak phases are fully negligible in the SM). The observables used in the fit for this type of final states are extracted from the ratios of the WS and RS decay rates [52–54], given by

$$\begin{aligned} \frac{\Gamma(D^0 \rightarrow \bar{f}, t)}{\Gamma(D^0 \rightarrow f, t)} &= (r_D^f)^2 + r_D^f y_f'^+ \tau + \frac{(x_f'^+)^2 + (y_f'^+)^2}{4} \tau^2, \\ \frac{\Gamma(\bar{D}^0 \rightarrow f, t)}{\Gamma(\bar{D}^0 \rightarrow \bar{f}, t)} &= (r_D^f)^2 + r_D^f y_f'^- \tau + \frac{(x_f'^-)^2 + (y_f'^-)^2}{4} \tau^2, \end{aligned} \quad (15)$$

up to second order in the charm mixing and CP -violating parameters and with $\tau = \Gamma t$. The coefficients $x_f'^\pm$ and $y_f'^\pm$ can be written as a rotation of an angle ϕ_2 of their CP -conserving limits, which we denote with x_f' and y_f' , respectively, as

$$\begin{pmatrix} x_f'^\pm \\ y_f'^\pm \end{pmatrix} = - \left| \frac{q}{p} \right|^{\pm 1} \begin{pmatrix} \cos \phi_2 & \pm \sin \phi_2 \\ \mp \sin \phi_2 & \cos \phi_2 \end{pmatrix} \begin{pmatrix} x_f' \\ y_f' \end{pmatrix}, \quad (16)$$

with

$$\begin{aligned} x_f' &= x \cos \delta_D^f + y \sin \delta_D^f, \\ y_f' &= y \cos \delta_D^f - x \sin \delta_D^f. \end{aligned} \quad (17)$$

The linear and quadratic terms in Eq. (15) can be decomposed into their CP -conserving (c_f, c_f') and CP -violating parts ($\Delta c_f, \Delta c_f'$), as follows:

$$\begin{aligned} y_f'^\pm &= c_f \pm \Delta c_f, \\ \frac{(x_f'^\pm)^2 + (y_f'^\pm)^2}{4} &= c_f' \pm \Delta c_f', \end{aligned} \quad (18)$$

where the coefficients on the right-hand side of Eq. (18) are given in terms of the charm mixing parameters as

$$\begin{aligned} c_f &= x_{12} \cos \phi_2^M \sin \delta_D^f - y_{12} \cos \phi_2^\Gamma \cos \delta_D^f, \\ \Delta c_f &= -x_{12} \sin \phi_2^M \cos \delta_D^f - y_{12} \sin \phi_2^\Gamma \sin \delta_D^f, \\ c_f' &= \frac{x_{12}^2 + y_{12}^2}{4} + \frac{(r_D^f)^2}{4} (y_{12}^2 - x_{12}^2), \\ \Delta c_f' &= \frac{1}{2} x_{12} y_{12} \sin \phi_{12}. \end{aligned} \quad (19)$$

The separation of the CP -violating from the CP -conserving contributions entering the WS/RS ratios of $D^0 \rightarrow K^\pm \pi^\mp$ decays has been performed for the first time this year by LHCb [55], combining both the Run 1 and Run 2 results. However, while the Run 1 measurements can be used directly to extract the coefficients in Eq. (19), the Run 2 data use experimental information from $D^0 \rightarrow K^+ K^-$ decays to correct a detection asymmetry. This operation leaves invariant c_f and c_f' , while it changes the expressions of the CPV observables as

$$\begin{aligned} \Delta \tilde{c}_f &= \Delta c_f - c_f a_D^{KK} - 2r_D^f \Delta Y^{KK}, \\ \Delta \tilde{c}_f' &= \Delta c_f' - 2c_f' a_D^{KK} - 2r_D^f c_f \Delta Y^{KK}, \end{aligned} \quad (20)$$

and the parameters r_D^f are replaced by $r_D^f (1 \mp a_D^{KK})$ in Eq. (15). Here, a_D^{KK} is the direct CP asymmetry in $D^0 \rightarrow K^+ K^-$ decays and we introduced the parameter ΔY^{KK} . They were measured in Refs. [71–73, 76–79] and [66–70], and are defined in terms of decay amplitudes and decay rates in Eqs. (23) and (26), respectively, when describing the observables involving SCS decays to CP eigenstates.

We consider in the combination also observables coming from the analysis of quantum correlated $D^0 - \bar{D}^0$ pairs (see, e.g., [50]). Here, one of the D mesons decays to the so-called tagging mode, while the other decays to the signal mode, typically $K^- \pi^+$. Then, the following asymmetries are measured:

$$A_D(f^+) = \frac{\mathcal{B}(D_{f^+} \rightarrow K^- \pi^+) - \mathcal{B}(D_{f^-} \rightarrow K^- \pi^+)}{\mathcal{B}(D_{f^+} \rightarrow K^- \pi^+) + \mathcal{B}(D_{f^-} \rightarrow K^- \pi^+)}, \quad (21)$$

where $\mathcal{B}(D_{f^\pm} \rightarrow K^- \pi^+)$ are the branching fractions of neutral charm state decays to $K^- \pi^+$ when tagged by CP -even or multibody modes, also known as quasi- CP eigenstates, D_{f^+} , or CP -odd modes D_{f^-} . For example, we could have $f^+ = \pi^+ \pi^- (\pi^0)$ and $f^- = K_S^0 \pi^0$. In the CP -conserving limit, the asymmetry in Eq. (21) depends on the charm parameters as

$$A_D(f^+) = \frac{F_+^{f^+}(y - 2r_D^{K\pi} \cos \delta_D^{K\pi})}{1 + (r_D^{K\pi})^2 + (1 - F_+^{f^+})(y + 2r_D^{K\pi} \cos \delta_D^{K\pi})}, \quad (22)$$

up to $\mathcal{O}(x, y, (r_D^{K\pi})^2)$ corrections. Here, we have introduced the so-called CP -even fraction $F_+^{f^+}$ of the mode f^+ , which is a non-negative real number needed for quasi- CP eigenstates that reaches its maximum of one for CP -even eigenstates. A definition of $F_+^{f^+}$ in terms of decay amplitudes is given in Sec. III C, in the context of multibody final states.

B. SCS decays to CP eigenstates

SCS decays to CP eigenstates f , such as $D^0 \rightarrow \pi^+\pi^-$ or $D^0 \rightarrow K^+K^-$, are described in terms of just two of the four amplitudes in Eq. (1) and their magnitudes can be parametrized as

$$1 - a_D^f = \left| \frac{A_D^f}{A_D^f} \right|, \quad (23)$$

where a_D^f is the direct CP asymmetry, already introduced in Eq. (20). CP -conserving and CP -violating ratios of time-dependent decay rates are measured. The first type is obtained by calibrating the signal through the CF mode $K^\mp \pi^\pm$ as

$$\frac{\Gamma(D^0 \rightarrow f, t) + \Gamma(\overline{D}^0 \rightarrow f, t)}{\Gamma(D^0 \rightarrow K^-\pi^+, t) + \Gamma(\overline{D}^0 \rightarrow K^+\pi^-, t)} \propto 1 - \tau \tilde{y}_{CP}, \quad (24)$$

where we have neglected the quadratic terms in the charm mixing and CP -violating parameters that come with higher powers in τ . Then, the slope \tilde{y}_{CP} is extracted from Eq. (24) through a linear fit [56–65], and we have:

$$2\tilde{y}_{CP} = y \cos \phi_2 \left(\left| \frac{q}{p} \right| + \left| \frac{p}{q} \right| \right) - x \sin \phi_2 \left(\left| \frac{q}{p} \right| - \left| \frac{p}{q} \right| \right) - r_D^{K\pi} \left[\cos \phi_2 (y \cos \delta_D^{K\pi} + x \sin \delta_D^{K\pi}) \left(\left| \frac{q}{p} \right| + \left| \frac{p}{q} \right| \right) - \sin \phi_2 (x \cos \delta_D^{K\pi} - y \sin \delta_D^{K\pi}) \left(\left| \frac{q}{p} \right| - \left| \frac{p}{q} \right| \right) \right]. \quad (25)$$

CP -violating ratios can be defined through the time-dependent asymmetries between the rates of D^0 and \overline{D}^0 decays to CP -even final states as follows:

$$A_D^{CP}(f, t) = \frac{\Gamma(D^0 \rightarrow f, t) - \Gamma(\overline{D}^0 \rightarrow f, t)}{\Gamma(D^0 \rightarrow f, t) + \Gamma(\overline{D}^0 \rightarrow f, t)} = a_D^f + \tau \Delta Y^f. \quad (26)$$

Experiments measure the slopes ΔY^f of Eq. (26) for the modes $D^0 \rightarrow K^+K^-$ and $D^0 \rightarrow \pi^+\pi^-$ [66–70], and we can define the following combinations:

$$\begin{aligned} \Delta Y^{KK} - \Delta Y^{\pi\pi}, \\ \Delta Y = \frac{1}{2}(\Delta Y^{KK} + \Delta Y^{\pi\pi}). \end{aligned} \quad (27)$$

We use the relations in Eq. (27) to constrain the parameter ΔY^{KK} , which enters the observables coming from the WS/RS ratios of $D^0 \rightarrow K^\pm \pi^\mp$ in Eq. (20). Moreover, since in the U-spin symmetric limit the final state-dependent contributions to ΔY^f are equal and opposite in sign for K^+K^- and $\pi^+\pi^-$ [82], averaging the two modes allows ΔY to be expressed in the framework of approximate universality as

$$\Delta Y = \frac{1}{2} \left[x \sin \phi_2 \left(\left| \frac{q}{p} \right| + \left| \frac{p}{q} \right| \right) - y \cos \phi_2 \left(\left| \frac{q}{p} \right| - \left| \frac{p}{q} \right| \right) \right], \quad (28)$$

up to quadratic corrections in the charm mixing and CPV parameters.

Notice that we do not include the latest measurement of ΔY^f from $D^0 \rightarrow \pi^+\pi^-\pi^0$ decays by LHCb [131] in the combination, as it is less precise than the observables in Eq. (27). Moreover, the application of approximate universality in this case is not as good as for ΔY since there is no U-spin cancellation of final-state dependent contributions.

We consider in the combination also measurements of the time-integrated CP asymmetry for the mode K^+K^- [71–73,76–79] which, from Eq. (26), can be written as

$$A_D^{CP}(f) = a_D^f + \langle \tau \rangle_E^f \Delta Y^f, \quad (29)$$

where we introduced the so-called average decay time $\langle \tau \rangle_E^f$, which depends on the final state f and on the experimental environment E . We assume $\langle \tau \rangle_{B\text{facts}}^{\pi\pi} = \langle \tau \rangle_{B\text{facts}}^{KK} = 1$ for measurements performed at the B factories.

