Author Accepted Manuscript version of paper
CAPUTO, A.C., FEDERICI, L., PELAGAGGE, P., SALINI, P., On the design of shell-and-tube heat exchangers under uncertain operating conditions, Applied Thermal Engineering, Vol. 212, 2022,
https://doi.org/10.1016/j.applthermaleng.2022.118541

ON THE DESIGN OF SHELL-AND-TUBE HEAT EXCHANGERS UNDER UNCERTAIN OPERATING CONDITIONS

Antonio C. Caputo
Department of Industrial, Electronic and Mechanical Engineering, University Roma Tre
Via Vito Volterra, 62, Roma, 00146, Italy.
acaputo@uniroma3.it

Alessandro Federici
Pacifico M. Pelagagge
Paolo Salini (corresponding author)
Department of Industrial Engineering, Information and Economics, University of L'Aquila,
Zona industriale di Pile, L'Aquila, 67100, Italy
alessandro.federici@graduate.univaq.it
pacifico.pelagagge@univaq.it
paolo.salini@univaq.it

Abstract. This paper addresses the problem of designing shell and tube heat exchangers operating under uncertain conditions. Traditional design methods assume constant process conditions and known value of thermal design factors. Thus may fail to meet specifications when operational conditions change. Use of safety factors or worst-case design often leads to conservative design and equipment oversizing. No guideline is available in the literature for choosing a design approach. In order to provide designers better awareness on this issue, we investigate when uncertainty should be taken into account, what are the advantages and drawbacks of available approaches for design under uncertainty, and which is the preferred method to be applied in specific problem instances. To answer the above research questions, comparative sizing of equipment is carried out under consistent assumptions adopting five alternative design methods; namely design for nominal reference condition with verification of off-design performances, worst case design, use of safety margins, design to optimize a prescribed objective function, and robust design. Either random uncertainty in external process conditions, and epistemic uncertainty in heat transfer coefficients and fouling resistances are considered. Different types of specifications (more/less is better or nominal is better) are accounted for. Probability of meeting specifications and resulting geometrical characteristics are compared. It is the first time that a similar comparison is carried out, and results discussion allows to identify merits and limitations of the considered approaches. It is found that no single approach is superior, as any method can be suited to a design situation but perform poorly in other cases. Justification for explaining the observed behaviour is used in order to provide guidance to designers in choosing the preferable design method for a specific instance resulting in more cost-effective equipment configuration with higher probability of meeting specifications.

Keywords*: heat exchanger design, variable operating conditions, design under uncertainty, robust design.*

1. INTRODUCTION

Shell and tube heat exchangers are the most frequent type of heat transfer equipment utilized in process plants. By providing and removing heat to and from process streams, as well as allowing for heat recovery and energy integration of different process streams, heat exchangers play a key role in effective feedstock transformation while promoting efficient energy utilization and significant resources and economic savings. However, many external and internal sources of uncertainty and variability affect heat exchangers operations in actual plants, making thermal and mechanical design of this kind of equipment rather complex, despite their apparent structural simplicity. Among external sources of variability, either deterministic or random changes of input streams conditions may occur, namely variation in hot and cold stream flow rate and temperature, changes in their composition and uncertainty in the fluids physical properties. Internal sources of variability include uncertainty deriving from imprecise heat transfer and pressure drop correlations, flow maldistribution effects, and fouling phenomena [1].

Sources of uncertainty are often beyond the control of the designer, and could affect different aspects [2]:

- Variations of the environmental and operating condition: whereas during the design phase some quantities could be assumed as deterministic and constant, during the operating time their value could change. This is the case of input parameters and different noise signal that influence the performances of the system.
- Manufacturing tolerances: some design solution can suffer more or less than others the departure from the geometrical theoretical configuration.
- Output evaluation: the model and the equations used in order to evaluate the performances of the equipment are not the exact description of the reality. In fact, assumptions and simplifications which are necessary to reduce the modeling effort could introduce other types of uncertainty and neglect some phenomena.
- Constraint uncertainties: the feasibility space, i.e. the set of the possible solutions, could be smaller than the one theoretically envisaged, due to model incapability to determine the actual feasibility region.

