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Cryptocurrencies in equity portfolios: trend or opportunity?

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Abstract

This study develops a large-scale framework to evaluate whether, and under what conditions, adding cryptocurrencies to equity investment universes improves portfolio performance. We apply four long-only portfolio strategies, Global Minimum Variance, Risk Parity, Most Diversified Portfolio, and Equally Weighted, to 10,000 randomly generated investment universes. These universes consist of baskets containing either only equities or varying combinations of equities and cryptocurrencies. We conduct an out-of-sample analysis on real-world data from 2018 to 2023 to assess the influence of cryptocurrencies on portfolio outcomes. The empirical findings reveal that portfolios constructed from mixed equity and cryptocurrency universes provide a better risk-return profile compared to purely equity-based portfolios, particularly for Risk Parity, Most Diversified, and Equally Weighted.

Keywords Portfolio Optimization · Cryptocurrencies · Performance Evaluation · Diversification · Equity Risk Premium

1 Introduction and literature review

This research aims to examine the potential influence of cryptocurrencies when incorporated into equity portfolios. The study does not delve into the technical aspects of cryptocurrencies but instead focuses on their financial implications. By treating cryptocurrencies as a distinct asset class, we explore and test different portfolio selection strategies within investment universes that encompass both equities and cryptocurrencies. Despite their young age, cryptocurrencies have rapidly revolutionized the

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investment landscape. This emerging market has attracted interest and subsequent capital from various investor profiles seeking to diversify their portfolios. Consequently, the cryptocurrency market has experienced remarkable growth in recent years, marked by considerable price fluctuations, substantial gains, and, inevitably, significant losses. Over recent years, the total market value of cryptocurrencies has exceeded an impressive 1.5 trillion.

This asset class has attracted both technology enthusiasts and academic researchers. A significant amount of research is focused on the characterization of the price dynamics of cryptocurrencies. This includes identifying the main drivers (Kristoufek 2015; Figa-Talamanca and Patacca 2019; Ozdamar et al. 2022; Cai and Zhao 2024; Zhang et al. 2021; Bazán-Palomino and Svogun 2023; Lan and Frömmel 2025; Cretarola et al. 2021); analyzing their bubble behavior (Kyriazis et al. 2020; Cretarola and Figà-Talamanca 2021, 2020; Chen et al. 2024; Chowdhury and Damianov 2024; Podhorsky 2024); and modeling the interdependence between different cryptocurrencies (Bistarelli et al. 2018, 2019; Figà-Talamanca et al. 2021; Jia et al. 2023; Tiwari et al. 2020; Nguyen et al. 2020; Bouri and Jalkh 2023).

A second stream of research examines the connections between various asset classes and cryptocurrencies to determine their potential for diversification and safe-haven properties. Corbet et al. (2018) explore the connection between three major cryptocurrencies and traditional financial assets (such as equities, bonds, and commodities). Their empirical results reveal that cryptocurrencies are potentially good financial assets for improving portfolio diversification, as they exhibit a low correlation with traditional financial assets (see also Gil-Alana et al. 2020).Bouri et al. (2017) examine Bitcoin's connection with various asset classes and show that Bitcoin serves as a good diversifier in most cases, but it only exhibits hedging and safe-haven properties in a few instances. Furthermore, Li and Miu (2023), exploiting a regime-switching approach, find that the correlation between four cryptocurrencies and two equity market indices is typically low, but becomes significantly high during market distress. Therefore, cryptocurrencies may not be a good choice during market downturns. Harb et al. (2024) find that Bitcoin, Ethereum, and Litecoin exhibit significant volatility spillovers, with Ripple serving as a primary transmitter of shocks. The study also reveals that cryptocurrencies are largely decoupled from the stock market but maintain connections to the bond market. These dynamics have important implications for portfolio diversification and risk management, suggesting that Ethereum may act as a hedge during crises and offering policymakers avenues for targeted intervention. Finally, Hsu et al. (2024) investigate how the hedging, and safe-haven characteristics of Bitcoin and Ethereum are influenced by structural changes evidencing that cryptocurrencies play different roles against specific asset markets in periods separated by structural changes.

Over the years, several studies have attempted to evaluate the behavior of portfolio selection strategies applied to investment universes that include cryptocurrencies. Eisl et al. (2015) analyze the mean-CVaR approach applied to an investment universe consisting of several international indices and Bitcoin. They highlight the improvement in the portfolio risk-return profile due to the presence of Bitcoin (see also Kajtazi and Moro 2019). Brauneis and Mestel (2019) test eight different long-only portfolio strategies applied to an investment universe that includes the 500 most capitalized cryptocurrencies. They show that considering better-known cryptocurrencies with lesser-known ones helps reduce overall portfolio risk (see also Ahelegbey et al. 2022). Petukhina et al. (2021) investigate the behavior of several portfolio selection approaches applied to an investment universe of 52 cryptocurrencies and 16 traditional assets. They conduct an out-of-sample analysis using a rolling-time-window scheme with three different rebalancing frequencies, providing valuable insights into the performance of different investment strategies within the context of mixed portfolios (see also Ma et al. 2020; Trimborn et al. 2020). Similar studies can be found in Akhtaruzzaman et al. (2020) and Platanakis and Urquhart (2020), both of which focus on the inclusion of Bitcoin alongside a limited number of traditional assets. Han et al. (2024) explore the out-of-sample diversification benefits of incorporating cryptocurrency factors into a range of asset allocation strategies. Their findings indicate that, in particular, size and momentum-related cryptocurrency factors can significantly improve the risk-adjusted performance of conventional equity-bond portfolios.

We align with this strand of literature and explore several portfolio selection strategies applied to randomized investment universes that include both cryptocurrencies and equities. Our research specifically targets the integration of cryptocurrencies into equity portfolios, intentionally excluding other asset classes such as commodities and bonds. This focus reflects the increasing interest among institutions to include cryptocurrencies within their equity investments (see, e.g., Ey-Parthenon 2024; Reuters 2025; Lan and Frömmel 2025).

To more robustly test the potential influence of cryptocurrencies when incorporated into equity portfolios, we examine the Global Minimum Variance, Risk Parity, Most Diversified, and Equally Weighted approaches applied to randomized and mixed investment universes. It is important to note that the distinctions from prior studies primarily relate to our experimental methodology. More precisely, to assess the actual impact of including cryptocurrencies on portfolio choices, we consider four baskets: one containing only equities and three with different compositions of equities and cryptocurrencies. For each basket type, we randomly generate 10,000 investment universes to obtain general results that are not dependent on the specific set of assets and cryptocurrencies on which to apply portfolio strategies. In brief, this procedure mitigates potential bias resulting from asset and crypto cherry-picking, thereby enhancing the generalizability of our findings.

Such investment universes consist of a total of 40 randomly selected assets from an equity market, represented by the components of a market index and a selection of 12 cryptocurrencies. This selection is guided by two main considerations: the limited availability of cryptocurrencies and the necessity for a diverse and representative range of mixed compositions. We then apply the four portfolio selection strategies to each set of 10,000 randomized investment universes per basket type. This process yields 10,000 optimal portfolios for every strategy and basket combination.

We conduct an out-of-sample analysis using several standard performance metrics, evaluating the average outcomes across the 10,000 optimal portfolios for each strategy and basket. Additionally, we construct quantile-based intervals to determine whether performance differences between mixed and equity-only portfolios are relevant. To the best of our knowledge, this methodological approach constitutes a novel contribution to existing literature. Our experiments draw on the constituents of four equity market indices (two from the US, one from the UK, and one from Europe) alongside a set of 12 large-cap cryptocurrencies.

The remainder of the paper is organized as follows. Section 2 presents the portfolio selection approaches used. Section 3 describes the dataset (Sect. 3.1), the experimental setup and the performance measures used to evaluate portfolio outcomes (Sect. 3.2). It then reports and discusses the computational results across equity markets, basket compositions, and portfolio strategies (Sect. 3.3). Section 3.4 provides further analysis to strengthen the interpretation of the results and summarizes the key findings across different portfolio strategies, time horizons, and robustness checks. Section 4 concludes the paper and outlines the main implications and possible directions for future research.

2 Portfolio selection strategies

In this section, we describe the portfolio selection approaches adopted to assess the impact of including cryptocurrencies in equity portfolios in terms of risk and profitability.

Let us first introduce some notations and assumptions. We consider an investment universe of n risky assets with linear returns, represented by the random variables $R_1, R_2, \ldots, R_n$. Furthermore, we identify the long-only portfolio, in case of full investment, with the vector $x \in \Delta = \left\{x \in \mathbb{R}^n : \sum_{i=1}^n x_i = 1, x_i \geq 0, i = 1, \ldots, n\right\}$, where x_i is the percentage of capital invested in asset i. Thus, the portfolio linear return is defined as $R_P(x) = \sum_{i=1}^n x_i R_i$. We assume that the asset returns are defined on a discrete probability space $\{\Omega, \mathcal{F}, P\}$, with $\Omega = \{\omega_1, \ldots, \omega_T\}$, $\mathcal{F}$ a σ-field and $P(\omega_t) = \pi_t$, with $t = 1, ..., T$. We use a look-back approach, where the realizations of the discrete random returns correspond to past realized data. More precisely, the investment decision is performed in an in-sample window of T historical scenarios, each with probability $\pi_t = 1/T$, if there are no ties (see, e.g., Carleo et al. 2017; Cesarone 2020; Bellini et al. 2021 and references therein). Then, if p_{it} is the realized price of asset i at time t, the realized return of asset i at time t is denoted by $r_{it} = \frac{p_{it} - p_{i(t-1)}}{p_{i(t-1)}}$ and the realized portfolio return at time t is $r_{P\,t}(x) = \sum_{i=1}^n x_i r_{it}$.

For this analysis, we consider three risk-focused portfolio selection approaches, widely used both by academics and practitioners (see, e.g., Anderson et al. 2012; Asness et al. 2012; Bai et al. 2016; Chaves et al. 2011; Corkery et al. 2013; Dagher 2012; Lee et al. 2011; Tobam 2025; Financial Times 2025) along with the Equally Weighted (EW) portfolio, which is often used in practice (see Benartzi and Thaler 2001; Windcliff and Boyle 2004), and some authors claim that its practical out-of-sample performance is hard to beat on real-world data sets (DeMiguel et al. 2009).

The first is the global Minimum Variance (MV) portfolio, which focuses on determining the fraction x_i of a given capital to be invested in each asset i belonging to a given investment universe in order to minimize the risk of the entire portfolio, identified with its variance. The MV portfolio is a special case of the classical Mean-Variance portfolio selection model (Markowitz 1952, 1959, 1987), which is widely regarded as a milestone in Modern Portfolio Theory and as a benchmark strategy for asset allocation (Kolm et al. 2014).

The Risk Parity (RP) strategy aims to achieve a balanced portfolio in terms of risk. This risk diversification approach is based on the general notion of an equal risk contribution of each asset. It can be shown that the risk of the RP portfolio is bounded between the risk of the minimum-risk portfolio and that of the EW portfolio (Cesarone and Tardella 2017; Cesarone et al. 2020; Cesarone and Colucci 2018). The latter strategy, where each asset is assigned a weight of $1/n$, is also analyzed for comparative purposes.