The first signal of direct CP violation in the charm system [75] was observed in the difference between the asymmetries in Eq. (29) for the modes K^+K^- and $\pi^+\pi^-$, which we fit as

$$\Delta A_D^{CP} = a_D^{KK} - a_D^{\pi\pi} + \langle \tau \rangle_E^{KK} \Delta Y^{KK} - \langle \tau \rangle_E^{\pi\pi} \Delta Y^{\pi\pi}. \quad (30)$$

C. Multibody final states

The most precise observables for multibody final states constraining together the charm mixing and CP -violating parameters come from the Dalitz plot analyses of $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ decays [37,38]. The reconstruction of the two-dimensional phase space and the decay time is performed, in a model-independent way, by partitioning them into bins,

which we indicate as $\pm i$ and j , respectively, with i and $-i$ connected by a CP transformation. For each of the bins, ratios analogous to WS/RS in Eq. (15) are measured:

$$\frac{\Gamma_{-ij}(D^0 \rightarrow K_S^0 \pi^+ \pi^-)}{\Gamma_{ij}(D^0 \rightarrow K_S^0 \pi^+ \pi^-)}, \quad \frac{\Gamma_{ij}(\overline{D}^0 \rightarrow K_S^0 \pi^+ \pi^-)}{\Gamma_{-ij}(\overline{D}^0 \rightarrow K_S^0 \pi^+ \pi^-)}. \quad (31)$$

Fitting the quantities in Eq. (31) with a quadratic expansion in τ of the decay rates, the following observables are determined:

$$\begin{aligned} 2x_{CP} &= x \cos \phi_2 \left(\left| \frac{q}{p} \right| + \left| \frac{p}{q} \right| \right) + y \sin \phi_2 \left(\left| \frac{q}{p} \right| - \left| \frac{p}{q} \right| \right), \\ 2y_{CP} &= y \cos \phi_2 \left(\left| \frac{q}{p} \right| + \left| \frac{p}{q} \right| \right) - x \sin \phi_2 \left(\left| \frac{q}{p} \right| - \left| \frac{p}{q} \right| \right), \\ 2\Delta x &= x \cos \phi_2 \left(\left| \frac{q}{p} \right| - \left| \frac{p}{q} \right| \right) + y \sin \phi_2 \left(\left| \frac{q}{p} \right| + \left| \frac{p}{q} \right| \right), \\ 2\Delta y &= y \cos \phi_2 \left(\left| \frac{q}{p} \right| - \left| \frac{p}{q} \right| \right) - x \sin \phi_2 \left(\left| \frac{q}{p} \right| + \left| \frac{p}{q} \right| \right). \end{aligned} \quad (32)$$

Notice that for $D \rightarrow K_S^0 \pi \pi$ measurements performed at B factories we take into account $K - \bar{K}$ mixing following Ref. [82] [see Eq. (144)]. Besides the Dalitz plot analysis, the phase space dependence of the D mesons decay amplitudes to a multibody final state f , such as $K_S^0 K^- \pi^+$, can be parametrized through the following two quantities:

$$\begin{aligned} (r_D^f)^2 &= \frac{\int d\Phi_D |\mathcal{A}_D^f|^2}{\int d\Phi_D |\mathcal{A}_D^{\bar{f}}|^2} = \frac{\int d\Phi_D |\mathcal{A}_D^{\bar{f}}|^2}{\int d\Phi_D |\mathcal{A}_D^f|^2}, \\ \kappa_D^f e^{-i\delta_D^f} &= \frac{\int d\Phi_D \mathcal{A}_D^f \mathcal{A}_D^{f*}}{\sqrt{\int d\Phi_D |\mathcal{A}_D^f|^2} \sqrt{\int d\Phi_D |\mathcal{A}_D^{\bar{f}}|^2}} \\ &= \frac{\int d\Phi_D \mathcal{A}_D^{\bar{f}} \mathcal{A}_D^{f*}}{\sqrt{\int d\Phi_D |\mathcal{A}_D^{\bar{f}}|^2} \sqrt{\int d\Phi_D |\mathcal{A}_D^f|^2}}, \end{aligned} \quad (33)$$

where direct CP violation has been neglected. The integrals in Eq. (33) are performed over the D phase space coordinates Φ_D . The parameter in the upper row is a straightforward generalization for multibody final states of the ratios introduced in Eqs. (14) and (23) for two-body decays. The interference integral in the bottom row is described simply through its magnitude κ_D^f , also known as the coherence factor, and its integrated strong phase δ_D^f . For quasi- CP eigenstates, such as $\pi^+ \pi^- \pi^0$, the imaginary part of the interference integral over the full phase space in Eq. (33) is vanishing. Then, instead of the coherence factor and the integrated strong phase, it is customary to use one single real parameter: the CP -even fraction F_{+}^f , defined as

$$F_{+}^f = \frac{1}{2} \left[1 + \kappa_D^f \cos \delta_D^f \right], \quad (34)$$

which has already been introduced in Eq. (22). From its definition in Eq. (34), the CP -even fraction is non-negative and it is unity for two-body CP -even states.

The decay parameters for $K_S^0 K^\pm \pi^\mp$, $K^\pm \pi^\mp$, and $K^\pm \pi^\mp \pi^0$ final states are measured from the ratio between the branching fractions of the DCS and CF decays [34,42,43], given by

$$\frac{\mathcal{B}(D^0 \rightarrow \bar{f})}{\mathcal{B}(D^0 \rightarrow f)} = \frac{(r_D^f)^2 - \kappa_D^f r_D^f (y \cos \delta_D^f - x \sin \delta_D^f)}{1 - \kappa_D^f r_D^f (y \cos \delta_D^f + x \sin \delta_D^f)}, \quad (35)$$

where we have neglected CP violation and the nonlinear terms in the mixing parameters.

IV. BEAUTY OBSERVABLES

Several charm mixing and decay parameters also enter time-integrated measurements of the so-called B meson cascade decays to $D^0 - \overline{D}^0$ mixed states [123,124,132]. We show how the CKM angle γ appears in the equations describing the most used and precise observables available to date. We also discuss the extraction of γ from the CP -violating phase of the interference between neutral B decays to charmed mesons with and without mixing.

A. Cascade decays

Beauty cascade decays [83–104,112–122] are processes where a B meson goes to an hadron state h and a D^0 or \overline{D}^0 meson, which subsequently oscillates and decays to one of the final states f introduced in Sec. III. In the following, we indicate with B a positively charged B^+ or a neutral meson $B_{(s)}^0$.

Since the final states f are accessible to both the neutral D mesons, the total amplitude for the cascade decays is given by the coherent sum of the two paths in Fig. 1. Then, the CKM angle γ can be measured from the interference between the favoured $b \rightarrow c$ and suppressed $b \rightarrow u$ quark transitions and the corresponding decay amplitudes can be arranged as

$$\begin{aligned} (r_B^{Dh})^2 &= \frac{\int d\Phi_B |\mathcal{A}_B^{Dh}|^2}{\int d\Phi_B |\mathcal{A}_B^{\overline{Dh}}|^2}, \\ \kappa_B^{Dh} e^{-i(\delta_B^{Dh} + \gamma)} &= \frac{\int d\Phi_B \mathcal{A}_B^{Dh} (\mathcal{A}_B^{\overline{Dh}})^*}{\sqrt{\int d\Phi_B |\mathcal{A}_B^{Dh}|^2} \sqrt{\int d\Phi_B |\mathcal{A}_B^{\overline{Dh}}|^2}}, \end{aligned} \quad (36)$$

with the integrals performed over the B meson phase space Φ_B . For two-body final states, the expressions in Eq. (36) simplify since there is no phase space dependence and the coherence factor is unity.

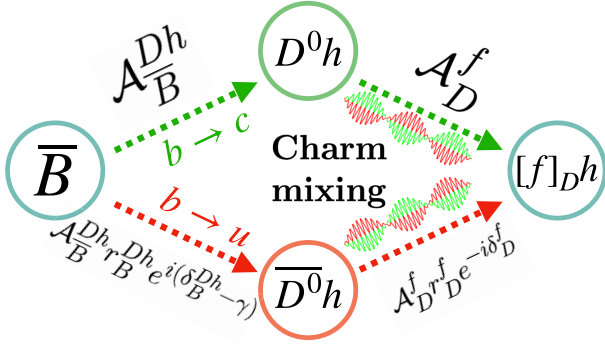


FIG. 1. Schematic representation of $B \rightarrow D$ cascade decays to two-body final states. The CKM angle γ is extracted from the interference between the green and the red paths, whose amplitudes must be summed coherently. See the text for the definitions of the parameters and a generalization for multibody final states.

Then, the time-integrated rate of the cascade decay $\bar{B} \rightarrow [f]_D h$ is found to be

$$\Gamma(\bar{B} \rightarrow [f]_D h) \propto 1 + (r_D^f r_B^{Dh})^2 + 2\kappa_D^f \kappa_B^{Dh} r_D^f r_B^{Dh} \cos(\delta_B^{Dh} - \delta_D^f - \gamma) + \Gamma_{\text{mix}}(x, y, \gamma, f, h), \quad (37)$$

where $\Gamma_{\text{mix}}(x, y, \gamma, f, h)$ takes into account $D^0 - \bar{D}^0$ mixing and it is given by

$$\Gamma_{\text{mix}}(x, y, \gamma, f, h) = -\alpha y \left[\kappa_D^f r_D^f \cos \delta_D^f (1 + (r_B^{Dh})^2) + \kappa_B^{Dh} r_B^{Dh} \cos(\delta_B^{Dh} - \gamma) (1 + (r_D^f)^2) \right] + \alpha x \left[\kappa_D^f r_D^f \sin \delta_D^f ((r_B^{Dh})^2 - 1) + \kappa_B^{Dh} r_B^{Dh} \sin(\delta_B^{Dh} - \gamma) (1 - (r_D^f)^2) \right], \quad (38)$$

up to quadratic corrections in the charm mixing and CP -violating parameters and neglecting direct CP violation. The α coefficient in Eq. (38) describes the impact on $\Gamma_{\text{mix}}(x, y, \gamma, f, h)$ of the nontrivial dependence of the signal selection efficiency on time and it is one when the latter is constant [132].

Since at this order in the charm mixing and CP -violating parameters the decay rate of the CP conjugated process is obtained simply by replacing γ with $-\gamma$ in Eq. (37), the difference between $\Gamma(\bar{B} \rightarrow [f]_D h)$ and $\Gamma(B \rightarrow [\bar{f}]_D \bar{h})$ gives access to $\sin \gamma$. This is commonly referred to as the time-integrated CP asymmetry, which is an example of the so-called Gronau-London-Wyler (GLW) [133,134] and Atwood-Dunietz-Soni (ADS) [135,136] observables that we use in our combination. A complete list of the GLW and ADS modes considered in this work is reported in Table II, while their parametrizations are derived in Appendixes A and B. We have taken the coherence factors for the modes $B \rightarrow DK^*$ from [87] and [88] for charged and neutral B

TABLE II. GLW and ADS observables coming from B mesons cascade decays. The XX notation denotes both $K^+ K^-$ and $\pi^+ \pi^-$ states, while we use $K\pi$ for $K^\mp \pi^\pm$. h stands for both K and π mesons. We refer to $\pi^+ \pi^- \pi^+ \pi^-$ and $K^\mp \pi^\pm \pi^+ \pi^-$ final states as 4π and $K3\pi$, respectively. We use f^\pm as a shorthand notation for CP -even and CP -odd eigenstates, respectively. The observables are described in details in Appendixes A and B.