Furthermore, uncertainty can be divided into a) aleatory uncertainty and b) epistemic uncertainty. The former comes from the random variability of the input and it is not reducible, whereas the latter is imputable to non-perfect knowledge of the physical phenomena and unperfect correlations, and it is theoretically reducible.

Traditional design methods ignore variability, in that they assume constant and known values of process conditions, namely temperature and flow rate of input process streams, their composition and thermophysical properties, as well as heat transfer and pressure loss coefficients obtained from empirical correlations. Equipment is designed referring to nominal operating conditions, often assuming average values in case a variability range is expected, and providing ex post verification that minimum heat duty specifications are satisfied when operating parameters are changed within a likely range. Nevertheless. uncertainty propagation and sensitivity analysis in heat exchanger is a topic which gathered some attention from the academics. Statistical methods to understand the behavior of an equipment are known and used since ’50 [3]. In particular, methods for assessing internal parameters uncertainty and the capability of designs to satisfy requirements are available [4]. In order to protect against uncertainty and obtain a desired degree of confidence in satisfying specifications design handbooks suggest additional pressure drop and surface area to be factored in as a safety factor, determined by statistical combination of the variance of uncertain factors [1, 5].

Otherwise, design is made assuming worst case conditions. This may lead to excessively conservative design, with equipment oversizing or failure to meet design specification under some operational conditions.

To counter uncertainty in a reactive manner, one could design the equipment incorporating some structural flexibility, to be used if and when necessary (i.e. a bypass stream or additional heat transfer area to be put into service on request). However, this may be often unpractical given that heat exchangers are designed with a predefined structural architecture when the geometrical parameters are fixed in the design phase, and can not be changed during operation. As an alternative, some kind of equipment regulation and control could be allowed. Another approach could be to design allowing output variability, i.e. allowing operational states where specifications are not strictly met, provided that some performance measure is optimized.

When the above options are not feasible, the goal of designing a robust heat exchanger may be sought. Robustness is the ability of the equipment to maintain its operational capability and meeting design specification under different operating circumstances. Technically it involves a design able to optimize the mean response and reduce variability of the output in the face of input variability and noise factors by preventing variability propagation rather than reducing the variability of the input [6]. Therefore, robustness is a manner of passively managing uncertainty by choosing an equipment architecture which ensures adequate performance over a wider range of conditions [7]. However, while methods to assess the effects of uncertainty propagation in heat exchangers performances are available in the literature, methods for robust design of heat exchangers under uncertain operating conditions are scarce.

Overall, as shown in the subsequent literature review, optimal heat exchangers design methods, in general, neglect variability and uncertainty. Manual methods, instead, address uncertainty in empirical manners or adopting penalizing approaches such as design for average conditions, worst case design or oversizing, but no sound justification is offered to support their utilization. Papers addressing uncertainty in heat exchangers mostly focus on uncertainty propagation instead of design method to counter uncertainty. The designer is left without guidance about how to deal with uncertainty. Therefore, a knowledge gap exist as far as equipment design under uncertainty is concerned.

In order to contribute to a solution to this relevant design problem, in this paper an attempt is made to answer the following research questions: should uncertainty always be taken into account when designing heat exchangers or can it be neglected? What are the available approaches for design under uncertainty and what are their advantages and limitations? Which is the preferred method to be applied in specific problem instances?

The paper is organized as follows. The literature about optimal heat exchanger design is reviewed at first, including the case of uncertainty quantification and equipment design under uncertainty. Subsequently the problem of heat exchangers design under uncertainty is stated, identifying five possible alternative approaches including robust design. Afterwards, a numerical experiment is carried out to compare the five methods on a consistent basis, and to analyze their performances, applicability domains and limitations. This is the first time that alternative methods for heat exchanger design under uncertainty are compared, and another novelty consists in examining the possibility of designing robust shell and tube heat exchangers adopting a metaheuristic optimization method based on a Genetic Algorithm (GA). In this case the GA is used to determine the required equipment configuration to minimize the distance of output stream temperature from the specification. Guidelines to help designer in selecting the best approach according to the specific problem instances are derived. The paper is concluded by discussing its limitations and planning future research work.