Finally, we also test the Most Diversified Portfolio (MDP), which consists of maximizing the relative distance between the portfolio risk in the worst-case scenario, namely when risk is additive, and the generic portfolio risk. Note that risk is additive when the risk drivers are highly dependent (see Cesarone et al. 2025; Ararat et al. 2024; Bellini et al. 2021).

We point out that although short-selling positions are allowed on some cryptocurrency exchanges, this can be very expensive. Furthermore, the short-selling position is not always feasible in the equity market; therefore, throughout the paper, we focus on long-only optimal portfolios.

2.1 The minimum variance portfolio

The Minimum Variance long-only portfolio (Markowitz 1952, 1959) is the Pareto-optimal solution of the Markowitz efficient frontier with minimum risk, measured by the portfolio variance $\sigma_p^2(x) = \sum_{i=1}^n \sum_{j=1}^n \sigma_{i,j} x_i x_j$, where $\sigma_{i,j}$ is the covariance of returns of asset i and asset j with $i, j = 1, \dots n$. It can be obtained by solving the following convex quadratic programming problem:

$$\begin{aligned} & \underset{x}{\text{minimize}} \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{i,j} x_i x_j \\ & \text{s.t.} \quad x \in \Delta = \left\{ x \in \mathbb{R}^n : \sum_{i=1}^{n} x_i = 1, x_i \geq 0, i = 1, \dots, n \right\}, \end{aligned} \tag{1}$$

where the first constraint represents the full investment constraint while the nonnegativity constraints do not allow short selling.

2.2 The risk parity portfolio

The Risk Parity approach aims to construct a portfolio where each asset equally contributes to the overall portfolio risk (see, e.g., Qian 2005, 2011; Maillard et al. 2010). Therefore, this approach, by construction, selects all the assets in the investment universe (see, e.g., Clarke et al. 2013; Roncalli 2014).

In this study, we use volatility as a portfolio risk measure that is convex and positively homogeneous. Thus, exploiting Euler's homogeneous function theorem, we can decompose the overall risk of a given portfolio into the sum of the risk contributions of each asset as follows

$$\sigma_p(x) = \sum_{i=1}^{n} TRC_i^{\sigma},$$

where the risk contribution of the i^{th} asset $TRC_i^{\sigma} = x_i \cdot \partial_{x_i}\sigma_p(x)$, x_i is the fraction of capital invested in asset i, $\partial_{x_i}\sigma_p(x) = \frac{1}{\sigma_p(x)}(\Sigma x)_i$ is its marginal risk, and Σ represents the covariance matrix.

The Risk Parity portfolio can be obtained by requiring equal risk contribution from each asset, namely

$$TRC_i^{\sigma} = TRC_j^{\sigma} \quad \Longrightarrow \quad x_i(\Sigma x)_i = x_j(\Sigma x)_j \qquad \forall \quad i, j = 1, \ldots, n.$$

As described in Cesarone and Tardella (2017), a possible way to practically find the RP portfolio is to solve the following system of equations and inequalities

$$\begin{cases} TRC_i^{\sigma} = \lambda \\ x \in \Delta \\ \lambda \in \mathbb{R}, \end{cases} \tag{2}$$

which has a unique solution when the covariance matrix of the asset returns is positive definite (Cesarone et al. 2020).

2.3 The most diversified portfolio

An alternative strategy based on risk allocation has been introduced by Choueifaty and Coignard (2008) and consists in maximizing the so-called diversification ratio

$$DR(x) = \frac{\sum_{i=1}^{n} x_i \sigma_i}{\sigma_p(x)}, \tag{3}$$

which represents the relative distance between the portfolio volatility in the worst case (i.e., when the asset returns are perfectly positively correlated) and the generic portfolio volatility for any correlation structure of the investment universe, namely $\sigma_p(x) = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{i,j} x_i x_j}$. Using the convexity property of volatility, it is straightforward to observe that $DR(x) \geq 1$. The Most Diversified Portfolio (MDP) is the optimal portfolio that maximizes the diversification ratio (3), or, equivalently, minimizes its reciprocal $\frac{1}{DR(x)}$ as follows

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & \frac{1}{DR(x)} = \frac{\sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{ij} x_i x_j}}{\sum_{i=1}^{n} x_i \sigma_i} \\ \text{s.t.} \quad & \\ & x \in \Delta. \end{aligned} \tag{4}$$

As shown by Choueifaty et al. (2013), MDP can be found by solving the following (convex) quadratic programming problem

$$\begin{array}{ll} \underset{y}{\text{minimize}} & \sum_i \sum_j y_i y_j \sigma_{ij} \\ \text{s.t.} & \\ & \sum_{i=1}^n \sigma_i y_i = 1 \\ & y_i \geq 0 \qquad \forall i = 1, \ldots, n\,. \end{array} \tag{5}$$

Then, since the portfolio volatility $\sigma_p(x)$ is a homogenous function of degree 1, the portfolio weights that maximize the diversification ratio in Problem (4) can be found by simply normalizing the solution y^* of Problem (5). Thus, $x_i^{MDP} = \frac{y_i^*}{\sum_{k=1}^n y_k^*}$, for $i = 1, \ldots, n$.

3 Empirical analysis

In this section, we present an extensive empirical analysis using real data sets. More precisely, Sects. 3.1 and 3.2 describe the datasets and the methodology adopted, respectively. Section 3.3 discusses the results obtained.

All experiments were executed on a workstation with an Intel(R) Xeon(R) CPU E5-2623 v4 (2.6 GHz, 64 GB RAM) running Windows 10 Pro, using MATLAB R2023a.

3.1 Description of the datasets

For the experiments, we use the constituents of four equity market indices and a set of 12 cryptocurrencies. The dataset comprises almost six years of daily price observations, spanning January 1, 2018, to October 24, 2023. This period offers an optimal trade-off between the number of cryptocurrencies included and the length of the available time series. Extending the time series further would enhance the robustness of the analysis, but would restrict the dataset to only the most established cryptocurrencies, such as Bitcoin and Ethereum.

The equity indices considered are:

- EuroStoxx50 (EuroStoxx), composed of 48 assets;
- FTSE100 (FTSE), composed of 96 assets;
- NASDAQ100 (NASDAQ), composed of 90 assets;
- S&P100 (S&P), composed of 100 assets.

Daily equity prices were retrieved from LSEG Data & Analytics (formerly Refinitiv-Datastream, see Lseg 2023), whereas daily cryptocurrency prices were obtained from CoinMarketCap (Coinmarketcap 2023).

Table 1 reports the 25 cryptocurrencies with the largest market capitalization as reported by CoinMarketCap on October 27, 2023. From this set, we exclude stablecoins and Wrapped Bitcoin, retaining only cryptocurrencies with data available for the full sample period. This leads to a universe of 12 large-cap cryptocurrencies: Bitcoin

(BTC), Ethereum (ETH), Binance (BNB), Ripple (XRP), Cardano (ADA), Dogecoin (DOGE), TRON (TRX), Chainlink (LINK), Litecoin (LTC), Stellar (XLM), Monero (XMR), and Bitcoin Cash (BCH).

The cryptocurrency universe is selected on the basis of market capitalization and the availability of complete data over the full sample period. These criteria provide a transparent and economically grounded sampling rule. However, this choice may introduce survivorship bias, as the analysis is restricted to larger, longer-lived cryptocurrencies. Hence, the evidence should be interpreted as referring to established large-cap crypto assets.

Since equity markets are not traded on weekends, whereas cryptocurrency markets operate continuously, Saturday and Sunday observations are removed from the cryptocurrency series to align the two asset classes on common trading days (see e.g., Adelopo et al. (2025); Gil-Alana et al. (2020)). This adjustment may slightly understate cryptocurrency volatility, with possible effects on volatility-sensitive allocations, but it avoids interpolation-based adjustments and ensures that correlations and portfolio weights are computed on synchronized data.

3.2 Experimental setup and performance measures description

To assess the effect of cryptocurrencies for constructing portfolios on equity-crypto investment universes, we define 4 types of *baskets*, with the following compositions:

- 100% equity - 0% crypto;
- 90% equity - 10% crypto;
- 80% equity- 20% crypto;
- 70% equity- 30% crypto.

For each basket, we generate 10,000 random investment universes. Each universe consists of 40 distinct assets, drawn without replacement from the constituents of an equity index and a set of 12 large-cap cryptocurrencies. On the one hand, the "without replacement" rule assures that no asset appears more than once within the same universe. Clearly, it does not restrict the subsequent portfolio optimization, which may still allocate a large fraction of capital, or possibly a zero fraction, to a subset of assets when implied by the selected strategy. On the other hand, the large number of random universes lowers sensitivity to specific asset combinations, limiting ad hoc selection effects and strengthening the robustness and generalizability of our empirical results. The composition of each universe is determined by the basket definition. The 100–0 basket contains 40 equities; the 90–10 basket consists of 36 equities and 4 cryptocurrencies; the 80–20 basket holds 32 equities and 8 cryptocurrencies; and the 70–30 basket comprises 28 equities and 12 cryptocurrencies. This structure addresses two practical considerations: (i) the limited size of the cryptocurrency pool and (ii) the need to cover a sufficiently broad range of allocation mixes so that cryptocurrencies form a meaningful portion of each mixed basket. Maintaining a fixed total of 40 assets also enables straightforward comparison across baskets and equity markets with varying index sizes. Overall, this sampling approach serves a dual purpose. First, it allows for conclusions that are not dependent on a particular, potentially favorable, asset selection, thereby reducing selection bias and enhancing the generalizability of

Table 1 Top 25 cryptocurrencies by market capitalization according to CoinMarketCap on October 27, 2023. The table reports the name, symbol, market capitalization, and price of each cryptocurrency. The last column (*Active*) indicates inclusion in our dataset: Y = included; N = excluded because no price history is available from January 2018 onward; S = excluded because the asset is a stablecoin or a Bitcoin-pegged asset

Name	Symbol	Market Capitalization	Price	Active
Bitcoin	BTC	$662,115,721,497.01	$33,909.80	Y
Ethereum	ETH	$214,081,832,972.05	$1,780.05	Y
Tether USDt	USDT	$84,532,221,644.03	$1.0003	S
BNB	BNB	$34,053,939,398.41	$224.48	Y
XRP	XRP	$29,249,745,336.61	$0.5461	Y
USDC	USDC	$24,974,208,191.23	$0.9999	S
Solana	SOL	$13,303,382,707.03	$31.74	N
Cardano	ADA	$10,200,467,608.35	$0.2895	Y
Dogecoin	DOGE	$9,611,213,300.79	$0.06788	Y
TRON	TRX	$8,316,742,120.00	$0.09363	Y
Toncoin	TON	$7,062,114,772.18	$2.0578	N
Chainlink	LINK	$6,239,471,021.73	$11.20	Y
Polygon	MATIC	$5,676,505,589.15	$0.6097	N
Wrapped Bitcoin	WBTC	$5,553,972,331.25	$33,865.85	S
Dai	DAI	$5,348,674,533.37	$1.0001	S
Polkadot	DOT	$5,246,856,630.42	$4.1394	N
Litecoin	LTC	$4,949,332,977.49	$67.06	Y
Bitcoin Cash	BCH	$4,679,350,946.60	$239.44	Y
Shiba Inu	SHIB	$4,541,005,599.62	$0.00000771	N
Avalanche	AVAX	$3,763,086,672.01	$10.60	N
UNUS SED LEO	LEO	$3,720,743,826.02	$4.0063	N
TrueUSD	TUSD	$3,354,560,960.16	$0.9997	S
Stellar	XLM	$3,119,586,970.35	$0.1121	Y
Monero	XMR	$2,952,322,970.53	$160.89	Y
OKB	OKB	$2,680,859,131.21	$44.68	N

the results. Second, keeping the universe size constant across all baskets ensures that any differences between equity-only and mixed portfolios are due to the presence and proportion of cryptocurrencies, rather than to variations in portfolio dimensionality.