GLW observables	ADS observables	$B \rightarrow D^0$ decays	D^0 decays	References
$A_B^{CP}(XX\pi^0, h)$,	$A^{\text{fav.sup}}(K\pi\pi^0, h)$,	$B^\pm \rightarrow Dh^\pm$	$D \rightarrow K\pi\pi^0$,	[91]
$R^{CP}(XX\pi^0, K\pi\pi^0, K, \pi)$	$R_\pm(K\pi\pi^0, h)$		$D \rightarrow XX\pi^0$	
$A_B^{CP}(XX, h\pi\pi)$,	$A^{\text{fav}}(K\pi, h\pi\pi)$,	$B^\pm \rightarrow Dh^\pm \pi\pi$	$D \rightarrow K\pi$,	[86]
$R^{CP}(XX, K\pi, K\pi\pi, \pi\pi\pi)$	$R_\pm(K\pi, h\pi\pi)$		$D \rightarrow XX$	
$A_B^{CP}(XX, K^{*\pm})$, $R(XX, K\pi, K^{*\pm})$	$A^{\text{fav.sup}}(K\pi, K^{*\pm})$, $R^{\text{ADS}}(K\pi, K^{*\pm})$	$B^\pm \rightarrow DK^{*\pm}$	$D \rightarrow K\pi$, $D \rightarrow XX$,	[96]
$A_B^{CP}(4\pi, K^{*\pm})$, $R(4\pi, K3\pi, K^{*\pm})$	$A^{\text{fav.sup}}(K3\pi, K^{*\pm})$, $R^{\text{ADS}}(K3\pi, K^{*\pm})$		$D \rightarrow K3\pi$, $D \rightarrow 4\pi$	
$A_B^{CP}(XX, K^{*0})$, $R(XX, K\pi, K^{*0})$	$A^{\text{fav}}(K\pi, K^{*0})$, $R_\pm(K\pi, K^{*0})$	$B^0 \rightarrow DK^{*0}$	$D \rightarrow K\pi$, $D \rightarrow XX$,	[94]
$A_B^{CP}(4\pi, K^{*0})$, $R(4\pi, K3\pi, K^{*0})$	$A^{\text{fav}}(K3\pi, K^{*0})$, $R_\pm(K3\pi, K^{*0})$	$B_s^0 \rightarrow D\bar{K}^{*0}$	$D \rightarrow K3\pi$, $D \rightarrow 4\pi$	
$A_B^{CP}(XX, h)$, $R^{CP}(XX, K\pi, K, \pi)$	$A^{\text{fav}}(K\pi, K)$, $R_\pm(K\pi, h)$	$B^\pm \rightarrow Dh^\pm$	$D \rightarrow K\pi$, $D \rightarrow XX$	[89]
$A_B^{CP}(XX, h(\gamma, \pi^0))$,	$A^{\text{fav}}(K\pi, K(\gamma, \pi^0))$,	$B^\pm \rightarrow [D\pi^0]_D h^\pm$	$D \rightarrow K\pi$,	[89]
$R^{CP}(XX, K\pi, K(\gamma, \pi^0), \pi(\gamma, \pi^0))$	$R_\pm(K\pi, h(\gamma, \pi^0))$	$B^\pm \rightarrow [D\gamma]_D h^\pm$	$D \rightarrow XX$	
	$A^{\text{fav.sup}}(K_S^0 K\pi, h)$,			

(Table continued)

TABLE II. (*Continued*)

GLW observables	ADS observables	$B \rightarrow D^0$ decays	D^0 decays	References
	$R^{\text{fav.sup}}(K_S^0 K \pi, K, \pi)$,	$B^\pm \rightarrow D h^\pm$	$D \rightarrow K_S^0 K \pi$	[90],
	$R^{\text{ADS}}(K_S^0 K \pi, \pi)$			[92]
$A_B^{CP}(XX\pi\pi, h)$, $R^{CP}(XX\pi\pi, K3\pi, K, \pi)$		$B^\pm \rightarrow D h^\pm$	$D \rightarrow XX\pi\pi$	[101]
	$R^{\text{ADS}}(K\pi, h)$, $A^{\text{sup}}(K\pi, h)$	$B^\pm \rightarrow D h^\pm$	$D \rightarrow K\pi$	[84]
	$R^{\text{ADS}}(K\pi\pi^0, h)$, $A^{\text{sup}}(K\pi\pi^0, h)$	$B^\pm \rightarrow D h^\pm$	$D \rightarrow K\pi\pi^0$	[85]
$A_B^{CP}(K^+K^-, K)$, $R^{CP}(K^+K^-, K\pi, K, \pi)$		$B^\pm \rightarrow D h^\pm$	$D \rightarrow K^+K^-, D \rightarrow K\pi$,	[93]
$A_B^{CP}(K_S^0\pi^0, K)$, $R^{CP}(K_S^0\pi^0, K\pi, K, \pi)$			$D \rightarrow K_S^0\pi^0$	
$A_B^{CP}(f^\pm, K(\pi^0))$,		$B^\pm \rightarrow [D\pi^0]_D, h^\pm$	$D \rightarrow f^\pm$,	[83]
$R^{CP}(f^\pm, K\pi, K(\pi^0), \pi(\pi^0))$			$D \rightarrow K\pi$	
$A_B^{CP}(f^\pm, K)$, $R^{CP}(f^\pm, K\pi, K, \pi)$		$B^\pm \rightarrow DK^\pm$	$D \rightarrow f^\pm, D \rightarrow K\pi$	[115]
$A_B^{CP}(f^\pm, K)$, $R^{CP}(f^\pm, K\pi, K, \pi)$		$B^\pm \rightarrow D^*K^\pm$	$D \rightarrow f^\pm, D \rightarrow K\pi$	[113]
$A_B^{CP}(f^\pm, K^{*\pm})$, $R(f^\pm, K\pi, K^{*\pm})$	$A^{\text{sup}}(K\pi, K^{*\pm})$, $R^{\text{ADS}}(K\pi, K^{*\pm})$	$B^\pm \rightarrow DK^{*\pm}$	$D \rightarrow f^\pm, D \rightarrow K\pi$	[114]
$A_B^{CP}(\pi^+\pi^-\pi^0, K)$		$B^\pm \rightarrow DK^\pm$	$D \rightarrow \pi^+\pi^-\pi^0$	[112]
	$[R_-(K\pi, h(\gamma, \pi^0)) + R_+(K\pi, h(\gamma, \pi^0))]/2$	$B^\pm \rightarrow [D\pi^0]_D, h^\pm$	$D \rightarrow K\pi$	[116]
	$\frac{R_-(K\pi, h(\gamma, \pi^0)) - R_+(K\pi, h(\gamma, \pi^0))}{R_-(K\pi, h(\gamma, \pi^0)) + R_+(K\pi, h(\gamma, \pi^0))}$	$B^\pm \rightarrow [D\gamma]_D, h^\pm$		
	$[R_-(K\pi, h) + R_+(K\pi, h)]/2$	$B^\pm \rightarrow D h^\pm$	$D \rightarrow K\pi$	[116]
	$\frac{R_-(K\pi, h) - R_+(K\pi, h)}{R_-(K\pi, h) + R_+(K\pi, h)}$			
	$R_\pm(K\pi\pi^0, K)$	$B^\pm \rightarrow DK^\pm$	$D \rightarrow K\pi\pi^0$	[117]
$A_B^{CP}(XX, K)$, $R^{CP}(XX, K\pi, K, \pi)$		$B^\pm \rightarrow DK^\pm$	$D \rightarrow XX, D \rightarrow K\pi$	[118]
	$A^{\text{sup}}(K\pi, h)$, $R^{\text{ADS}}(K\pi, h)$	$B^\pm \rightarrow D h^\pm$	$D \rightarrow K\pi$	[119]

mesons, respectively. For cascade decays with multibody final states of the charm [137–139], as $K_S^0 K^+ K^-$, $K_S^0 \pi^+ \pi^-$, $\pi^+ \pi^- K^+ K^-$, and $\pi^+ \pi^- K^+ K^-$, analyses of the variation of the decay rates across the phase space are available [96–104]. The Dalitz plots of these events are fitted using a model for the D meson decay amplitudes or by binning the phase space and solving a system of linear equations in a model-independent way. The first method is more precise statistically with respect to the latter but introduces an additional systematic uncertainty due to the model.

In both cases, the following cartesian observables are obtained:

$$x_\pm^{Dh} = r_B^{Dh} \cos(\delta_B^{Dh} \pm \gamma), \quad y_\pm^{Dh} = r_B^{Dh} \sin(\delta_B^{Dh} \pm \gamma). \quad (39)$$

Measuring the quantities in Eq. (39) for beauty decays with a pion in the final state is particularly hard due to the small value of $r_B^{D\pi} \sim \mathcal{O}(10^{-3})$.

In this case, the cartesian observables for $\bar{B} \rightarrow [f]_D \pi$ are written as rotated with respect to the ones entering $\bar{B} \rightarrow [f]_D K$ through the transformation

$$\begin{pmatrix} x_\pm^{D\pi} \\ y_\pm^{D\pi} \end{pmatrix} = \begin{pmatrix} x_\xi^{D\pi} & -y_\xi^{D\pi} \\ y_\xi^{D\pi} & x_\xi^{D\pi} \end{pmatrix} \begin{pmatrix} x_\pm^{DK} \\ y_\pm^{DK} \end{pmatrix}, \quad (40)$$

where $x_\xi^{D\pi}$ and $y_\xi^{D\pi}$ are given by

$$x_\xi^{D\pi} = \frac{r_B^{D\pi}}{r_B^{DK}} \cos(\delta_B^{D\pi} - \delta_B^{DK}), \quad y_\xi^{D\pi} = \frac{r_B^{D\pi}}{r_B^{DK}} \sin(\delta_B^{D\pi} - \delta_B^{DK}). \quad (41)$$

Then, the coefficients in Eq. (41) are extracted simultaneously with x_\pm^{DK} and y_\pm^{DK} . The cartesian observables used in our combination are reported in Table III.

It is worth noting that the expressions in Eqs. (37)–(39), and (41) are still valid for cascade decays with the D^0 or \bar{D}^0 meson produced together with a π^0 meson or a photon γ by the decay of a D^* intermediate state, simply replacing the strong phase δ_D^f with $\delta_{D^*}^f$ or $\delta_{D^*}^f + \pi$, respectively.

TABLE III. Bondar-Poluektov-Giri-Grossman-Soffer-Zupan cartesian observables used in the combination. Here, XX stands for both the K^+K^- and $\pi^+\pi^-$ states. The dependence of the observables on the fit parameters are reported in Eqs. (39) and (41).