2. LITERATURE REVIEW ON HEAT EXCHANGER DESIGN, OPTIMIZATION AND UNCERTAINTY MANAGEMENT

The traditional design approach for heat exchangers assumes stationary operating conditions. An iterative procedure is usually followed where the designer, on the basis of experience, changes the value of design variables until a satisfactory equipment configuration is found which meets the specifications while satisfying commonly accepted rules of thumbs deriving from good engineering practice [8]. Details on this approach are available in textbooks [9] and reference handbooks [1, 10]. Design case studies and practical numerical examples are also available in case of shell and tube exchangers [11] as well as for other equipment configurations [12, 13]. Computer-aided tools for equipment design are available on the market but are still relying on manual inputs and based on the above iterative approach which is time-consuming and does not guarantee an optimal solution [14].

Numerical design methods, based on optimization algorithms, have been recently made available and are reviewed elsewhere [15, 16]. Numerical methods have the capability of handling both discrete and continuous variables as well as of efficiently searching the multidimensional design space. Nevertheless, while a number of approaches have been proposed to automate and optimize the heat exchangers design, several issues still remain to be solved. For instance, most of computer aided optimal design procedures assume steady state operations thus neglecting either deterministic and stochastic variability in the operating conditions. The uncertainty in the heat transfer coefficients correlations is also neglected, and penalties for off-design performances are usually not included in the evaluation of economic objective functions. As an alternative, in case of changing operating conditions, a worst case design is attempted, or heat transfer coefficients are corrected through arbitrary fouling factors, thus penalizing performances when average operating conditions apply. Finally, the cost correlations used for economic optimization are not sensitive to the constructive details of the equipment as determined by the design algorithm, but only to the overall heat exchange area. This makes them unsuitable for detailed equipment optimization. This was pointed out by Caputo et al. [17] who also develop more detailed cost estimating models. As a consequence some authors [18] raise concerns about the use of apparently precise computer based methods, given the inherent fuzziness of the problem and the uncertainty in operating conditions or in the adopted design correlations.

On the other hand, design of processes under uncertainty is a research field in itself, extensively studied from a long time [19] especially in the area of chemical process plant design and process synthesis, as reviewed elsewhere [20, 21].

However, Costa and Bagajewicz [22] point out that despite advances in computational tools the basic design of process equipment is still carried out in industry resorting to trial-and-verification procedures guided by heuristic rules. This is mainly imputable to the circumstance that the academic literature about process design under uncertainty is dominated by mixed-integer nonlinear models (see for instance[23, 24]) which, regardless form the utilized solution techniques (i.e. stochastic or mathematical programming-based), have practical limitations that have restrained practitioners from abandoning the time tested heuristics and simplified computer-aided tools.

Nevertheless, the literature specific about heat exchanger design under variable operating conditions and parameters uncertainty is comparatively scarce, and mainly focused on either assessing the equipment performance variation under uncertainty [25], the effect on designs [5] or suggesting guidelines to account for variability [4], rather than in providing optimal design solutions and methods. However, it can be argued that general methods developed for optimal probabilistic design of processes could be applied as well to equipment design. In particular, Haseler et al. [26] as well as Clarke et al. [27] investigated the sensitivity of the overall heat exchanger calculations and operational performances to uncertainties in individual fluid properties, also using Monte Carlo techniques, but they did not consider the effects of uncertainties in process specifications, and did not addressed any optimization problem. This kind of analysis was also extended to cross-flow heat exchangers[28]. The Monte Carlo method was also used by Affan Badar et al.[29] and Mahbub Uddin and Bell [5] to appraise the impact of uncertainties on heat transfer coefficient. Prasad and Bharadwaj [30] analysed the performance of a counterflow concentric tube heat exchanger estimating the uncertainties in the temperature response caused by uncertainties in the fluids inlet temperatures and the overall heat transfer coefficients using a two-point distribution technique. Kayansayan [31] developed an analytical method to estimate thermal behaviour of heat exchangers in off-design conditions. Knetsch and Hauptmanns [32] investigated the dynamic response of heat exchangers under process and parameters uncertainty resorting to a Monte Carlo variability propagation. Lambert and Gosselin[33] explored the impact of correlations uncertainty on heat exchangers sizing and cost. Bounds on thermal systems performances when operating in uncertain conditions were investigated by Taylor, Hodge and James [34]. Abdelaziz and Radermacher [35], investigated the effect of manufacturing tolerances and flow maldistribution on the performance of compact heat exchangers. Specific investigation on uncertainties in heat exchangers performances owing to fouling phenomena have been carried out by Khan and Zubair [36, 37], Zubair and Qureshi [38], while Yeap et al. [39] develop specific heat transfer and pressure drop fouling models to allow estimation of cost-effective extra area to heat exchangers. Lemos et al. [40, 41] included velocity dependent fouling factors and a threshold fouling model into a design optimization procedure.