The portfolio selection strategies, described in Sect. 2, are then applied to the 10,000 randomized investment universes of each basket type.

To evaluate the performance of the selected optimal portfolios, we conduct an out-of-sample analysis using a rolling time-window approach. More precisely, we allow for rebalancing the portfolio composition at fixed time intervals during the holding period. In this study, we set one year (250 observations) for the rolling estimation window (in-sample), and we compute the daily portfolio returns over the next month (20 days, namely the out-of-sample window); the portfolio strategy is then rebalanced (every 20 days) according to new parameter estimates. For each selected portfolio strategy, we end up with nearly five years of out-of-sample daily returns r_{Pt}^{out}.

The portfolio performance is assessed through the following measures typically adopted in Portfolio Selection, assuming a risk-free rate $r_f = 0$ (see, e.g., Cesarone and Colucci (2018); Cesarone et al. (2019, 2023, 2022); Ararat et al. (2024), and references therein).

- The average of out-of-sample portfolio returns $\mu^{out} = \dfrac{1}{T - T^{in}} \sum_{t=T^{in}+1}^{T} r_{Pt}^{out}$, where T^{in} indicates the length of the in-sample window.
- The standard deviation of the out-of-sample portfolio returns,

$$\sigma^{out} = \sqrt{\frac{1}{T - T^{in}} \sum_{t=T^{in}+1}^{T} (r_{Pt}^{out} - \mu^{out})^2}$$

.

- The Sharpe ratio (Sharpe 1966) measures the gain per unit of risk assumed. It is computed as

$$SR^{out} = \frac{\mu^{out} - r_f}{\sigma^{out}}.$$

A higher Sharpe ratio indicates superior risk-adjusted portfolio performance.
- The Sortino ratio (Sortino and Satchell 2001) represents a variation of the Sharpe ratio, where, instead of considering the standard deviation as a risk measure, it considers the Target Downside Deviation (TDD), $TDD = \sqrt{\frac{1}{T-T^{in}} \sum_{t=T^{in}+1}^{T} ((r_{Pt}^{out} - r_f)_-)^2}$. Thus, the Sortino ratio is defined as

$$SOR^{out} = \frac{\mu^{out} - r_f}{TDD},$$

Higher values of this ratio are preferable.
- The Maximum Drawdown (see Chekhlov et al. 2005) measures the maximal out-of-sample loss from the observed peak, and it is defined as:

$$MaxDD = \min_{T^{in}+1 \le t \le T} dd_t,$$

where dd_t represents the Drawdown of the portfolio at time t:

$$dd_t = \frac{W_t - \max\limits_{T^{in}+1 \le \tau \le t} W_\tau}{\max\limits_{T^{in}+1 \le \tau \le t} W_\tau}.$$

Here, $W_t = W_{t-1}(1 + r_{Pt}^{out})$ is the portfolio wealth at time t, with $t \in \{T^{in} + 1, \ldots, T\}$ and $W_{T^{in}} = 1$. Since this measure is always non-positive, higher values are desirable.

- The Ulcer Index (see Martin and Mccann 1989) is a technical indicator that evaluates the depth and the duration of drawdowns in prices over the out-of-sample period, and it is defined as:

$$UI = \sqrt{\frac{\sum_{t=T^{in}+1}^{T} dd_t^2}{T - T^{in}}}.$$

  Lower values of this metric are better.
- The Rachev ratio (see Biglova et al. 2004) measures the upside potential by comparing the right and left tails of the out-of-sample returns. It is computed as

$$Rachev = \frac{CVaR_\alpha(r_f - R_P^{out})}{CVaR_\alpha(R_P^{out} - r_f)},$$

  where $\alpha = 5\%, 10\%$. Higher values indicate better performance.
- The Turnover (see, e.g., DeMiguel et al. 2009) evaluates the amount of trading required to perform in practice the portfolio strategy, and it is computed as:

$$Turn = \frac{1}{Q}\sum_{q=1}^{Q}\sum_{i=1}^{n} \mid x_{q,i} - x_{q-1,i} \mid,$$

  where Q represents the number of total rebalances and x the vector of portfolio weights of the n assets before $(q-1)$ and after (q) rebalancing. In our case, we do not consider trades due to changes in asset prices between rebalances, but only those accounted for in the proposed models. Lower values of this indicator, which serves as a proxy for transaction costs, are preferable.
- The Return on Investment (ROI) measures the time-by-time return generated by each portfolio strategy over a given time horizon $\Delta\tau$ and is defined as follows (see, e.g., Cesarone et al. 2022, 2025):

$$\mathrm{ROI}_{P,\tau} = \frac{W_{P,\tau} - W_{P,\tau-\Delta\tau}}{W_{P,\tau-\Delta\tau}}, \quad \tau = \Delta\tau + 1, \ldots, T. \tag{6}$$

  Here, $W_{P,\tau-\Delta\tau}$ denotes the amount of capital invested at the beginning of the investment horizon, $W_{P,\tau} = W_{P,\tau-\Delta\tau}\prod_{t=\tau-\Delta\tau+1}^{\tau}(1 + r_{P\,t}^{out})$ indicates the portfolio wealth and T is the number of historical scenarios. In our experiment, we fix $\Delta\tau$ equal to 250 days (1 year) and 750 days (3 years).

3.3 Computational results

The following tables show the results of our analysis, which are discussed separately for each portfolio selection criterion described in Sect. 2, as well as for the Equally Weighted portfolio. The first three columns of Tables 2 through 11 show the average value, the 10^{th}, and the 90^{th} percentile of performance indicators of the portfolio

Table 2 Out-of-sample performance results for the MV portfolios on Crypto-EuroStoxx and Crypto-FTSE. See Remark 1 for the full color and style coding

Eurostoxx	**Minimum Variance**					
	100-0			**90-10**	**80-20**	**70-30**
	Av. Value	**10perc**	**90perc**			
μ^{out}	0,033%	0,027%	0,038%	0,036%	**0,038%**	**0,040%**
σ^{out}	0,0097	0,0095	0,0102	0,0099	0,0101	*0,0102*
Sharpe	0,0338	0,0282	0,0388	0,0368	0,0380	0,0388
Sortino	0,0456	0,0378	0,0524	0,0496	0,0515	**0,0527**
Mdd	-0,3255	-0,3390	-0,3143	-0,3309	-0,3336	-0,3366
Ulcer	0,0864	0,0781	0,0951	0,0861	0,0862	0,0867
Rachev5	0,8705	0,8591	0,8829	**0,8830**	**0,8909**	**0,8978**
Rachev10	0,9149	0,8985	0,9270	0,9235	**0,9283**	**0,9333**
Turnover	0,2168	0,2046	0,2269	0,2141	0,2086	**0,2020**

FTSE	**Minimum Variance**					
	100-0			**90-10**	**80-20**	**70-30**
	Av. Value	**10perc**	**90perc**			
μ^{out}	0,027%	0,016%	0,037%	0,030%	0,032%	0,033%
σ^{out}	0,0092	0,0088	0,0096	0,0094	0,0095	*0,0096*
Sharpe	0,0290	0,0178	0,0406	0,0326	0,0337	0,0345
Sortino	0,0399	0,0241	0,0561	0,0447	0,0463	0,0474
Mdd	-0,2811	-0,3145	-0,2492	-0,2832	-0,2861	-0,2896
Ulcer	0,0758	0,0574	0,0964	0,0763	0,0779	0,0803
Rachev5	0,8920	0,8537	0,9293	0,8979	0,9003	0,9030
Rachev10	0,9321	0,8999	0,9645	0,9397	0,9429	0,9453
Turnover	0,2557	0,2373	0,2745	0,2486	0,2380	**0,2267**

strategies built on equities only (the 100-0 basket). The next three columns display the average value of each performance measure in the 90-10, 80-20, and 70-30 baskets, respectively.

For an immediate comparison, the outcomes are displayed with different colors: for each performance indicator (row), the range of colors goes from deep green to deep red, where deep green represents the best *average* performances and deep red represents the worst ones, the average being computed across the 10,000 investment universes of the basket. We point out that the 10^{th}-90^{th} percentile band of the performance measures within the 100-0 basket is reported to gauge whether outcomes in mixed baskets are empirically meaningful relative to the equity-only benchmark. However, it is important to note that this approach is not a formal statistical hypothesis test but rather an empirical evaluation of the relative positioning of the out-of-sample performance outcomes. More precisely, by examining the 10^{th}-90^{th} percentile band of the 100-0 basket, we can empirically assess whether the inclusion of cryptocurrencies in an investment universe can generate performance improvements relative to portfolios composed exclusively of equities.

Below is an example of how to interpret the results presented in Tables 2 through 11. For instance, if we examine Table 2 concerning Eurostoxx, we find that the 10^{th} and 90^{th} percentiles of μ^{out} for the equity-only basket are 0.027% (column 2) and 0.038% (column 3), respectively. The average μ^{out} values for the 90–10 (column 4) and 80–20 (column 5) baskets fall within this 10^{th}–90^{th} percentile band, suggesting that their out-of-sample mean returns are not empirically distinguishable from potential outcomes observed under the equity-only benchmark. By contrast, the average μ^{out} for the 70–30 basket (column 6) is equal to 0.040%, which is above the 90^{th} percentile of the equity-only distribution. This indicates an empirically notable improvement in μ^{out} relative to what is possibly achieved by equity-only portfolios.