BPGGSZ cartesian observables	$B \rightarrow D^0$ decays	D^0 decays	References
$x_{\pm}^{DK^{*0}}, y_{\pm}^{DK^{*0}}$	$B^0 \rightarrow DK^{*0}$	$D \rightarrow K_S^0 XX$	[104]
$x_{\pm}^{DK}, x_{\xi}^{D\pi}$	$B^{\pm} \rightarrow DK^{\pm}$	$D \rightarrow K_S^0 \pi^+ \pi^- \pi^0$	[98]
$y_{\pm}^{DK}, y_{\xi}^{D\pi}$	$B^{\pm} \rightarrow D\pi^{\pm}$		
$x_{\pm}^{DK}, x_{\xi}^{D\pi}$	$B^{\pm} \rightarrow DK^{\pm}$	$D \rightarrow K_S^0 XX$	[99,100]
$y_{\pm}^{DK}, y_{\xi}^{D\pi}$	$B^{\pm} \rightarrow D\pi^{\pm}$		
$x_{\pm}^{DK}, x_{\xi}^{D\pi}$	$B^{\pm} \rightarrow DK^{\pm}$	$D \rightarrow K^+ K^- \pi^+ \pi^-$	[101]
$y_{\pm}^{DK}, y_{\xi}^{D\pi}$	$B^{\pm} \rightarrow D\pi^{\pm}$		
$x_{\pm}^{[D\pi^0]_{D^*K}}, x_{\pm}^{[D\gamma]_{D^*K}}$	$B^{\pm} \rightarrow [D\pi^0]_{D^*} K^{\pm}$	$D \rightarrow K_S^0 \pi^+ \pi^-$	[97]
$y_{\pm}^{[D\pi^0]_{D^*K}}, y_{\pm}^{[D\gamma]_{D^*K}}$	$B^{\pm} \rightarrow [D\gamma]_{D^*} K^{\pm}$		
$x_{\pm}^{D^*K}, x_{\xi}^{D^*\pi}$	$B^{\pm} \rightarrow D^* K^{\pm}$	$D \rightarrow K_S^0 XX$	[102,103]
$y_{\pm}^{D^*K}, y_{\xi}^{D^*\pi}$	$B^{\pm} \rightarrow D^* \pi^{\pm}$		
$x_{\pm}^{DK^{*\pm}}, y_{\pm}^{DK^{*\pm}}$	$B^{\pm} \rightarrow DK^{*\pm}$	$D \rightarrow K_S^0 XX$	[96]
$x_{\pm}^{DK}, x_{\xi}^{D\pi}$	$B^{\pm} \rightarrow DK^{\pm}$	$D \rightarrow K^{\mp} \pi^{\pm} \pi^+ \pi^-$	[95]
$y_{\pm}^{DK}, y_{\xi}^{D\pi}$	$B^{\pm} \rightarrow D\pi^{\pm}$		
$\sqrt{(x_{\pm}^{DK} - (2F_{\pi\pi\pi^0} - 1))^2 + (y_{\pm}^{DK})^2}$	$B^{\pm} \rightarrow DK^{\pm}$	$D \rightarrow \pi^+ \pi^- \pi^0$	[112] ^a
$\arctan[y_{\pm}^{DK} / (x_{\pm}^{DK} - (2F_{\pi\pi\pi^0} - 1))]$			
$x_{\pm}^{DK}, y_{\pm}^{DK}, x_{\pm}^{D^*K}, y_{\pm}^{D^*K}$	$B^{\pm} \rightarrow D^{(*)} K^{\pm}$	$D \rightarrow K_S^0 XX$	[121]
$x_{\pm}^{DK^{*\pm}}, y_{\pm}^{DK^{*\pm}}$	$B^{\pm} \rightarrow DK^{*\pm}$	$D \rightarrow K_S^0 \pi\pi$	[121]
$x_{\pm}^{DK^{*\pm}}, y_{\pm}^{DK^{*\pm}}$	$B^{\pm} \rightarrow DK^{*\pm}$	$D \rightarrow K_S^0 \pi\pi$	[120]
$x_{\pm}^{DK^{*0}}, y_{\pm}^{DK^{*0}}$	$B^0 \rightarrow DK^{*0}$	$D \rightarrow K_S^0 \pi\pi$	[122]

^aWe thank Matthew W. Kenzie for providing us with the results of the analysis in terms of cartesian observables.

B. Neutral B decays to charmed mesons

The CKM angle γ can be obtained from the CP -violating phase of the interference between neutral B decays to charmed mesons f (e.g., $f = D^{(*)-}\pi^+, D^-\rho^+, D_S^-K^+$, and $D_S^-K^+\pi^+\pi^-$) with and without mixing, as shown schematically in Fig. 2. The decay amplitudes for these processes can be organized into ratios of magnitudes, coherence factors and strong phases through the following integrals over the phase space $\Phi_{B(s)}$:

$$(r_{B(s)}^f)^2 = \frac{\int d\Phi_{B(s)} |\mathcal{A}_{B(s)}^f|^2}{\int d\Phi_{B(s)} |\mathcal{A}_{B(s)}^f|^2},$$

$$\kappa_{B(s)}^f e^{i(\delta_{B(s)}^f + \varphi_{(s)})} = \frac{\int d\Phi_{B(s)} \mathcal{A}_{B(s)}^f \mathcal{A}_{B(s)}^{f*}}{\sqrt{\int d\Phi_{B(s)} |\mathcal{A}_{B(s)}^f|^2} \sqrt{\int d\Phi_{B(s)} |\mathcal{A}_{B(s)}^f|^2}}, \quad (42)$$

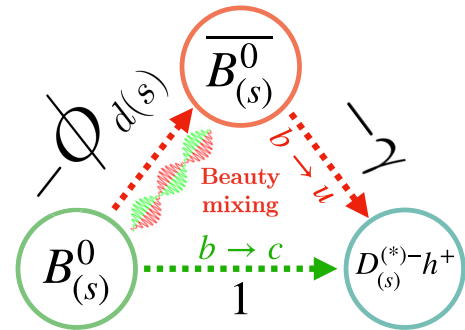


FIG. 2. Schematic representation of the extraction of the CKM angle γ from the CP -violating phase of the interference between neutral B decays to charmed mesons with and without mixing. Here, h^+ stands for π^+ and ρ^+ (K^+ and $K^+\pi^+\pi^-$) for $B(s)$ decays.

TABLE IV. Observables coming from time-dependent analyses of neutral B mesons decays to charmed mesons. The observables are defined in Eq. (44).

Observables	$B_{(s)}^0 \rightarrow D_{(s)}^\mp h^\pm$ decays	References
$C_{D_s^\mp K^\pm}, G_{D_s^\mp K^\pm}, S_{D_s^\mp K^\pm}$	$B_s^0 \rightarrow D_s K$	[107]
$S_{D^\mp \pi^\pm}$	$B^0 \rightarrow D \pi$	[105]
$C_{D_s^\mp K^\pm \pi\pi}, G_{D_s^\mp K^\pm \pi\pi}, S_{D_s^\mp K^\pm \pi\pi}$	$B_s^0 \rightarrow D_s K \pi \pi$	[106]
$-(S_{D^- \pi^+} + S_{D^+ \pi^-})/2, (S_{D^- \pi^+} - S_{D^+ \pi^-})/2$	$B^0 \rightarrow D \pi$	[109,110]
$-(S_{D^{*-} \pi^+} + S_{D^{*+} \pi^-})/2, (S_{D^{*-} \pi^+} - S_{D^{*+} \pi^-})/2$	$B^0 \rightarrow D^* \pi$	[108–111]
$-(S_{D^- \rho^+} + S_{D^+ \rho^-})/2, (S_{D^- \rho^+} - S_{D^+ \rho^-})/2$	$B^0 \rightarrow D \rho$	[109]

where $\varphi_{(s)}$ are the weak phases entering the decay, which are given in the SM by a combination of the relevant CKM matrix elements. Then, the time-dependent rates of neutral $B_{(s)}^0$ meson decays can be written as [140,141]

$$\Gamma(B_{(s)}^0 \rightarrow f, t) \propto \cosh(t\Delta\Gamma_{(s)}/2) - G_f \sinh(t\Delta\Gamma_{(s)}/2) + C_f \cos(t\Delta m_{(s)}) - S_f \sin(t\Delta m_{(s)}), \quad (43)$$

where $\Delta m_{(s)}$ and $\Delta\Gamma_{(s)}$ are the differences in masses and lifetimes between the heavier and lighter eigenstates of the $B_{(s)}^0$ mixing Hamiltonian. The observables C_f , G_f , and S_f are fitted from the data [105–111] and are given by

$$C_f = \frac{1 - (r_{B_{(s)}^0}^f)^2}{1 + (r_{B_{(s)}^0}^f)^2},$$

$$G_f = \frac{-2\eta_{(s)}\kappa_{B_{(s)}^0}^f r_{B_{(s)}^0}^f}{1 + (r_{B_{(s)}^0}^f)^2} \cos\left(\delta_{B_{(s)}^0}^f - (\phi_{d(s)} + \gamma)\right),$$

$$S_f = \frac{2\eta_{(s)}\kappa_{B_{(s)}^0}^f r_{B_{(s)}^0}^f}{1 + (r_{B_{(s)}^0}^f)^2} \sin\left(\delta_{B_{(s)}^0}^f - (\phi_{d(s)} + \gamma)\right), \quad (44)$$

with $\eta = -1$ and $\eta_s = 1$. Here, we have neglected CP violation in pure $B_{(s)}^0 - \bar{B}_{(s)}^0$ mixing, as well as direct CP violation. The phases $\phi_{(d,s)}$ are obtained in the SM from the CKM angles $\beta = \arg(-V_{td}^* V_{cd} V_{cb}^* V_{tb})$ and $\beta_s = \arg(-V_{cs}^* V_{ts} V_{tb}^* V_{cb})$ as $\phi_d = 2\beta$ and $\phi_s = -2\beta_s$, up to corrections to the fourth power of the sine of the Cabibbo angle. Similar observables to the ones in Eq. (44) can be obtained for the CP conjugate final states by changing the sign of $\eta_{(s)}$ and of the CP -violating phases between mixing and decay $-(\phi_{d(s)} + \gamma)$. Beyond the SM, the relations between $\phi_{d(s)}$ and $\beta_{(s)}$ could be altered, but one can still use the experimental values of $\phi_{(d,s)}$ to take into account $B_{(s)}^0$ mixing effects in the extraction of γ . The measurements used in the combination are reported in Table IV, while inputs for ϕ_d and ϕ_s are taken from the UTfit summer 2024 update [142] (including also the theoretical uncertainty [143]) and from the HFLAV summer 2024 average of $B_s^0 \rightarrow J/\psi\phi$ modes [126], respectively.

For the purpose of using γ in the unitarity triangle analysis, the correlation between the results for γ and the

inputs used for $\phi_{(d,s)}$ should be taken into account. However, in practice, for current experimental uncertainties such correlation is lost in the global fit, although it might move the central value of γ at the degree level when only B_s decays are considered.