From the design point of view Buckley [3] was the first to use a statistical approach to the sizing of process equipment. Starting from a design based on nominal values of design parameters, and assuming a normal distribution of uncertain parameters, he combined the impact of the standard deviation of each variable parameter to determine the overall variability of required exchanger heat transfer surface area. Then fixed the oversizing level in order to obtain a desired confidence level. A similar design approach based on specifying the probability of meeting a prescribed duty in case of process uncertainty, using the root sum squares method, is described by Cho [4] and Affan Badar et al. [29], but without attempting an optimal solution. As far as optimal exchanger design under uncertainty is considered, Caputo et al. [42, 43] optimized shell and tube exchangers design considering variability in the input streams conditions (flow rate and temperature) or uncertainties in the heat transfer coefficient, adopting an economic objective function. Saldanha et al. [44] used multi-objective particle swarm optimization and scenario construction to optimize shell and tube heat exchanger considering uncertainties in the tube-side inlet and outlet temperatures, pressure drop factor, friction factor, Nusselt number, as well as shell-side heat transfer coefficient and pressure drop while the objective function minimized heat transfer area and pressure drop.

Finally, a risk assessment method to evaluate the consequences of process uncertainties is provided by Shilling et al. [45]. An application of robust design of a heat exchanger in the automotive sector is given by Arner [46]. Design under uncertainty was also applied to heat exchangers network, HEN,[47]. More recently Al Khulaifi and Al Mutairi [48] developed a method for optimal synthesis of HENs while Floquet et al. [49] presented a method to assess the robustness of a HEN.

Overall, the literature review shows that the research on heat exchanger design has been more focused on assessing the impact of uncertain parameters and estimating uncertainty propagation. This allows to rate the goodness of an equipment configuration but does not directly help in designing the exchanger, The established research stream on optimal heat exchanger design generally neglects variability and uncertainty. Statistical design methods aimed at ensuring a desired confidence level on expected performances have been suggested, but empirical manual methods based on nominal condition design with verification for off-design conditions, or empirical techniques based on worst case conditions, safety factors and oversizing are used in practice. No justification for such empirical method has been elaborated.

Methods for optimal stochastic design of heat exchangers are quite scarce. Overall, the heat exchangers design community lacks a consistent comparison of design methods addressing uncertainty, and guidelines helping to choose the proper design method. This paper attempts to give a contribution to fill this knowledge gap.

3. PROBLEM STATEMENT OF HEAT EXCHANGERS DESIGN UNDER UNCERTAINTY

Propagation of uncertainty and design of engineering system under uncertainty is a well known problem. De Rocquigny provides a general method for quantitative uncertainty management [71] while application examples are discussed in [50]. Recent advances in uncertainty quantification are reviewed in [51]. Some authors focused their research on large systems [52], characterized by a large number of components. As shown in Figure 1 variability associated to input variables, random events and to internal parameters uncertainty, propagate through the system making the output performance measures variable as well. The output variability may exceed some acceptability range, bounded by predefined performance levels thresholds. As an example [32] when a stream feeding a catalytic reactor exceeds a maximum threshold the catalyst may be damaged, while if it drops below a minimum threshold the required quality of the effluent is not guaranteed. Risk is a quantification of the system output deviation from the expected value in term of frequency of occurrence and magnitude of consequences.