To further facilitate the reading of the results, Figure 1 summarizes the average values of the Sharpe ratio for each type of basket and strategy across the four analyzed equity markets. More precisely, squares represent the average Sharpe ratio for the 100-0 basket, while dots represent the averages for the mixed baskets. A filled dot indicates that the average Sharpe ratio of the mixed basket lies outside the 10^{th}–90^{th} percentile band of the equity-only benchmark, that is, above (or below) the typical range observed for the 100–0 basket. Conversely, an unfilled dot indicates that the

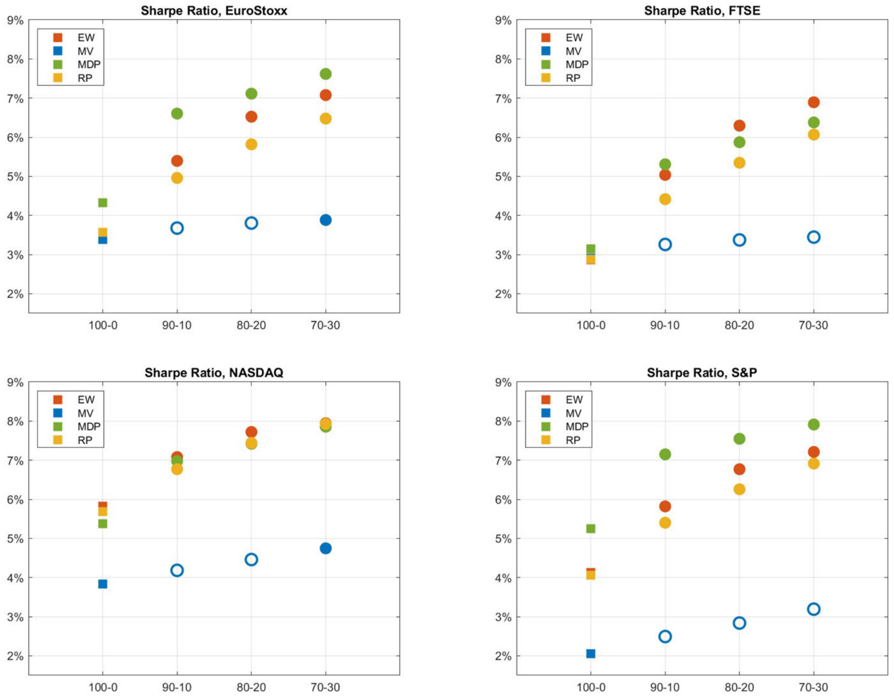

Fig. 1 Average Sharpe ratio values for the 100–0, 90–10, 80–20, and 70–30 baskets, using the 10^{th}–90^{th} percentile band of the equity-only benchmark

Table 3 Out-of-sample performance results for the MV portfolios on Crypto-NASDAQ and Crypto-S&P. See Remark 1 for the full color and style coding.

NASDAQ	Minimum Variance					
	100-0			90-10	80-20	70-30
	Av. Value	10perc	90perc			
μ^{out}	0,045%	0,034%	0,056%	0,050%	0,053%	**0,057%**
σ^{out}	0,0118	0,0113	0,0122	0,0119	0,0120	0,0121
Sharpe	0,0384	0,0294	0,0474	0,0418	0,0446	**0,0475**
Sortino	0,0539	0,0409	0,0669	0,0588	0,0628	**0,0671**
Mdd	-0,3022	-0,3300	-0,2737	-0,3037	-0,3022	-0,3015
Ulcer	0,0653	0,0540	0,0783	0,0659	0,0664	0,0672
Rachev5	0,9525	0,9232	0,9835	0,9574	0,9654	0,9750
Rachev10	0,9749	0,9473	1,0036	0,9810	0,9888	0,9973
Turnover	0,2406	0,2184	0,2628	0,2349	0,2260	**0,2161**

S&P	Minimum Variance					
	100-0			90-10	80-20	70-30
	Av. Value	10perc	90perc			
μ^{out}	0,023%	0,009%	0,037%	0,028%	0,032%	0,036%
σ^{out}	0,0109	0,0104	0,0115	0,0110	0,0111	0,0112
Sharpe	0,0205	0,0082	0,0328	0,0249	0,0284	0,0319
Sortino	0,0289	0,0115	0,0464	0,0352	0,0402	0,0454
Mdd	-0,2934	-0,3319	-0,2549	-0,2934	-0,2950	-0,2973
Ulcer	0,0771	0,0601	0,0966	0,0773	0,0776	0,0775
Rachev5	0,9617	0,9332	0,9902	0,9709	0,9800	**0,9903**
Rachev10	0,9575	0,9341	0,9811	0,9665	0,9753	**0,9849**
Turnover	0,2817	0,2579	0,3061	0,2732	0,2626	**0,2506**

mixed-basket average falls within the 10^{th}–90^{th} percentile band of the equity-only basket.

See also the summary figures 8 and 9 in Appendix A for μ^{out} and σ^{out}, respectively.

We also present graphs representing the percentage of cryptocurrencies selected by the strategies during the rebalancing periods. These graphs depict the trends of the median and the two percentiles (5th and 95th) from the 10,000 values obtained. Furthermore, we provide analogous graphs for each strategy to illustrate the ROI trends over two different time horizons: one year and three years.

Table 4 Out-of-sample performance results for MDP on Crypto-EuroStoxx and Crypto-FTSE. See Remark 1 for the full color and style coding.

Eurostoxx	Most Diversified Portfolio					
	100-0			90-10	80-20	70-30
	Av. Value	10perc	90perc			
μ^{out}	0,047%	0,043%	0,052%	**0,087%**	**0,099%**	**0,112%**
σ^{out}	0,0109	0,0106	0,0115	*0,0130*	*0,0139*	*0,0147*
Sharpe	0,0433	0,0397	0,0469	**0,0660**	**0,0711**	**0,0762**
Sortino	0,0588	0,0538	0,0638	**0,0984**	**0,1100**	**0,1215**
Mdd	-0,3407	-0,3507	-0,3303	*-0,3629*	*-0,3658*	*-0,3688*
Ulcer	0,0709	0,0642	0,0790	0,0767	*0,0790*	*0,0811*
Rachev5	0,8824	0,8664	0,8979	**1,0170**	**1,0699**	**1,1218**
Rachev10	0,9232	0,9111	0,9352	**1,0511**	**1,0931**	**1,1319**
Turnover	0,2325	0,2217	0,2428	0,2419	*0,2480*	*0,2479*

FTSE	Most Diversified Portfolio					
	100-0			90-10	80-20	70-30
	Av. Value	10perc	90perc			
μ^{out}	0,032%	0,022%	0,042%	**0,060%**	**0,070%**	**0,079%**
σ^{out}	0,0101	0,0096	0,0108	*0,0113*	*0,0118*	*0,0123*
Sharpe	0,0315	0,0215	0,0419	**0,0531**	**0,0587**	**0,0638**
Sortino	0,0436	0,0296	0,0583	**0,0772**	**0,0871**	**0,0961**
Mdd	-0,3216	-0,3539	-0,2912	-0,3314	-0,3349	-0,3381
Ulcer	0,0817	0,0623	0,1031	0,0842	0,0861	0,0883
Rachev5	0,9126	0,8748	0,9516	**0,9971**	**1,0315**	**1,0630**
Rachev10	0,9554	0,9240	0,9869	**1,0391**	**1,0667**	**1,0918**
Turnover	0,2447	0,2274	0,2626	0,2470	0,2453	0,2398

Table 5 Out-of-sample performance results for MDP on Crypto-NASDAQ and Crypto-S&P. See Remark 1 for the full color and style coding.

NASDAQ	Most Diversified Portfolio					
	100-0			90-10	80-20	70-30
	Av. Value	10perc	90perc			
μ^{out}	0,070%	0,055%	0,086%	**0,104%**	**0,118%**	**0,131%**
σ^{out}	0,0131	0,0125	0,0137	*0,0148*	*0,0158*	*0,0166*
Sharpe	0,0537	0,0423	0,0649	**0,0697**	**0,0742**	**0,0785**
Sortino	0,0763	0,0599	0,0925	**0,1065**	**0,1179**	**0,1286**
Mdd	-0,3015	-0,3344	-0,2693	-0,3117	-0,3134	-0,3160
Ulcer	0,0652	0,0534	0,0793	0,0711	0,0741	0,0773
Rachev5	0,9761	0,9473	1,0064	**1,0669**	**1,1165**	**1,1586**
Rachev10	1,0009	0,9738	1,0287	**1,0730**	**1,1112**	**1,1448**
Turnover	0,2196	0,2009	0,2383	0,2377	*0,2411*	*0,2400*

S&P	Most Diversified Portfolio					
	100-0			90-10	80-20	70-30
	Av. Value	10perc	90perc			
μ^{out}	0,065%	0,046%	0,086%	**0,105%**	**0,119%**	**0,133%**
σ^{out}	0,0124	0,0119	0,0129	*0,0146*	*0,0157*	*0,0168*
Sharpe	0,0525	0,0375	0,0685	**0,0715**	**0,0755**	**0,0791**
Sortino	0,0748	0,0531	0,0981	**0,1135**	**0,1267**	**0,1390**
Mdd	-0,3062	-0,3387	-0,2753	-0,3048	-0,3061	-0,3085
Ulcer	0,0733	0,0546	0,0961	0,0803	0,0813	0,0834
Rachev5	0,9871	0,9525	1,0216	**1,1080**	**1,1689**	**1,2277**
Rachev10	0,9999	0,9704	1,0303	**1,1056**	**1,1542**	**1,2000**
Turnover	0,2726	0,2545	0,2909	0,2800	0,2808	0,2771

To facilitate the interpretation of the computational results reported in Tables 2-13, the following remark summarizes the color and style coding adopted throughout the results section.

Remark 1 (Table color and style coding) The color intensity reflects the average performance across the 10,000 random investment universes, with deeper green indicating better outcomes and deeper red indicating worse outcomes. For mixed baskets, values are shown in bold when they fall outside the 10th-90th percentile band of the equity-only basket and correspond to better results, in italics when they fall outside that band and correspond to worse results, and in regular font when they fall within the equity-only range.

3.3.1 Minimum variance portfolios

Tables 2 and 3 present the computational results obtained for the MV portfolios. A similar trend emerges across all four markets. Generally, a moderate increase in portfolio profitability (in terms of μ^{out}, Sharpe, Sortino, and Rachev ratio) is observed in mixed baskets (90-10, 80-20, 70-30). However, the values of these performance measures typically fall within a band considered non-relevant compared to those generated by the equity-only basket.

Similarly, mixed portfolios show increased risk measures (e.g., σ^{out}, Mdd, and Ulcer index) that fall within the equity-only range. Turnover levels also remain within this range and generally improve as the proportion of cryptocurrencies in the universe increases. Notably, only in the 70-30 case does turnover fall outside the 10th–90th

Table 6 Out-of-sample performance results for the RP portfolios on Crypto-EuroStoxx and Crypto-FTSE. See Remark 1 for the full color and style coding.

Eurostoxx	Risk Parity					
	100-0			90-10	80-20	70-30
	Av. Value	10perc	90perc			
μ^{out}	0,045%	0,042%	0,047%	0,064%	0,078%	0,091%
σ^{out}	0,0126	0,0122	0,0129	0,0129	0,0134	0,0140
Sharpe	0,0357	0,0335	0,0377	0,0496	0,0582	0,0648
Sortino	0,0488	0,0459	0,0517	0,0682	0,0804	0,0900
Mdd	-0,3887	-0,3995	-0,3783	-0,3952	-0,4005	-0,4059
Ulcer	0,0959	0,0916	0,1005	0,0964	0,0991	0,1037
Rachev5	0,8771	0,8692	0,8854	0,8925	0,9168	0,9370
Rachev10	0,9215	0,9156	0,9276	0,9551	0,9768	0,9927
Turnover	0,0360	0,0346	0,0370	0,0381	0,0394	0,0407

FTSE	Risk Parity					
	100-0			90-10	80-20	70-30
	Av. Value	10perc	90perc			
μ^{out}	0,031%	0,026%	0,037%	0,050%	0,063%	0,075%
σ^{out}	0,0109	0,0104	0,0115	0,0113	0,0117	0,0124
Sharpe	0,0288	0,0239	0,0338	0,0441	0,0535	0,0607
Sortino	0,0398	0,0329	0,0469	0,0614	0,0747	0,0853
Mdd	-0,3411	-0,3632	-0,3197	-0,3440	-0,3478	-0,3524
Ulcer	0,0856	0,0739	0,0981	0,0878	0,0921	0,0974
Rachev5	0,9038	0,8837	0,9242	0,9247	0,9449	0,9685
Rachev10	0,9450	0,9293	0,9608	0,9758	1,0000	1,0173
Turnover	0,0453	0,0431	0,0474	0,0467	0,0475	0,0482

Table 7 Out-of-sample performance results for the RP portfolios on Crypto-NASDAQ and Crypto-S&P. See Remark 1 for the full color and style coding.