V. RESULTS

We combine the observables in Tables I–IV in a Bayesian framework, employing as likelihood the product of Gaussian distributions for each set of correlated measurements. The fit parameters are assumed to follow uniform distributions in a sufficiently large range. It is worth noting that the beauty observables described in Sec. IV and Appendixes A and B are symmetric under the simultaneous transformation $\gamma \rightarrow \gamma \pm \pi$ and $\delta_B^{Dh} \rightarrow \delta_B^{Dh} \pm \pi$, for each of the B modes considered, and we have limited our study to just one of these solutions. Other solutions can be found by adding $\pm\pi$ to our results. The posterior probability density functions (PDFs) of the parameters are determined through a Markov chain Monte Carlo (MCMC) algorithm¹ implemented using the Bayesian Analysis Toolkit (BAT) software package [145]. We perform four combinations using different subsets of the beauty observables, divided by the species of the B mesons: all the modes together, only charged B , only neutral B and B_s modes. Furthermore, we perform a combination of the charm observables only, to compare the results with the ones obtained by HFLAV and to assess the impact of the combined analysis.

We report in Table V the charm mixing and CP -violating parameters obtained using all the inputs or excluding the beauty modes, while the corresponding PDFs are shown in Fig. 3. Two-dimensional contours are depicted in Fig. 4. The charm parameters are compatible with the estimates found by the 2023 analysis by HFLAV² [126]. An interesting byproduct of our fit is the value of

¹The fit code is available at <https://github.com/silvest/GammaDDbar> [144].

²See the “No subleading ampl. for CF/DCS decays” fit at https://hflav-eos.web.cern.ch/hflav-eos/charm/CKM23/results_mix_cp.html.

TABLE V. Results for the charm part of the combination when using all the observables (“All modes”) or excluding the beauty measurements (“Charm only”). The half-width of the smallest interval containing at least 68% probability (Uncertainty) and its center (Value) are reported for each quantity of interest. The smallest interval containing at least 95% probability is reported as well.

Parameter	All modes			Charm modes only		
	Value	Uncertainty	95.4% probability	Value	Uncertainty	95.4% probability
$\phi_2^M [^\circ]$	0.13	± 0.70	[-1.27, 1.54]	0.07	± 0.73	[-1.42, 1.53]
$\phi_2^\Gamma [^\circ]$	2.1	± 1.6	[-1.0, 5.2]	1.9	± 1.6	[-1.1, 4.9]
$\phi_2 [^\circ]$	-1.5	± 1.1	[-3.7, 0.6]	-1.3	± 1.1	[-3.5, 0.7]
$ q/p - 1 [^\circ]$	-1.6	± 1.5	[-4.5, 1.4]	-1.4	± 1.5	[-4.3, 1.5]
$x_{12} \simeq x [^\circ]$	4.01	± 0.43	[3.16, 4.83]	3.96	± 0.43	[3.12, 4.80]
$y_{12} \simeq y [^\circ]$	6.10	± 0.17	[5.77, 6.45]	6.28	± 0.23	[5.84, 6.74]
$a_D^{KK} [^\circ]$	0.40	± 0.53	[-0.63, 1.43]	0.40	± 0.53	[-0.64, 1.44]
$a_D^{\pi\pi} [^\circ]$	2.34	± 0.60	[1.15, 3.53]	2.34	± 0.61	[1.15, 3.52]
$\phi_{12} [^\circ]$	-2.0	± 1.8	[-5.6, 1.7]	-1.8	± 1.9	[-5.5, 1.9]
$r_D^{K\pi} [^\circ]$	5.8573	± 0.0094	[5.8387, 5.8758]	5.8601	± 0.0099	[5.8406, 5.8796]
$\delta_D^{K\pi} [^\circ]$	191.4	± 2.4	[186.6, 196.0]	194.1	± 3.2	[187.6, 200.3]

the direct CP asymmetries in $D^0 \rightarrow K^+K^-$ and $D^0 \rightarrow \pi^+\pi^-$ decays within the framework of approximate universality:

$$a_D^{KK} = (4.0 \pm 5.3) \times 10^{-4}, \quad a_D^{\pi\pi} = (23.4 \pm 6.0) \times 10^{-4},$$

$$\rho = 57\%. \quad (45)$$

We report their two-dimensional PDF in Fig. 5. The values in Eq. (45) provide evidence for direct CP violation in

$D^0 \rightarrow \pi^+\pi^-$ at approximately 4σ , together with a sizable deviation from the U-spin expectation of $a_D^{KK} \sim -a_D^{\pi\pi}$. Notice that HFLAV and LHCb fit a_D^{KK} and $a_D^{\pi\pi}$ neglecting final-state-dependent contributions to the linear parts entering the time-integrated CP asymmetries in Eqs. (29) and (30) (i.e., $\Delta Y^f = \Delta Y$). These terms are not known at present and could be non-negligible since they have a relative size of the order of the U-spin breaking parameter with respect to

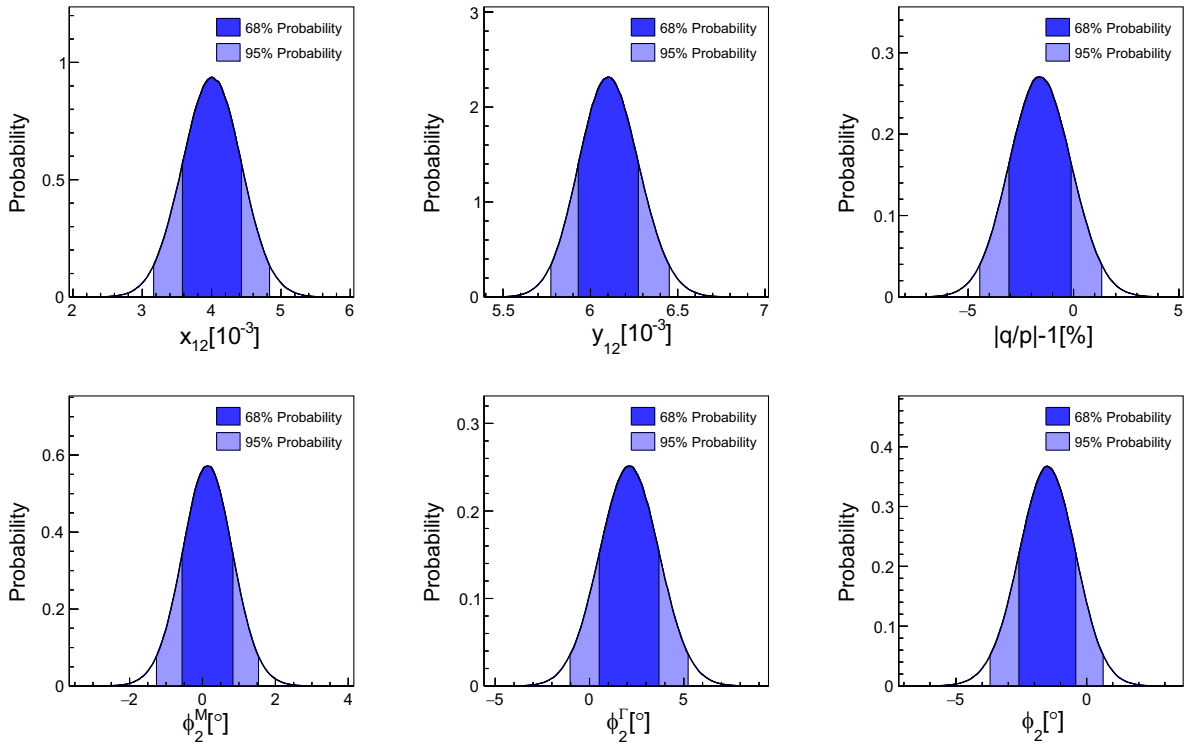


FIG. 3. PDFs of the charm mixing and CP -violating parameters obtained using all the observables.

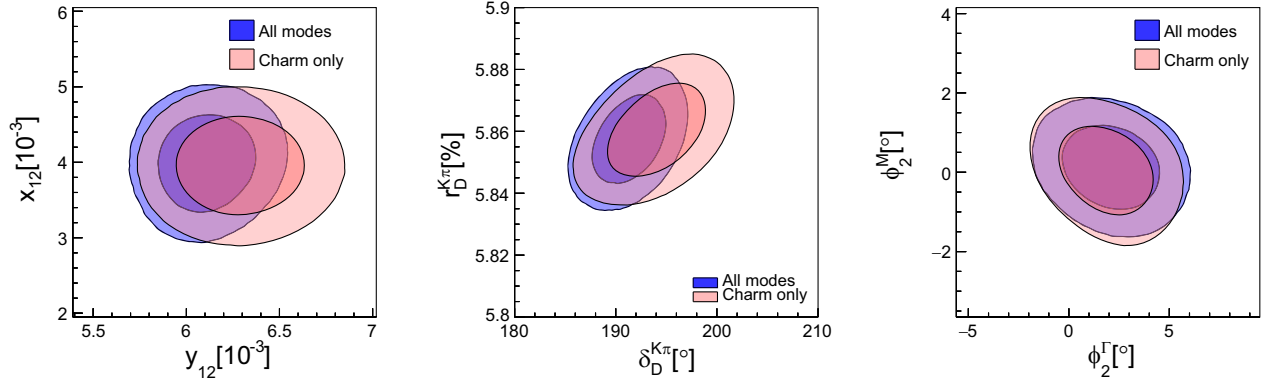


FIG. 4. PDFs of the charm mixing and CP -violating parameters obtained using all the observables (“All modes”) or using only the charm modes (“Charm only”). Darker (lighter) regions correspond to 68% (95%) probability.

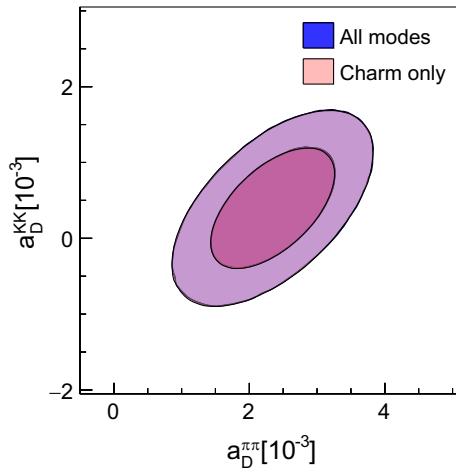


FIG. 5. Pdf of the parameters describing the direct CP violation in $D^0 \rightarrow K^+K^-$ (a_D^{KK}) and $D^0 \rightarrow \pi^+\pi^-$ ($a_D^{\pi\pi}$) decays. Darker (lighter) regions corresponds to 68% (95%) probability.

the final-state independent part [82]. On the other hand, the relations we adopted exploit the average between $D^0 \rightarrow K^+K^-$ and $D^0 \rightarrow \pi^+\pi^-$ decays to get an additional suppression of $\mathcal{O}(\epsilon)$ of the final-state-dependent contributions. At this level of precision, the probability intervals found for a_D^{KK} and $a_D^{\pi\pi}$ are compatible between the two fitting strategies, except for small shifts of their central values. As the precision of $A_D^{CP}(f)$ measurements improves, differences between the fits might become relevant.