NASDAQ	Risk Parity					
	100-0			90-10	80-20	70-30
	Av. Value	10perc	90perc			
μ^{out}	0,078%	0,069%	0,086%	0,096%	0,109%	0,122%
σ^{out}	0,0137	0,0131	0,0143	0,0141	0,0147	0,0154
Sharpe	0,0568	0,0518	0,0619	0,0677	0,0744	0,0793
Sortino	0,0809	0,0735	0,0882	0,0966	0,1065	0,1139
Mdd	-0,3012	-0,3200	-0,2826	-0,3081	-0,3159	-0,3250
Ulcer	0,0750	0,0633	0,0876	0,0826	0,0900	0,0982
Rachev5	0,9460	0,9306	0,9614	0,9586	0,9730	0,9857
Rachev10	0,9785	0,9655	0,9915	0,9967	1,0064	1,0154
Turnover	0,0390	0,0361	0,0413	0,0406	0,0416	0,0429

S&P	Risk Parity					
	100-0			90-10	80-20	70-30
	Av. Value	10perc	90perc			
μ^{out}	0,051%	0,043%	0,058%	0,069%	0,084%	0,097%
σ^{out}	0,0124	0,0120	0,0129	0,0128	0,0134	0,0140
Sharpe	0,0406	0,0350	0,0462	0,0540	0,0626	0,0691
Sortino	0,0571	0,0491	0,0652	0,0763	0,0888	0,0987
Mdd	-0,3290	-0,3494	-0,3087	-0,3342	-0,3378	-0,3413
Ulcer	0,0757	0,0640	0,0883	0,0810	0,0852	0,0900
Rachev5	0,9351	0,9193	0,9508	0,9499	0,9673	0,9836
Rachev10	0,9549	0,9445	0,9653	0,9789	0,9944	1,0105
Turnover	0,0365	0,0348	0,0382	0,0382	0,0397	0,0413

Table 8 Out-of-sample performance results for the EW portfolios on Crypto-EuroStoxx and Crypto-FTSE. See Remark 1 for the full color and style coding.

Eurostoxx	Equally Weighted					
	100-0			90-10	80-20	70-30
	Av. Value	10perc	90perc			
μ^{out}	0,048%	0,046%	0,051%	0,078%	0,108%	0,138%
σ^{out}	0,0137	0,0134	0,0141	0,0145	0,0166	0,0195
Sharpe	0,0353	0,0333	0,0373	0,0539	0,0652	0,0708
Sortino	0,0487	0,0458	0,0515	0,0748	0,0909	0,0990
Mdd	-0,4072	-0,4175	-0,3970	-0,4201	-0,4363	-0,4535
Ulcer	0,1042	0,0999	0,1085	0,1150	0,1417	0,1768
Rachev5	0,8946	0,8885	0,9011	0,9145	0,9365	0,9545
Rachev10	0,9331	0,9283	0,9380	0,9758	0,9830	0,9836

FTSE	Equally Weighted					
	100-0			90-10	80-20	70-30
	Av. Value	10perc	90perc			
μ^{out}	0,035%	0,030%	0,040%	0,066%	0,097%	0,129%
σ^{out}	0,0121	0,0115	0,0128	0,0131	0,0155	0,0187
Sharpe	0,0287	0,0246	0,0328	0,0504	0,0629	0,0689
Sortino	0,0401	0,0344	0,0460	0,0710	0,0889	0,0975
Mdd	-0,3628	-0,3852	-0,3404	-0,3686	-0,3858	-0,4111
Ulcer	0,0950	0,0837	0,1068	0,1110	0,1422	0,1778
Rachev5	0,9330	0,9130	0,9532	0,9641	0,9748	0,9708
Rachev10	0,9650	0,9499	0,9802	1,0105	1,0132	1,0077

percentile band of the equity-only basket, but on the side corresponding to better results. It is also worth noting that even when the number of cryptocurrencies within the basket is increased, the overall weight of selected cryptos remains very low across all baskets and markets (see Figure 2 for Crypto-FTSE, and figures in Supplementary Material S1 for the other markets). These graphs show the median, 5^{th}, and 95^{th} percentile of the distribution of cryptocurrency weights selected by the optimization across the 10,000 random investment universes. Overall, the MV approach, with the exception of some periods, tends to allocate only a small fraction to cryptocurrencies, typically around 2% and 3%, even when the basket composition makes a larger crypto set available.

Therefore, for "Minimum Variance" investors, there seems to be no advantage in including cryptocurrencies in an equity investment universe.

Table 9 Out-of-sample performance results for the EW portfolios on Crypto-NASDAQ and Crypto-S&P. See Remark 1 for the full color and style coding.

NASDAQ	Equally Weighted					
	100-0			90-10	80-20	70-30
	Av. Value	10perc	90perc			
μ^{out}	0,090%	0,080%	0,100%	**0,116%**	**0,142%**	**0,167%**
σ^{out}	0,0155	0,0147	0,0162	*0,0164*	*0,0183*	*0,0211*
Sharpe	0,0582	0,0532	0,0632	**0,0708**	**0,0772**	**0,0794**
Sortino	0,0832	0,0759	0,0904	**0,1011**	**0,1100**	**0,1130**
Mdd	-0,3185	-0,3385	-0,2988	*-0,3418*	*-0,3860*	*-0,4385*
Ulcer	0,1002	0,0860	0,1149	*0,1241*	*0,1572*	*0,1929*
Rachev5	0,9604	0,9457	0,9747	0,9475	0,9348	0,9344
Rachev10	0,9829	0,9727	0,9933	0,9899	0,9895	**0,9953**

S&P	Equally Weighted					
	100-0			90-10	80-20	70-30
	Av. Value	10perc	90perc			
μ^{out}	0,055%	0,048%	0,063%	**0,085%**	**0,114%**	**0,143%**
σ^{out}	0,0135	0,0129	0,0140	*0,0146*	*0,0168*	*0,0198*
Sharpe	0,0412	0,0358	0,0467	**0,0582**	**0,0677**	**0,0721**
Sortino	0,0581	0,0504	0,0658	**0,0821**	**0,0955**	**0,1018**
Mdd	-0,3481	-0,3697	-0,3266	-0,3582	*-0,3738*	*-0,4054*
Ulcer	0,0879	0,0748	0,1013	*0,1109*	*0,1465*	*0,1844*
Rachev5	0,9354	0,9221	0,9492	0,9331	0,9340	0,9361
Rachev10	0,9632	0,9549	0,9715	**0,9790**	**0,9827**	**0,9902**

Table 10 Out-of-sample performance results for the MV and MDP portfolios on Crypto-*EquityAll*. See Remark 1 for the full color and style coding.

EquityAll	Minimum Variance					
	100-0			90-10	80-20	70-30
	Av. Value	10perc	90perc			
μ^{out}	0,026%	0,011%	0,040%	0,029%	0,031%	0,033%
σ^{out}	0,0101	0,0095	0,0108	0,0102	0,0103	0,0105
Sharpe	0,0252	0,0107	0,0390	0,0278	0,0294	0,0311
Sortino	0,0347	0,0145	0,0538	0,0382	0,0405	0,0430
Mdd	-0,3288	-0,3673	-0,2891	-0,3286	-0,3293	-0,3314
Ulcer	0,0894	0,0675	0,1146	0,0898	0,0903	0,0912
Rachev5	0,9089	0,8700	0,9489	0,9128	0,9166	0,9207
Rachev10	0,9362	0,9034	0,9698	0,9410	0,9458	0,9508
Turnover	0,2445	0,2220	0,2674	0,2370	0,2276	**0,2179**

EquityAll	Most Diversified Portfolio					
	100-0			90-10	80-20	70-30
	Av. Value	10perc	90perc			
μ^{out}	0,050%	0,035%	0,065%	**0,079%**	**0,090%**	**0,101%**
σ^{out}	0,0113	0,0106	0,0121	*0,0128*	*0,0134*	*0,0142*
Sharpe	0,0439	0,0309	0,0573	**0,0615**	**0,0667**	**0,0714**
Sortino	0,0613	0,0427	0,0805	**0,0914**	**0,1024**	**0,1129**
Mdd	-0,3426	-0,3803	-0,3053	-0,3441	-0,3447	-0,3464
Ulcer	0,0850	0,0646	0,1095	0,0893	0,0911	0,0931
Rachev5	0,9337	0,8924	0,9763	**1,0187**	**1,0653**	**1,1098**
Rachev10	0,9771	0,9421	1,0131	**1,0489**	**1,0856**	**1,1200**
Turnover	0,2456	0,2262	0,2652	0,2526	0,2515	0,2476

Table 11 Out-of-sample performance results for the RP and EW portfolios on Crypto-*EquityAll*. See Remark 1 for the full color and style coding.

EquityAll	Risk Parity					
	100-0			90-10	80-20	70-30
	Av. Value	10perc	90perc			
μ^{out}	0,049%	0,041%	0,057%	**0,066%**	**0,079%**	**0,092%**
σ^{out}	0,0119	0,0113	0,0125	0,0123	*0,0129*	*0,0136*
Sharpe	0,0412	0,0345	0,0481	**0,0535**	**0,0615**	**0,0677**
Sortino	0,0575	0,0479	0,0674	**0,0748**	**0,0864**	**0,0957**
Mdd	-0,3581	-0,3824	-0,3340	-0,3615	-0,3645	-0,3685
Ulcer	0,0904	0,0760	0,1054	0,0956	0,1007	*0,1061*
Rachev5	0,9249	0,9045	0,9458	0,9357	**0,9508**	**0,9692**
Rachev10	0,9680	0,9515	0,9845	0,9835	**1,0025**	**1,0187**
Turnover	0,0431	0,0405	0,0456	0,0440	0,0448	*0,0459*

EquityAll	Equally Weighted					
	100-0			90-10	80-20	70-30
	Av. Value	10perc	90perc			
μ^{out}	0,055%	0,046%	0,065%	**0,084%**	**0,114%**	**0,143%**
σ^{out}	0,0131	0,0124	0,0138	*0,0143*	*0,0167*	*0,0198*
Sharpe	0,0422	0,0354	0,0492	**0,0589**	**0,0681**	**0,0723**
Sortino	0,0594	0,0497	0,0693	**0,0829**	**0,0960**	**0,1020**
Mdd	-0,3743	-0,4001	-0,3487	-0,3838	*-0,4032*	*-0,4313*
Ulcer	0,1014	0,0869	0,1162	*0,1239*	*0,1568*	*0,1921*
Rachev5	0,9427	0,9250	0,9604	0,9399	0,9451	0,9505
Rachev10	0,9797	0,9647	0,9944	**0,9957**	**0,9977**	**0,9993**

Table 12 Out-of-sample performance results for the MV and MDP portfolios in three cases: (1) the 100-0 basket, (2) the 70-30 basket (on Crypto-*EquityAll*), and (3) the basket with the same 28 equities as in the 70-30 basket but only Bitcoin representing the crypto universe. See Remark 1 for the full color and style coding.

	Minimum Variance				
	100-0			70-30	Equity (28) - BTC
	Av. Value	10perc	90perc		
μ^{out}	0,026%	0,011%	0,040%	0,033%	0,029%
σ^{out}	0,0101	0,0095	0,0108	0,0105	0,0105
Sharpe	0,0252	0,0107	0,0390	0,0311	0,0279
Sortino	0,0347	0,0145	0,0538	0,0430	0,0385
Mdd	-0,3288	-0,3673	-0,2891	-0,3314	-0,3316
Ulcer	0,0894	0,0675	0,1146	0,0912	0,0920
Rachev5	0,9089	0,8700	0,9489	0,9207	0,9156
Rachev10	0,9362	0,9034	0,9698	0,9508	0,9439
Turnover	0,2445	0,2220	0,2674	**0,2179**	**0,2156**

	Most Diversified Portfolio				
	100-0			70-30	Equity (28) - BTC
	Av. Value	10perc	90perc		
μ^{out}	0,050%	0,035%	0,065%	**0,101%**	0,065%
σ^{out}	0,0113	0,0106	0,0121	*0,0142*	0,0118
Sharpe	0,0439	0,0309	0,0573	**0,0714**	0,0550
Sortino	0,0613	0,0427	0,0805	**0,1129**	0,0764
Mdd	-0,3426	-0,3803	-0,3053	-0,3464	-0,3417
Ulcer	0,0850	0,0646	0,1095	0,0931	0,0930
Rachev5	0,9337	0,8924	0,9763	**1,1098**	0,9420
Rachev10	0,9771	0,9421	1,0131	**1,1200**	0,9912
Turnover	0,2456	0,2262	0,2652	0,2476	**0,2151**

Table 13 Out-of-sample performance results for the RP and EW portfolios in three cases: (1) the 100-0 basket, (2) the 70-30 basket (on Crypto-*EquityAll*), and (3) the basket with the same 28 equities as in the 70-30 basket but only Bitcoin representing the crypto universe. See Remark 1 for the full color and style coding.

	Risk Parity				
	100-0			70-30	Equity (28) - BTC
	Av. Value	10perc	90perc		
μ^{out}	0,049%	0,041%	0,057%	**0,092%**	0,055%
σ^{out}	0,0119	0,0113	0,0125	*0,0136*	0,0121
Sharpe	0,0412	0,0345	0,0481	**0,0677**	0,0458
Sortino	0,0575	0,0479	0,0674	**0,0957**	0,0638
Mdd	-0,3581	-0,3824	-0,3340	-0,3685	-0,3577
Ulcer	0,0904	0,0760	0,1054	*0,1061*	0,0947
Rachev5	0,9249	0,9045	0,9458	**0,9692**	0,9270
Rachev10	0,9680	0,9515	0,9845	**1,0187**	0,9702
Turnover	0,0431	0,0405	0,0456	*0,0459*	0,0417

	Equally Weighted				
	100-0			70-30	Equity (28) - BTC
	Av. Value	10perc	90perc		
μ^{out}	0,055%	0,046%	0,065%	**0,143%**	0,062%
σ^{out}	0,0131	0,0124	0,0138	*0,0198*	0,0133
Sharpe	0,0422	0,0354	0,0492	**0,0723**	0,0469
Sortino	0,0594	0,0497	0,0693	**0,1020**	0,0659
Mdd	-0,3743	-0,4001	-0,3487	*-0,4313*	-0,3737
Ulcer	0,1014	0,0869	0,1162	*0,1921*	0,1078
Rachev5	0,9427	0,9250	0,9604	0,9505	0,9407
Rachev10	0,9797	0,9647	0,9944	**0,9993**	0,9795

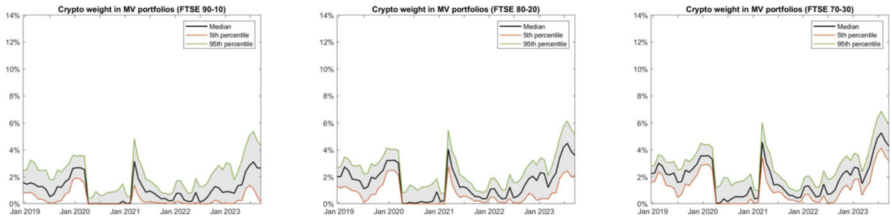

Fig. 2 Cryptocurrency weights selected over time from the MV approach on Crypto-FTSE

Furthermore, we compute the ROI trend with one-year and three-year investment horizons for the 100-0 and 70-30 baskets. For each time period considered, we compute the median, 5^{th}, and 95^{th} percentile of the distributions of the 10,000 ROIs.

By examining the one-year ROI for the MV portfolios (see figures in Supplementary Material S2), we note that the equity-only basket and the 70-30 basket exhibit very similar distributions of values. Indeed, the increase in ROI is quite limited since few cryptocurrencies are selected from this strategy.

On the other hand, if we consider a three-year investment horizon, with the inclusion of cryptocurrencies, we can observe an increase in ROI (see figures in Supplementary Material S3), albeit moderate compared to the other portfolio selection strategies.

In conclusion, although cryptocurrencies are often discussed as potential diversifiers in portfolio selection, our findings suggest that their effectiveness is more nuanced for the MV strategy. In the early part of the sample, when correlations between equities and cryptocurrencies were relatively low, cryptocurrencies could provide diversification benefits. The rolling correlation analysis in Sect. 3.4.3, however, shows that the average cross-group correlation between equities and cryptocurrencies increases markedly after 2020. This rise diminishes the diversification benefits that might otherwise compensate for the high standalone volatility of cryptocurrencies. As a result, the MV strategy generally assigns very small, often negligible, weights to cryptocurrencies. The limited presence of cryptocurrencies in MV portfolios is therefore attributable not only to their volatility but also to the increased correlation with equities observed after 2020.

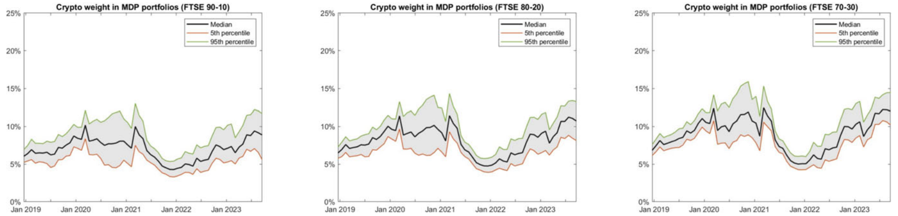

Fig. 3 Cryptocurrency weights selected over time from MDP on Crypto-FTSE

3.3.2 Most diversified Portfolios

In Tables 4 and 5, we report the results of the MDP strategy for the four equity markets considered in the analysis. We find that mixed portfolios exhibit a substantial increase in profitability, consistently higher than that of equity-only portfolios, and that this improvement strengthens as the percentage of cryptocurrencies in the basket increases. In many cases, portfolios built from the 70–30 basket display μ^{out}, Sharpe and Sortino ratios doubled compared to the equity-only portfolios (see Table 4 for the Crypto-EuroStoxx and Crypto-FTSE datasets). As expected, the inclusion of cryptocurrencies is accompanied by an increase in portfolio risk (i.e., σ^{out}, Mdd, and Ulcer index). However, the magnitude of the risk increase is generally modest when compared with the concurrent increase in return (see also Table 5 for the Crypto-NASDAQ and for Crypto-S&P datasets). Turnover also tends to increase with the inclusion of more cryptocurrencies, with the exception of the Crypto-FTSE dataset.

As shown in Figure 3, the MDP strategy is more aggressive than the MV strategy. Indeed, the percentage of cryptocurrencies selected by MDP indicates that it tends to select a higher portion of cryptocurrencies compared to the MV strategy. This percentage slightly increases as the number of available cryptocurrencies in the market grows (see also figures in Supplementary Material S1). However, with the exception of specific periods (see, for example, Crypto-S&P during 2020 in Supplementary Material S1), the overall weight of cryptocurrencies in the portfolios remains relatively limited, typically around 8% to 10%, in comparison to the actual availability of cryptocurrencies in the baskets.

Furthermore, we note that when a higher percentage of cryptos is selected (in 2020), the one-year ROI of the MDP strategy increases significantly (in 2021). In other periods, the one-year ROI distribution is similar between the 100-0 and 70-30 baskets, as shown in figures in Supplementary Material S2.

On the other hand, as illustrated in Figure 4, the three-year ROI of the mixed portfolios shows a significant increase (see also figures in Supplementary Material S3). In fact, the entire three-year ROI distribution in the 70-30 basket of MDP shifts upward.

Overall, the advantages of incorporating cryptocurrencies into an equity portfolio appear to be substantial when employing the MDP strategy.

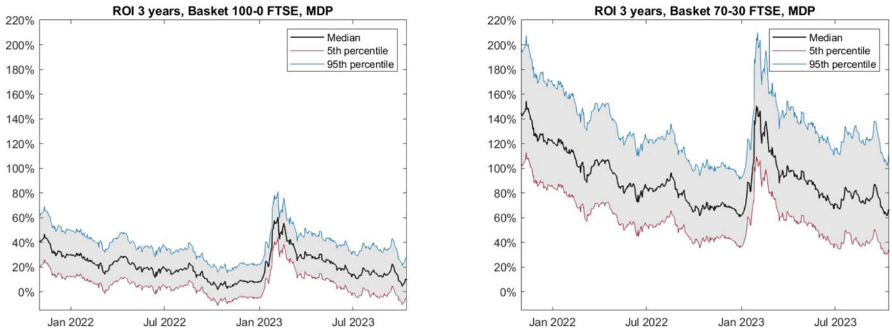

Fig. 4 Three-year ROI for MDP on equity-only FTSE (left) and Crypto-FTSE (right)

3.3.3 Risk parity portfolios

Tables 6 and 7 present the computational results of the Risk Parity strategy for Crypto-EuroStoxx and Crypto-FTSE, and for Crypto-NASDAQ and Crypto-S&P, respectively.

When the 100–0 basket is compared with the 70–30 basket, profitability measures generally shift upward for mixed universes. In many cases, the average values achieved by mixed portfolios lie above the 10^{th}–90^{th} percentile band of the equity-only (100–0) outcomes, indicating a clear improvement relative to what is typically observed when only equities are available. On the risk side, the effect of introducing cryptocurrencies is more contained. Drawdown-based indicators such as Mdd and the Ulcer Index often remain within the 10^{th}–90^{th} percentile band of the equity-only basket, while σ^{out} tends to move upward, albeit with a limited numerical increase. Overall, for the RP portfolios, mixed portfolios appear to have a better risk-return profile than equity-only portfolios, with higher Turnover levels that are outside the equity-only range. Sharpe, Sortino, Rachev5, and Rachev10 tend to improve as the cryptocurrency share in the investment universe increases, frequently exceeding the corresponding 10^{th}–90^{th} percentile band of the 100–0 basket. For this strategy, as the percentage of cryptocurrencies in the investment universe increases, the RP allocation to cryptocurrencies rises accordingly. In particular, in the 70-30 basket, the median cryptocurrency weight can reach peaks around 15%-20% (see figures in Supplementary Material S1).