Adding beauty observables to the combination mainly improves the determination of the strong phase $\delta_D^{K\pi}$, which in turn improves the determination of y_{12} from $D \rightarrow K\pi$ decays.³ Let us now discuss the results for the

CKM angle γ . The estimates extracted using separately measurements of beauty decays with different types of initial B mesons are reported in Table VI and their PDFs are shown in Fig. 6; we also report in Fig. 7 the PDFs obtained using the different categories of B^0 observables. The CKM angle γ obtained using all the inputs falls within the confidence intervals found by the latest LHCb combination [124], as the other beauty and charm parameters. The γ estimates extracted from different subsets of beauty observables are all compatible with each other, substantially reducing the moderate tension of 2.2σ between γ_{B^\pm} and γ_{B^0} observed in previous combinations [146,147]. This change is driven by the most precise measurement of $B^0 \rightarrow DK^{*0}$ cascade decays recently published by LHCb [94], which already in their work provided a γ estimate compatible with our result. The full results of the fit are reported in Tables VIII and IX and Fig. 8. Correlation between D meson mixing parameters and the CKM angle γ is below 5%, as reported in Table VII. Correlations between γ and $\phi_{d(s)}$ is 1% (−2%) when combining all the observables, while it is −5% (−8%) when considering only neutral $B_{d(s)}^0$ measurements.

TABLE VI. Probability intervals for γ when using all the measurements and when splitting the inputs of the combination: only charged B measurements (γ_{B^\pm}), only neutral B (γ_{B^0}), and B_s modes ($\gamma_{B_s^0}$). The central value (Value) and the half-width of the smallest interval containing at least 68% probability (Uncertainty) are reported.

B meson types	Value	Uncertainty
All modes: γ [°]	65.7	± 2.5
B^\pm : γ_{B^\pm} [°]	65.5	± 2.7
B^0 : γ_{B^0} [°]	57	± 13
B_s^0 : $\gamma_{B_s^0}$ [°]	77.4	± 9.6

³We are indebted to Tommaso Pajero for pointing this out to us.

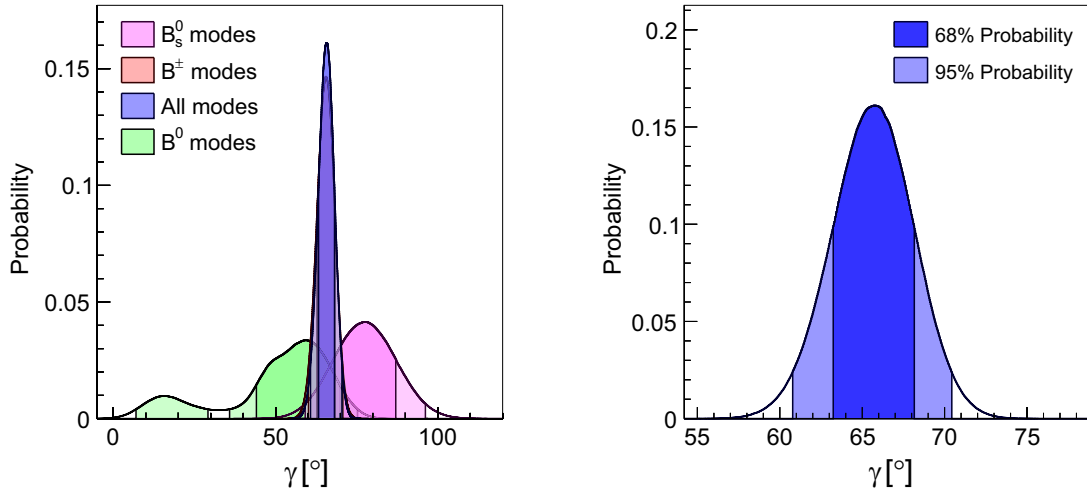


FIG. 6. Left: PDFs of the estimates of γ obtained combining all the beauty modes and using separately measurements involving only charged B , only B^0 , or B_s^0 mesons. Right: PDF of γ obtained by combining all the modes.

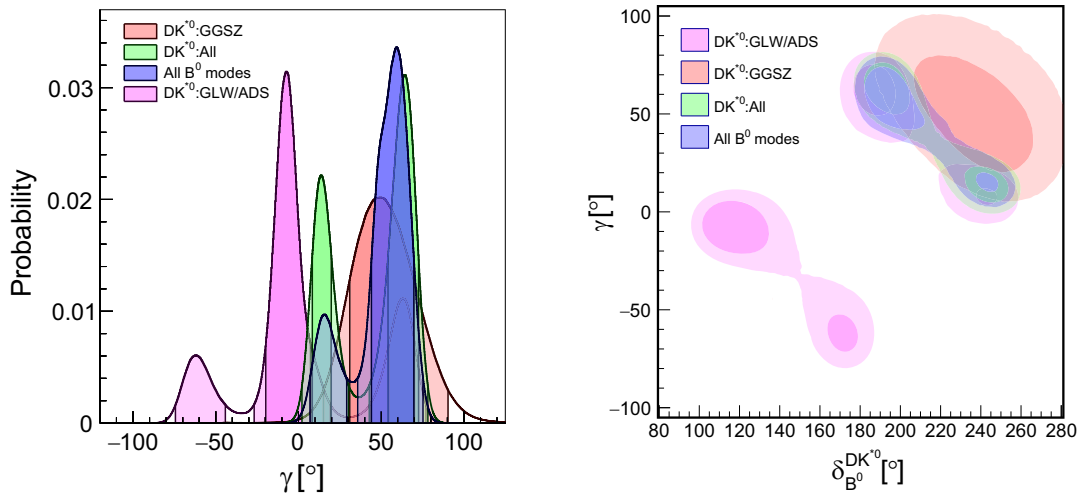


FIG. 7. Left: PDFs of γ obtained using different types of B^0 observables. Right: correlation between γ and the strong phase entering $B \rightarrow DK^{*0}$ decays.

TABLE VII. Correlation matrix for the most significant quantities in percent. Values smaller than 1% are not reported. Large correlations between $\phi_2^{M,\Gamma}$, $|q/p| - 1$ and ϕ_2 are due to the fact that they are not independent parameters, as shown in Eqs. (7) and (13).

	γ	r_B^{DK}	δ_B^{DK}	$r_B^{D\pi}$	$\delta_B^{D\pi}$	$r_D^{K\pi}$	$\delta_D^{K\pi}$	x_{12}	y_{12}	ϕ_2^M	ϕ_2^Γ	ϕ_2	$ q/p - 1$	a_D^{KK}	$a_D^{\pi\pi}$
γ	100	35	58	3	18	1	5		5	-1	-1	1			
r_B^{DK}		100	17	-6	2	-4	-9	1	-8	1	1	-1	-1		
δ_B^{DK}			100	-11	12	-14	-38	2	-33	4	5	-5	-3		
$r_B^{D\pi}$				100	60	7	10	-1	8	-1	-1	1	1		
$\delta_B^{D\pi}$					100	1	-9	-1	-10	1	2	-2	-1		
$r_D^{K\pi}$						100	38	36	3	-5	-3	7	-1	-1	-1
$\delta_D^{K\pi}$							100	6	88	-11	-13	14	8	-1	-1
x_{12}								100	8	1	-1	8	-4	1	1
y_{12}									100	-10	-10	11	6	-1	
ϕ_2^M										100	-18	1	53	43	42
ϕ_2^Γ											100	-98	-93	3	1
ϕ_2												100	84	-11	-9
$ q/p - 1$													100	14	15
a_D^{KK}														100	57
$a_D^{\pi\pi}$															100

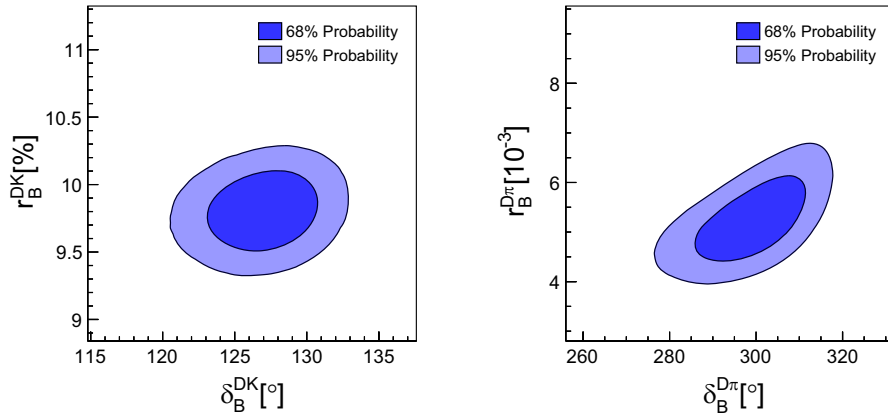


FIG. 8. Two-dimensional contours of the beauty decay parameters describing the most precise observables of the combination obtained by combining all the modes.

VI. CONCLUSIONS

We presented a Bayesian combination of charm and beauty observables to determine the CKM angle γ and the neutral D mixing and CP -violating parameters simultaneously in the framework of the SM and employing approximate universality for $D^0 - \bar{D}^0$ mixing.

The charm parameters are given by

$$x_{12} \simeq x = (0.401 \pm 0.043)\%, \quad y_{12} \simeq y = (0.610 \pm 0.017)\%, \quad \phi_2^M = (0.13 \pm 0.70)^\circ, \quad \phi_2^\Gamma = (2.1 \pm 1.6)^\circ, \\ \phi_2 = (-1.5 \pm 1.1)^\circ, \quad |q/p| - 1 = (-1.6 \pm 1.5)\%,$$

while the CKM angle γ is found to be

$$\gamma = (65.7 \pm 2.5)^\circ,$$

in perfect agreement with the indirect determination from the unitary triangle analysis $\gamma_{\text{UT}} = (65.6 \pm 1.4)^\circ$ [142]. The uncertainties on the two CP -violating phases $\phi_2^{M,\Gamma}$ are still one order of magnitude larger than what is needed to test the predictions $\phi_2^M \sim \phi_2^\Gamma \sim 0.1^\circ$ obtained from U-spin arguments in Eq. (10), and a factor of 7 larger than the upper bound of $|\phi_2^\Gamma| < 0.3^\circ$ in Eq. (11). The direct CP asymmetries in $D^0 \rightarrow K^+K^-$ and $D^0 \rightarrow \pi^+\pi^-$ decays within the framework of approximate universality are given by

$$a_D^{KK} = (4.0 \pm 5.3) \times 10^{-4}, \quad a_D^{\pi\pi} = (23.4 \pm 6.0) \times 10^{-4}. \quad (46)$$

To check the consistency of our combination of measurements sensitive to the angle γ , we extracted the value of γ separately from charged B , B^0 , and B_s^0 meson decays. We found three compatible estimates, given by

$$\gamma_{B^\pm} = (65.5 \pm 2.7)^\circ, \quad \gamma_{B^0} = (57 \pm 13)^\circ, \quad \gamma_{B_s^0} = (77.4 \pm 9.6)^\circ.$$

ACKNOWLEDGMENTS

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DATA AVAILABILITY

The data that support the findings of this article are openly available [144].

APPENDIX A: GLW OBSERVABLES

The GLW observables consist of CP -conserving and CP -violating ratios of time-integrated rates of B cascade decays when the final states f are (quasi-) CP eigenstates. Here, we report the definitions of the observables in the leftmost column of Table II and their expressions up to quadratic corrections in charm mixing and CP -violating parameters, neglecting direct CP violation.

The CP asymmetry $A_B^{CP}(f, h)$ is defined as

$$A_B^{CP}(f, h) = \frac{\Gamma(\bar{B} \rightarrow [f]_D h) - \Gamma(B \rightarrow [f]_D \bar{h})}{\Gamma(\bar{B} \rightarrow [f]_D h) + \Gamma(B \rightarrow [f]_D \bar{h})} = \frac{2r_B^{Dh} \kappa_B^{Dh} \sin \gamma \sin \delta_B^{Dh}}{(1 + (r_B^{Dh})^2) \frac{1 - (2F_+^f - 1)\alpha\gamma}{2F_+^f - 1 - \alpha\gamma} + 2r_B^{Dh} \kappa_B^{Dh} \cos \gamma \cos \delta_B^{Dh}}. \quad (A1)$$

The ratio of rates for decays in which the D meson is reconstructed as a CP eigenstate and decays with CF final states f^* , as $K^-\pi^+(\pi^0)$ or $K^-\pi^+\pi^+\pi^-$, is given by

$$R(f, f^*, h) = \frac{\mathcal{B}(D^0 \rightarrow f^*)}{\mathcal{B}(D^0 \rightarrow f)} \frac{\Gamma(\bar{B} \rightarrow [f]_D h) + \Gamma(B \rightarrow [f]_D \bar{h})}{\Gamma(\bar{B} \rightarrow [f^*]_D h) + \Gamma(B \rightarrow [f^*]_D \bar{h})}. \quad (A2)$$

In terms of charm parameters, Eq. (A2) can be written as

$$R(f, f^*, h) = \frac{(1 + (r_B^{Dh})^2)(1 - (2F_+^f - 1)\alpha y) + 2r_B^{Dh}\kappa_B^{Dh} \cos \gamma \cos \delta_B^{Dh}((2F_+^f - 1) - \alpha y)}{1 + (r_D^{f^*} r_B^{Dh})^2 + 2r_B^{Dh}\kappa_B^{Dh} r_D^{f^*} \kappa_D^{f^*} \cos \gamma \cos(\delta_B^{Dh} - \delta_D^{f^*}) + \Gamma_{\text{mix}}^+(x, y, \gamma, f^*, h)}, \quad (\text{A3})$$

with $\Gamma_{\text{mix}}^+(x, y, \gamma, f^*, h)$ arising due to the mixing terms in Eq. (38), given by

$$\begin{aligned} \Gamma_{\text{mix}}^+(x, y, \gamma, f^*, h) = & -\alpha y \left[r_D^{f^*} \kappa_D^{f^*} (1 + (r_B^{Dh})^2) \cos \delta_D^{f^*} + r_B^{Dh} \kappa_B^{Dh} (1 + (r_D^{f^*})^2) \cos \gamma \cos \delta_B^{Dh} \right] \\ & + \alpha x \left[r_B^{Dh} \kappa_B^{Dh} (1 - (r_D^{f^*})^2) \cos \gamma \sin \delta_B^{Dh} - r_D^{f^*} \kappa_D^{f^*} (1 - (r_B^{Dh})^2) \sin \delta_D^{f^*} \right]. \end{aligned} \quad (\text{A4})$$

Sometimes, it is useful to avoid the dependence on the charm branching fractions in Eq. (A2) by measuring directly the so-called double ratio, which is related to $R(f, f^*, h)$ for two different meson states h and h' as

$$R^{CP}(f, f^*, h, h') = \frac{R(f, f^*, h)}{R(f, f^*, h')}. \quad (\text{A5})$$

APPENDIX B: ADS OBSERVABLES

In this section, we give an overview of the ADS observables considered in the combination in Table II, expanding them in terms of the charm parameters and neglecting the nonlinear terms, as well as direct CP violation. The ADS observables are defined through time-integrated rates of B cascade decays, involving CF/DCS decays of the D mesons. As in Sec. III A, we refer to the CF mode of the D^0 meson as f .

The simplest observables that can be measured are the CP conjugate suppressed over favored ratios

$$R^-(f, h) = \frac{\Gamma(\bar{B} \rightarrow [\bar{f}]_D h)}{\Gamma(\bar{B} \rightarrow [f]_D h)}, \quad R^+(f, h) = \frac{\Gamma(B \rightarrow [f]_D \bar{h})}{\Gamma(B \rightarrow [\bar{f}]_D \bar{h})}. \quad (\text{B1})$$

The rates for the favored processes are reported in Eq. (37), while for the suppressed modes, we have

$$\Gamma(\bar{B} \rightarrow [\bar{f}]_D h) \propto (r_D^f)^2 + (r_B^{Dh})^2 + 2r_B^{Dh} r_D^f \kappa_B^{Dh} \kappa_D^f \cos(\delta_B^{Dh} + \delta_D^f - \gamma) + \Gamma_{\text{mix}}(-x, y, \gamma, f, h), \quad (\text{B2})$$

with $\Gamma_{\text{mix}}(x, y, \gamma, f, h)$ defined in Eq. (38). The rate for the CP conjugate mode can be obtained simply replacing γ with $-\gamma$ in Eq. (B2).

Suppressed (sup) and favored (fav) decays can be used also to define the following asymmetries:

$$A^{\text{sup}}(f, h) = \frac{\Gamma(\bar{B} \rightarrow [\bar{f}]_D h) - \Gamma(B \rightarrow [f]_D \bar{h})}{\Gamma(\bar{B} \rightarrow [\bar{f}]_D h) + \Gamma(B \rightarrow [f]_D \bar{h})}, \quad A^{\text{fav}}(f, h) = \frac{\Gamma(\bar{B} \rightarrow [f]_D h) - \Gamma(B \rightarrow [\bar{f}]_D \bar{h})}{\Gamma(\bar{B} \rightarrow [f]_D h) + \Gamma(B \rightarrow [\bar{f}]_D \bar{h})}. \quad (\text{B3})$$

In terms of charm parameters, $A^{\text{sup}}(f, h)$ reads

$$A^{\text{sup}}(f, h) = \frac{2r_D^f r_B^{Dh} \kappa_B^{Dh} \kappa_D^f \sin \gamma \sin(\delta_B^{Dh} + \delta_D^f) + \Gamma_{\text{mix}}^-(x, y, \gamma, f, h)}{(r_B^{Dh})^2 + (r_D^f)^2 + 2r_D^f r_B^{Dh} \kappa_B^{Dh} \kappa_D^f \cos \gamma \cos(\delta_B^{Dh} + \delta_D^f) + \Gamma_{\text{mix}}^-(x, y, \gamma, f, h)}, \quad (\text{B4})$$

where $\Gamma_{\text{mix}}^+(x, y, \gamma, f, h)$ has been already defined in Eq. (A4) for the GLW modes, while $\Gamma_{\text{mix}}^-(x, y, \gamma, f, h)$ is given by

$$\Gamma_{\text{mix}}^-(x, y, \gamma, f, h) = -\alpha r_B^{Dh} \kappa_B^{Dh} \sin \gamma \left[y \sin \delta_B^{Dh} (1 + (r_D^f)^2) - x \cos \delta_B^{Dh} (1 - (r_D^f)^2) \right]. \quad (\text{B5})$$

In the same way, we find for the favored asymmetry:

$$A^{\text{fav}}(f, h) = \frac{2r_D^f r_B^{Dh} \kappa_B^{Dh} \kappa_D^f \sin \gamma \sin(\delta_B^{Dh} - \delta_D^f) + \Gamma_{\text{mix}}^-(x, y, \gamma, f, h)}{1 + (r_D^f r_B^{Dh})^2 + 2r_B^{Dh} \kappa_B^{Dh} r_D^f \kappa_D^f \cos \gamma \cos(\delta_B^{Dh} - \delta_D^f) + \Gamma_{\text{mix}}^+(x, y, \gamma, f, h)}. \quad (\text{B6})$$

Favored and suppressed charm modes can be arranged into CP -conserving ratios as

$$R^{\text{ADS}}(f, h) = \frac{\Gamma(\bar{B} \rightarrow [\bar{f}]_D h) + \Gamma(B \rightarrow [f]_D \bar{h})}{\Gamma(\bar{B} \rightarrow [f]_D h) + \Gamma(B \rightarrow [\bar{f}]_D \bar{h})}, \quad (\text{B7})$$

or using two different channels for beauty decays, $\bar{B} \rightarrow [f]_D h$ and $\bar{B} \rightarrow [f]_D h'$, through the following observables:

$$R^{\text{sup}}(f, h, h') = \frac{\Gamma(\bar{B} \rightarrow [\bar{f}]_D h) + \Gamma(B \rightarrow [f]_D \bar{h})}{\Gamma(\bar{B} \rightarrow [\bar{f}]_D h') + \Gamma(B \rightarrow [f]_D \bar{h}')}, \quad R^{\text{fav}}(f, h, h') = \frac{\Gamma(\bar{B} \rightarrow [f]_D h) + \Gamma(B \rightarrow [\bar{f}]_D \bar{h})}{\Gamma(\bar{B} \rightarrow [f]_D h') + \Gamma(B \rightarrow [\bar{f}]_D \bar{h}')}. \quad (\text{B8})$$

The ADS ratio in Eq. (B7) can be expressed in terms of charm parameters as

$$R^{\text{ADS}}(f, h) = \frac{(r_B^{Dh})^2 + (r_D^f)^2 + 2r_D^f r_B^{Dh} \kappa_D^f \kappa_B^{Dh} \cos \gamma \cos(\delta_B^{Dh} + \delta_D^f) + \Gamma_{\text{mix}}^+(-x, y, \gamma, f, h)}{1 + (r_D^f r_B^{Dh})^2 + 2r_B^{Dh} \kappa_B^{Dh} r_D^f \kappa_D^f \cos \gamma \cos(\delta_B^{Dh} - \delta_D^f) + \Gamma_{\text{mix}}^+(x, y, \gamma, f, h)}. \quad (\text{B9})$$

The fit equation for $R^{\text{sup}}(f, h, h')$ is given by

$$R^{\text{sup}}(f, h, h') = \frac{\mathcal{B}(\bar{B} \rightarrow D^0 h) (r_B^{Dh})^2 + (r_D^f)^2 + 2r_D^f r_B^{Dh} \kappa_D^f \kappa_B^{Dh} \cos \gamma \cos(\delta_B^{Dh} + \delta_D^f) + \Gamma_{\text{mix}}^+(-x, y, \gamma, f, h)}{\mathcal{B}(\bar{B} \rightarrow D^0 h') (r_B^{Dh'})^2 + (r_D^f)^2 + 2r_D^f r_B^{Dh'} \kappa_D^f \kappa_B^{Dh'} \cos \gamma \cos(\delta_B^{Dh'} + \delta_D^f) + \Gamma_{\text{mix}}^+(-x, y, \gamma, f, h')}, \quad (\text{B10})$$

while for $R^{\text{fav}}(f, h, h')$ we have

$$R^{\text{fav}}(f, h, h') = \frac{\mathcal{B}(\bar{B} \rightarrow D^0 h) (1 + (r_D^f r_B^{Dh})^2 + 2r_B^{Dh} \kappa_B^{Dh} r_D^f \kappa_D^f \cos \gamma \cos(\delta_B^{Dh} - \delta_D^f) + \Gamma_{\text{mix}}^+(x, y, \gamma, f, h))}{\mathcal{B}(\bar{B} \rightarrow D^0 h') (1 + (r_D^f r_B^{Dh'})^2 + 2r_B^{Dh'} \kappa_B^{Dh'} r_D^f \kappa_D^f \cos \gamma \cos(\delta_B^{Dh'} - \delta_D^f) + \Gamma_{\text{mix}}^+(x, y, \gamma, f, h'))}. \quad (\text{B11})$$

APPENDIX C: FIT RESULTS

We report the results of the fit obtained by combining all the observables. The correlation matrix of the most physically relevant parameters is reported in Table VII. Tables VIII–X present the probability intervals found for the charm and beauty parameters. In Fig. 8, we report two-dimensional contours of the ratio of decay amplitudes and strong phases describing the most precise modes used in the combination: $B^\pm \rightarrow D\pi^\pm(K^\pm)$.