Concerning the analysis of the ROI distribution, a pattern emerges that is similar to that found for the MDP strategy. A pronounced increase in ROI is observed in 2021, whereas in the other sub-periods, ROI tends to remain broadly comparable to the equity-only portfolios. On the other hand, the three-year ROI distribution with cryptocurrencies included exhibits a significant upward shift. The one-year and three-year ROI distributions for the 70–30 and 100–0 baskets across all datasets are reported in Supplementary Material S2 and S3 respectively.

Overall, these findings suggest that incorporating cryptocurrencies into equity portfolios using the RP strategy is generally advantageous.

3.3.4 Equally weighted portfolios

Tables 8 and 9 report the out-of-sample performance of the EW portfolio for Crypto-EuroStoxx and Crypto-FTSE, and for Crypto-NASDAQ and Crypto-S&P, respectively.

The computational results indicate a significant growth in profitability measures when the percentage of cryptocurrencies in the selected portfolios rises (see, for example, μ^{out} and Sortino Ratio). However, this behavior is also associated with a considerable rise in risk measures (see, for example, σ^{out} and Ulcer).

The one-year ROI trend indicates significant differences between the 100–0 and 70–30 baskets in 2021. In other years, although the EW portfolio includes all available cryptocurrencies by construction, the one-year ROI of mixed portfolios typically increases only modestly relative to equity-only portfolios. In line with the evidence for the other strategies, the three-year ROI distribution for EW portfolios shifts upward when moving from the equity-only basket to the 70–30 basket. The figures illustrating the one-year and three-year ROI distributions of the EW strategy across all analyzed datasets are reported in Supplementary Material S2 and S3, respectively.

Overall, these results suggest that constructing an EW portfolio composed of cryptocurrencies and equities can also be beneficial, albeit at the cost of higher risk.

3.4 Financial interpretation, horizon effects, and robustness checks

The results in Sect. 3.3 show that the effect of cryptocurrencies on equity portfolios varies substantially across portfolio strategies. Mixed equity-cryptocurrency portfolios do not systematically dominate equity-only portfolios; the benefits of including cryptocurrencies depend on the specific allocation approach. For strategies focused on diversification or balanced risk contributions, such as the Most Diversified Portfolio (MDP) and Risk Parity (RP) methods, adding cryptocurrencies often leads to better returns and stronger risk-adjusted performance. At the same time, the increase in volatility and drawdowns tends to be modest. For Minimum Variance (MV) portfolios, by contrast, the role of cryptocurrencies is much more marginal, as their elevated volatility and increased correlation with equities typically lead to minimal portfolio weights.

This interpretation also helps position our contribution within the existing literature. Several studies have documented that cryptocurrencies may improve portfolio diversification, either because of their low correlation with traditional financial assets or because their inclusion can enhance portfolio risk-return profiles (see, e.g., Bouri et al. 2017; Corbet et al. 2018; Eisl et al. 2015; Kajtazi and Moro 2019; Petukhina et al. 2021; Han et al. 2024). At the same time, other evidence suggests that these benefits may weaken during periods of market stress or structural change, when correlations between cryptocurrencies and traditional assets tend to increase (see, e.g., Li and Miu 2023; Hsu et al. 2024). Our results qualify this literature by showing that the benefits of cryptocurrency inclusion are not unconditional. They depend on the portfolio construction approach, the investment horizon, and the evolving degree of comovement between cryptocurrencies and equity markets. Furthermore, the large-scale randomized design allows us to assess whether these effects persist across many alternative investment universes, thereby reducing the role of asset-selection effects.

Horizon analysis adds detail to these findings. Over a one-year period, the benefits of including cryptocurrency in a portfolio mostly appear during episodes of significant crypto appreciation, such as in 2020–2021. Outside of these periods, the one-year ROI distribution for mixed and equity-only portfolios tends to be similar. This indicates that short-term gains from crypto inclusion depend on favourable market conditions rather than being reliably persistent. In contrast, over a three-year horizon, the results are more robust: the ROI distributions of mixed portfolios consistently shift upward relative to the equity-only benchmark, particularly for RP and MDP. Longer investment horizons enable the higher-return potential of cryptocurrencies to compound, reducing the effect of short-term losses and making the diversification benefits more noticeable.

To assess the robustness and generalizability of our main findings, we implement three additional tests. First, we examine whether the results remain consistent when the equity allocation is drawn from a single pooled universe rather than market-specific indices (Sect. 3.4.1). Second, we compare the performance of a multi-cryptocurrency portfolio to a Bitcoin-only benchmark, evaluating the value of diversification within the crypto segment (Sect. 3.4.2). Third, we analyze the time-varying relationship between cryptocurrencies and equities using a rolling correlation analysis (Sect. 3.4.3). Finally, in Sect. 3.4.4, Table 14 summarize the key implications across strategies and scenarios.

3.4.1 Robustness to a pooled equity universe

A potential criticism is that the analysis reported in Sect. 3.3 treats the four equity markets separately, so the gains observed from adding cryptocurrencies may partly reflect market segmentation. We address this issue by constructing a pooled equity universe that includes all unique constituents of the EuroStoxx50, FTSE100, NASDAQ100, and S&P100 indices. After removing duplicates, this leads to 304 distinct equities (hereafter denoted by *EquityAll*). To ensure comparability with the original methodology, we apply the same portfolio construction procedure. For each allocation, we generate 10,000 random investment universes, each consisting of 40 assets drawn from *EquityAll* and the 12 selected cryptocurrencies. Portfolio compositions are unchanged: 100-0 (equities only), 90-10 (36 equities and 4 cryptocurrencies), 80-20 (32 equities and 8 cryptocurrencies), and 70-30 (28 equities and 12 cryptocurrencies).

Tables 10 and 11 are consistent with the results reported in Sect. 3.3. When the four equity markets are pooled into a single investment universe, mixed equity-cryptocurrency portfolios still tend to outperform the 100-0 benchmark in risk-adjusted terms, especially under the RP and MDP strategies, while incurring only modest increases in risk. The EW strategy also benefits from including cryptocurrencies, although with more pronounced increases in volatility and drawdowns. By contrast, the MV strategy shows only limited improvements.

These results suggest that the previous findings are not mainly driven by the division of EuroStoxx, FTSE, NASDAQ, and S&P into distinct equity universes. The diversification benefits associated with cryptocurrency inclusion remain visible when equities are selected from a broader pooled universe. Therefore, the main conclusion is preserved under this alternative definition of the equity investment universe.

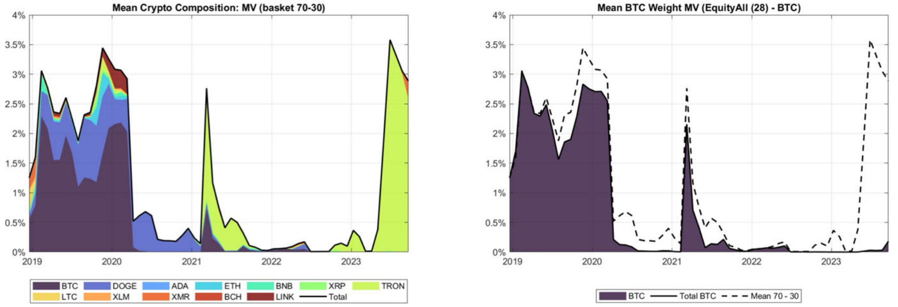

Fig. 5 Cryptocurrency weights selected over time from the MV approach in the 70-30 basket (left) and in the Equity (28) - BTC (right)

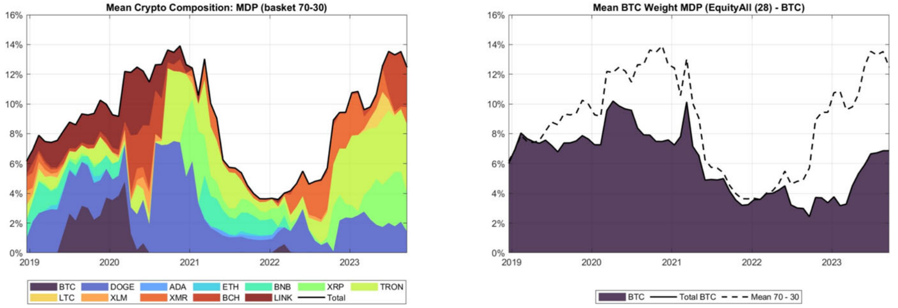

Fig. 6 Cryptocurrency weights selected over time from the MDP approach in the 70-30 basket (left) and in the Equity (28) - BTC (right)

3.4.2 The role of crypto diversification: Bitcoin-only vs. multi-crypto

A second potential issue concerns the role of diversification within the same cryptocurrency set. Since correlations among major cryptocurrencies are often high, one may wonder whether a diversified crypto basket truly adds value relative to a simpler Bitcoin-only exposure. To address this question, we compare the 70-30 mixed basket with a Bitcoin-only benchmark in which the equity component is held fixed at 28 assets from *EquityAll*, while Bitcoin is the only available cryptocurrency (hereafter, *Equity (28)-BTC*).

Tables 12 and 13 show that portfolios built on the 70-30 basket generally outperform the Bitcoin-only specification in terms of risk-adjusted measures. In many cases, the Bitcoin-only outcomes remain inside the 10th-90th percentile band of the equity-only benchmark, whereas the diversified multi-crypto specification moves outside that band on the side corresponding to better results. This indicates that high intra-crypto correlations do not eliminate diversification gains within the crypto universe. Correlations among cryptocurrencies are high but not perfect. Furthermore, cryptocurrencies exhibit differences in volatility, tail behavior, and co-movement with equities. These distinctions enable portfolio optimization strategies to exploit the heterogeneity among assets, rather than relying solely on a single Bitcoin position.

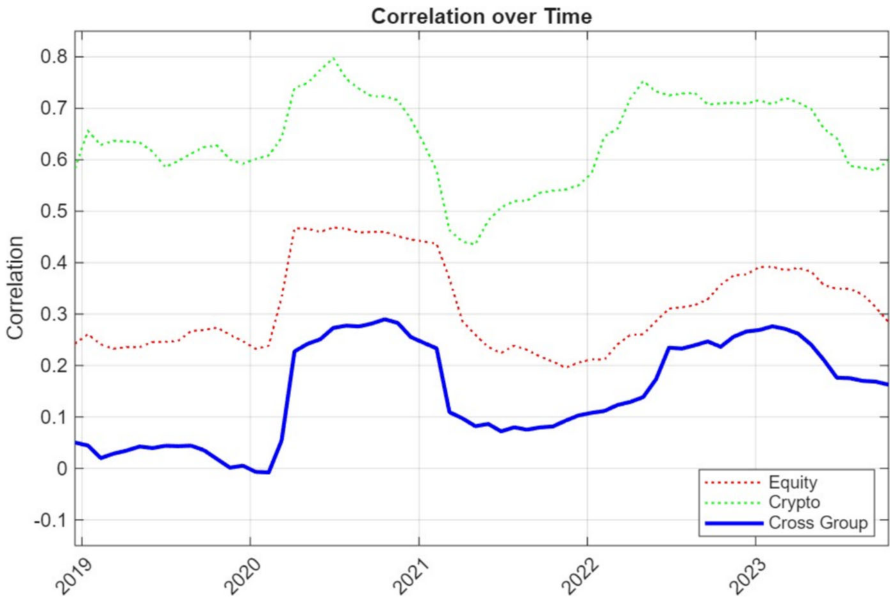

Fig. 7 Rolling correlation analysis of equity-only (dashed red), crypto-only (dashed green), and cross-group (solid blue)

Figures 5 and 6 provide further evidence for this view. When multiple cryptocurrencies are available, particularly when using the MDP and RP strategies, allocations tend to be spread across several cryptocurrencies rather than concentrated solely on Bitcoin. By contrast, the MV approach generally avoids selecting cryptocurrencies, except at the start of the sample period when equity-crypto correlations are low. These results suggest that a multi-crypto universe is preferable, and highlight the importance of comparing not just "crypto vs no crypto", but also "diversified crypto exposure vs single-asset crypto exposure".

3.4.3 Crypto-equity correlation analysis

Using the portfolio-based methodology of Aneja et al. (1989), we perform a rolling correlation analysis of equity-only, crypto-only, and cross-group. In Figure 7, we show the average correlation of equity-only (dashed red), crypto-only (dashed green), and cross-group (solid blue) over time. In the early part of the sample, this average correlation hovers near zero, suggesting that cryptocurrencies offer meaningful diversification benefits. But starting in 2020, the correlation increases sharply, reflecting a tighter connection between crypto and equity markets.

This fact aligns well with the MV portfolio results reported in Sect. 3.3.1. When correlations between equities and cryptocurrencies are low, the MV strategy allocates some crypto exposure, despite the high volatility these assets entail (see Figure 2). As correlations increase, MV assigns little or no weight to cryptocurrencies.

Conversely, the RP and MDP strategies, which are based on diversification concepts different from MV, still benefit from including both equities and cryptocurrencies (see e.g., Figure 3 and Figure S7 in Supplementary Material S1).

3.4.4 Summary of main results

Table 14 summarizes the key findings across different portfolio strategies, time horizons, and robustness checks. Taken as a whole, the evidence suggests that the influence of cryptocurrencies depends heavily on the chosen strategy, becomes more pronounced over longer time frames, and holds up even when the definitions of equity and crypto universes are broadened.

4 Conclusions

This study investigates the impact of cryptocurrencies on equity portfolio performance by direct inclusion in the investment universe. To address asset selection bias, a large-scale randomized approach is employed. For four basket types (100–0, 90–10, 80–20, 70–30), 10,000 random universes of fixed size are generated, each comprising 40 assets from four equity indices and twelve large-cap cryptocurrencies.

Empirical results reveal that cryptocurrency inclusion does not uniformly enhance portfolio construction. Benefits are pronounced in Risk Parity and Most Diversified Portfolio strategies, mixed in Equally Weighted portfolios, and minimal for Minimum Variance portfolios.

Across most markets and strategies, crypto inclusion yields higher out-of-sample returns and improved risk-adjusted performance, notably in Sharpe, Sortino, and Rachev ratios. Risk Parity and Most Diversified Portfolio see moderate increases in volatility and drawdown, while Equally Weighted portfolios gain returns but face greater risk exposure.

Horizon analysis indicates that the incremental impact of cryptocurrencies is irregular over one-year periods, peaking during 2020-2021. Over three-year horizons, crypto inclusion yields consistently higher ROI distributions than equity-only benchmarks, suggesting that return benefits arise when losses are recoverable over time.

Robustness checks confirm the main findings. Results hold when equities are sampled from pooled universes, indicating market differences do not explain outcomes. Diversified crypto exposure outperforms Bitcoin-only portfolios, implying that high intra-crypto correlations do not undermine diversification benefits. Increased equity-crypto comovement since 2020 weakens cryptocurrencies' role as minimum-variance diversifiers, but they remain relevant for diversification and risk-budgeting allocations.

Summing up, cryptocurrencies rarely benefit strict variance minimizers but enhance performance for investors who prioritize diversification, balanced risk, and longer investment horizons. Turnover suggests limited additional trading from crypto inclusion. However, while turnover is a proxy for transaction costs, it is important to note that the results are shown gross of transaction costs and liquidity frictions and should be interpreted with caution.

Table 14 Summary of the main findings across portfolio strategies, investment horizons, and robustness checks.

Strategy	Effect of crypto inclusion	Risk and crypto exposure	Horizon effect	Pooled equity universe	Bitcoin-only vs. multi-crypto	Overall implication
MV	Marginal improvements; most mixed-basket results remain within the equity-only range.	Slightly higher risk; very low crypto weights (about 2%-3%).	One-year and three-year gains remain limited.	Results are broadly unchanged under pooled equities.	Bitcoin-only and multi-crypto add little.	Weak support for crypto inclusion.
MDP	Strong gains in return and risk-adjusted performance as crypto availability increases.	Moderate increase in volatility and drawdown; crypto weights around 8%-10%.	Short-horizon gains are episodic; longer-horizon gains are clearer and more persistent.	Baseline results are confirmed.	Multi-crypto outperforms Bitcoin-only.	Strong support for crypto inclusion.
RP	Robust improvement in risk-adjusted performance; mixed portfolios often outperform the equity-only benchmark.	Limited increase in risk; crypto weights can reach 15%-20% in the 70-30 basket.	Benefits emerge already over shorter horizons and strengthen over longer ones.	Results remain strong under pooled equities.	Multi-crypto dominates Bitcoin-only.	Most robust evidence in favor of crypto inclusion.
EW	Higher average return and better risk-adjusted measures as crypto availability rises.	More pronounced increase in volatility, drawdown, and crypto exposure.	Short-horizon gains are uneven; three-year gains are more visible.	Positive effects persist, but with higher risk.	Multi-crypto generally outperforms Bitcoin-only.	Beneficial, though less controlled from a risk-management perspective.

Notes. "Pooled equity universe" refers to the specification in which equities are drawn from the combined set of EuroStoxx50, FTSE100, NASDAQ100, and S&P100 constituents after removing duplicates. "Bitcoin-only" denotes the mixed-universe specification in which Bitcoin is the only available cryptocurrency

Future research should address trading costs, liquidity constraints, and market-access frictions. Furthermore, a more comprehensive evaluation of the findings could be conducted by incorporating formal statistical tests alongside percentile-based assessments, exploring alternative cryptocurrency universes and sample periods, and applying bootstrap-based inference for the Sharpe ratio (Ledoit and Wolf 2008) or stochastic dominance tests (Kopa and Post 2015).

A Average values of μ^{out} and σ^{out} with empirical relevance

As described in Sect. 3.3, in Figures 8 and 9 we respectively illustrate the average values of μ^{out} and σ^{out} for each type of basket and strategy across the four analyzed equity markets. In these figures, squares represent the average values of the μ^{out} and σ^{out} for the 100-0 basket, while dots indicate the values for the mixed baskets. A filled dot indicates that the average value of μ^{out} and σ^{out} for the mixed baskets exceeds the 10^{th}-90^{th} percentile band, indicating an empirically meaningful improvement relative to the equity-only benchmark. Conversely, if the values of μ^{out} and σ^{out} falls within the 10^{th}-90^{th} percentile band, the dot is unfilled.

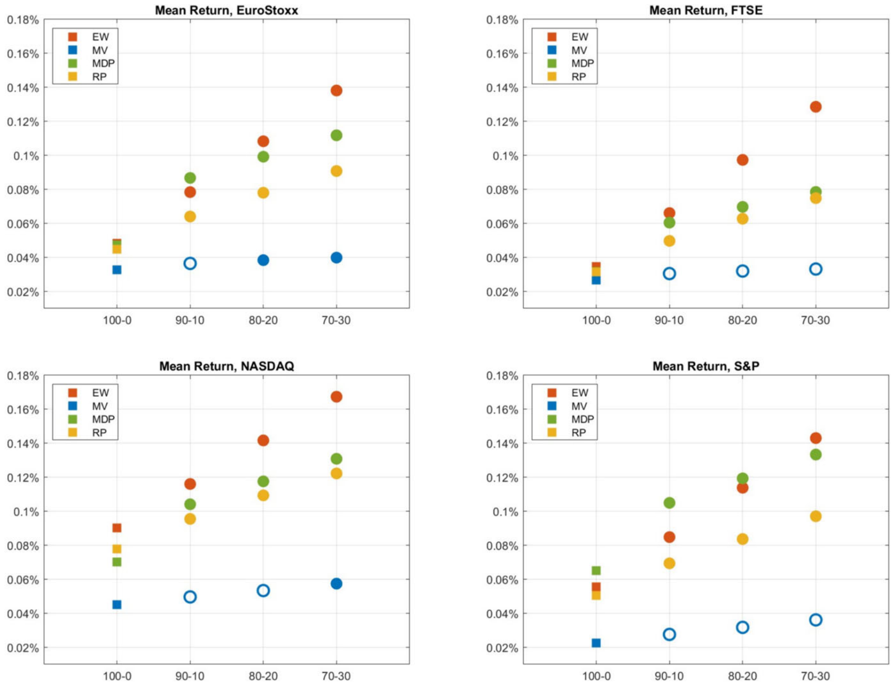

Fig. 8 Average μ^{out} values for the 100-0, 90-10, 80-20, and 70-30 baskets with empirical relevance (based on the equity-only percentile band)

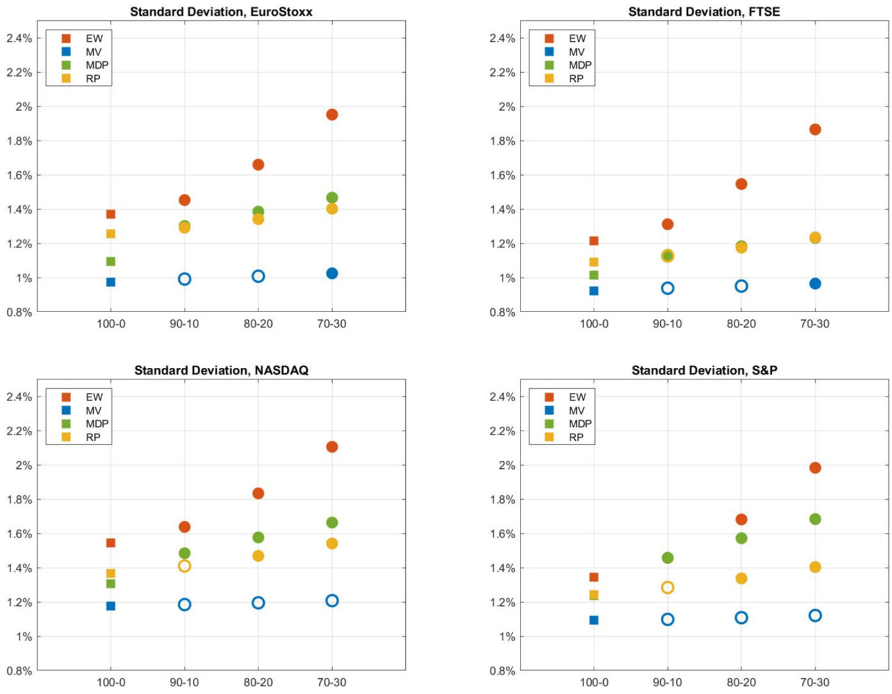

Fig. 9 Average σ^{out} values for the 100-0, 90-10, 80-20, and 70-30 baskets with empirical relevance (based on the equity-only percentile band)

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Declarations

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