TABLE VIII. Results for the charm part of the combination. For each of the parameters, we have reported the central value (“Value”) and the smallest intervals containing at least 68% (“Uncertainty”) and 95% probabilities.

Parameter	Value	Uncertainty	95.4% probability	Parameter	Value	Uncertainty	95.4% probability
$\phi_2^M [^\circ]$	0.13	± 0.70	[−1.27, 1.54]	$\phi_2^\Gamma [^\circ]$	2.1	± 1.6	[−1.0, 5.2]
$\phi_2 [^\circ]$	−1.5	± 1.1	[−3.7, 0.6]	$ q/p - 1 [\%]$	−1.6	± 1.5	[−4.5, 1.4]
$x_{12} \simeq x [\%]$	4.01	± 0.43	[3.16, 4.83]	$y_{12} \simeq y [\%]$	6.10	± 0.17	[5.77, 6.45]
$a_D^{KK} [\%]$	0.40	± 0.53	[−0.63, 1.43]	$a_D^{\pi\pi} [\%]$	2.34	± 0.60	[1.15, 3.53]
$\phi_{12} [^\circ]$	−2.0	± 1.8	[−5.6, 1.7]	$\kappa_D^{K3\pi}$	0.444	± 0.092	[0.261, 0.617]
$r_D^{K3\pi}$	0.05502	± 0.00070	[0.05364, 0.05636]	$\delta_D^{K3\pi} [^\circ]$	164	± 21	[126, 208]
$r_D^{K\pi} [\%]$	5.8573	± 0.0094	[5.8387, 5.8758]	$\delta_D^{K\pi} [^\circ]$	191.4	± 2.4	[186.6, 196.0]
$\kappa_D^{K\pi\pi^0}$	0.791	± 0.039	[0.713, 0.868]	$r_D^{K\pi\pi^0}$	0.04433	± 0.00099	[0.04239, 0.04626]
$\delta_D^{K\pi\pi^0} [^\circ]$	201.0	± 7.9	[185.2, 216.3]	$\kappa_D^{K_s^0 K\pi}$	0.824	± 0.094	[0.652, 0.995]
$r_D^{K_s^0 K\pi}$	0.6192	± 0.0052	[0.6090, 0.6297]	$\delta_D^{K_s^0 K\pi} [^\circ]$	13	± 14	[−15, 39]
$F_+^{4\pi}$	0.7444	± 0.0097	[0.7252, 0.7635]	$F_+^{KK\pi\pi}$	0.756	± 0.013	[0.732, 0.781]
$F_+^{\pi\pi\pi^0}$	0.9417	± 0.0040	[0.9337, 0.9496]	$F_+^{KK\pi^0}$	0.641	± 0.017	[0.608, 0.674]
$(\Delta Y^{KK} - \Delta Y^{\pi\pi}) [\%]$	0.24	± 0.26	[−0.27, 0.75]				

TABLE IX. Results for the beauty part of the combination. For each of the parameters, we have reported the central value (“Value”) and the smallest intervals containing at least 68% (“Uncertainty”) and 95% probabilities. For the phases $\delta_B^{DK\pi\pi}$ and $\delta_B^{D\pi\pi\pi\pi}$, we report the two disjoint intervals at 68.5% and 68.9% probabilities, respectively.

	Parameter	Value	Uncertainty	95.4% probability	Parameter	Value	Uncertainty	95.4% probability
CKM Angles	γ [°]	65.7	± 2.5	[60.8, 70.4]	$\phi_d/2$ [°]	22.13	± 0.60	[20.96, 23.32]
	ϕ_s [°]	-3.36	± 0.80	[-4.93, -1.79]				
B^\pm parameters	$\kappa_B^{DK^{*\pm}}$	0.950	± 0.043	[0.852, 1.001]	$r_B^{DK^{*\pm}}$	0.0994	± 0.0090	[0.0806, 0.1162]
	$\delta_B^{DK^{*\pm}}$ [°]	51	± 12	[30, 80]	$r_B^{DK} [\%]$	9.80	± 0.20	[9.42, 10.19]
	δ_B^{DK} [°]	127.0	± 2.5	[121.9, 131.8]	$r_B^{D^*K} [\%]$	5.18	± 0.56	[4.17, 6.38]
	$\delta_B^{D\pi}$ [°]	299.8	± 8.5	[282.0, 315.1]	$r_B^{D^*\pi}$	0.1078	± 0.0088	[0.0905, 0.1251]
	$\delta_B^{D^*K}$ [°]	-47.6	± 5.9	[-60.0, -36.7]	$r_B^{D^*\pi}$	0.0058	± 0.0030	[0.0001, 0.0123]
	$\delta_B^{D^*\pi}$ [°]	39	± 31	[-4, 133]	$\frac{\mathcal{B}(B \rightarrow DK)}{\mathcal{B}(B \rightarrow D\pi)}$	0.0782	± 0.0038	[0.0707, 0.0857]
	$r_B^{DK\pi\pi}$	0.048	± 0.031	[0.000, 0.096]	$\delta_B^{DK\pi\pi}$ [°]	[-226, -154] \cup [-75, 69] @68.5% probability		
	$\kappa_B^{DK\pi\pi}$	0.28	± 0.28	[0.00, 0.92]	$r_B^{D\pi\pi\pi}$	0.0099	± 0.0099	[0.0000, 0.0365]
	$\delta_B^{D\pi\pi\pi}$ [°]	[-204, -100] \cup [-72, 11] @68.9% probability			$\kappa_B^{D\pi\pi\pi}$	0.23	± 0.23	[0.00, 0.88]
	B^0 parameters	$r_{B^0}^{D\pi}$	0.0229	± 0.0090	[0.0058, 0.0439]	$\delta_{B^0}^{D\pi}$ [°]	32	± 31
$\kappa_{B^0}^{DK^{*0}}$		0.933	± 0.025	[0.885, 0.982]	$r_{B^0}^{DK^{*0}}$	0.231	± 0.015	[0.200, 0.259]
$\delta_{B^0}^{DK^{*0}}$ [°]		192.6	± 6.0	[180.9, 205.2]	$r_{B^0}^{D^*\pi}$	0.0244	± 0.0083	[0.0099, 0.0476]
$\delta_{B^0}^{D^*\pi}$ [°]		-34	± 31	[-74, 41]	$r_{B^0}^{D\rho}$	0.060	± 0.060	[0.000, 0.261]
$\delta_{B^0}^{D\rho}$ [°]		-77	± 27	[-265, 93]				
B_s^0 parameters		$r_{B_s^0}^{D_s K}$	0.331	± 0.035	[0.264, 0.402]	$\delta_{B_s^0}^{D_s K}$ [°]	-11.0	± 6.0
	$\kappa_{B_s^0}^{D_s K\pi\pi}$	0.76	± 0.14	[0.51, 1.00]	$r_{B_s^0}^{D_s K\pi\pi}$	0.453	± 0.077	[0.301, 0.605]
	$\delta_{B_s^0}^{D_s K\pi\pi}$ [°]	-14	± 12	[-39, 10]	$\kappa_{B_s^0}^{DK^{*0}}$	0.40	± 0.30	[0.00, 0.92]
	$r_{B_s^0}^{DK^{*0}}$	0.057	± 0.029	[0.000, 0.098]	$\delta_{B_s^0}^{DK^{*0}}$ [°]	72	± 58	[-44, 215]

TABLE X. Parameters corresponding to the average decay times entering the integrated CP asymmetries $A_D^{CP}(f, t)$ in Eq. (29) and ΔA_D^{CP} in Eq. (30). The column (Exp.) lists the name of the experiment, the tagging (-tag) mode employed, the calibration (-cal) method, and the reference describing the measurement.

Parameter	Value	Uncertainty	Exp.	Parameter	Value	Uncertainty	Exp.
$(\langle \tau \rangle_{\Delta A_D^{CP}}^{KK} + \langle \tau \rangle_{\Delta A_D^{CP}}^{\pi\pi})/2$	1.73	± 0.10	LHCb Run2, π -tag [75]	$\langle \tau \rangle_{\Delta A_D^{CP}}^{KK} - \langle \tau \rangle_{\Delta A_D^{CP}}^{\pi\pi}$	0.1350	± 0.0020	LHCb Run2, π -tag [75]
$(\langle \tau \rangle_{\Delta A_D^{CP}}^{KK} + \langle \tau \rangle_{\Delta A_D^{CP}}^{\pi\pi})/2$	1.210	± 0.010	LHCb Run2, μ -tag [75]	$\langle \tau \rangle_{\Delta A_D^{CP}}^{KK} - \langle \tau \rangle_{\Delta A_D^{CP}}^{\pi\pi}$	-0.0030	± 0.0010	LHCb Run2, μ -tag [75]
$\langle t \rangle_{A_D^{CP}}^{KK}$ [ps]	0.7315	± 0.0020	LHCb Run2, D^+ -cal [72]	$\langle t \rangle_{A_D^{CP}}^{KK}$ [ps]	0.6868	± 0.0014	LHCb Run2, D_s^+ -cal [72]
$\langle \tau \rangle_{\Delta A_D^{CP}}^{KK}$	2.152	± 0.016	LHCb Run1, π -tag [71]	$\langle \tau \rangle_{\Delta A_D^{CP}}^{\pi\pi}$	2.037	± 0.015	LHCb Run1, π -tag [71]
$\langle \tau \rangle_{\Delta A_D^{CP}}^{KK}$	1.0820	± 0.0041	LHCb Run1, μ -tag [71]	$\langle \tau \rangle_{\Delta A_D^{CP}}^{\pi\pi}$	1.0679	± 0.0041	LHCb Run1, μ -tag [71]
$\langle \tau \rangle_{A_D^{CP}}^{KK}$	2.239	± 0.019	LHCb Run1, π -tag [71]	$\langle \tau \rangle_{A_D^{CP}}^{KK}$	1.0510	± 0.0041	LHCb Run1, μ -tag [71]
$\langle \tau \rangle_{A_D^{CP}}^{KK}$	2.65	± 0.03	CDF [79]	$\langle \tau \rangle_{A_D^{CP}}^{\pi\pi}$	2.40	± 0.03	CDF [79]

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