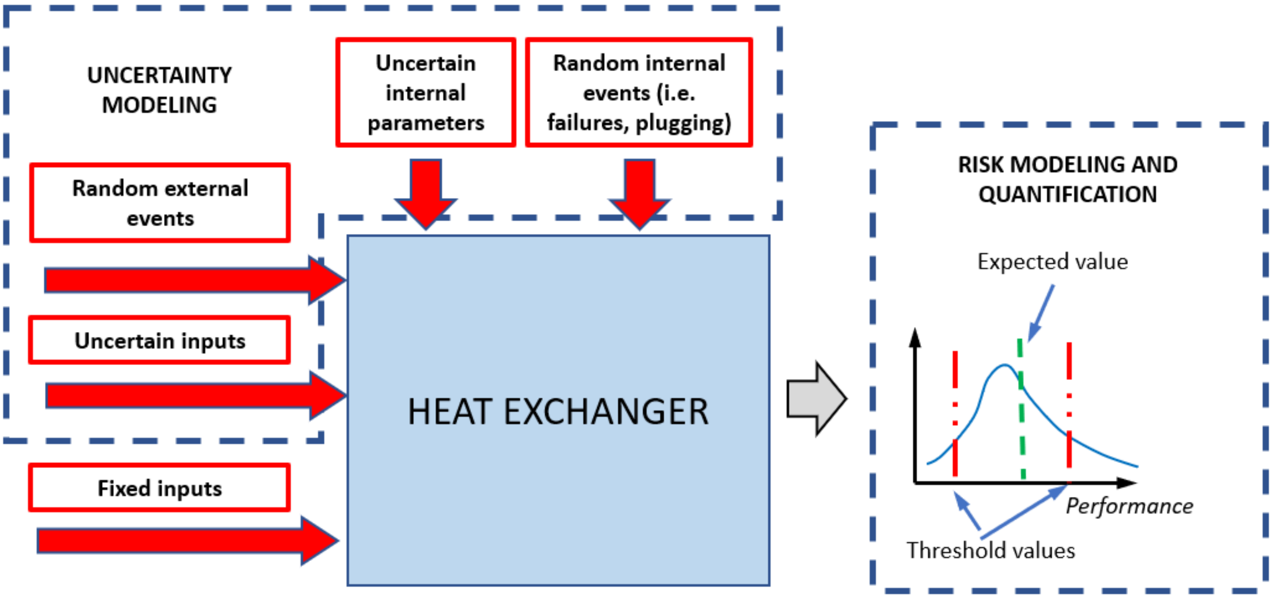

Figure 1 Conceptual schematization of system response under uncertainty.

One possible general formulation of a process design under uncertainty [53] is

$$\begin{aligned} Max \quad & f(u,q,x,\theta) \\ s.t. \quad & e(u,q,x,\theta)=0 \\ & g(u,q,x,\theta)\leq 0 \\ & g_q(y,y^*)\leq 0 \\ & u\in U, q\in Q, x\in X, y\in Y, \theta\in\Theta \end{aligned} \tag{1}$$

where f is a scalar function representing the performance measure to be optimized, u, q, x, θ are the vectors of design, control, state variables, and uncertain parameters respectively; y is the vector of quality related variables, having desired values $y*$, while e represent the model equations, g_q the quality constraints and g other constraints.

According to the literature [7, 54] there are basically three methods to protect a system against uncertainty, namely a) reducing and controlling input variability; b) active protection by making the system flexible. i.e. able to adapt to changing conditions [55]; c) passive protection so that the system is able to withstand the influence of uncertainty. This, in turn, may be pursued resorting to two opposite approaches, namely c1) including reserves and redundancy in system design (i.e. oversizing or designing for the worst case), as a cushion to buffer variability, or c2) building robustness in the system as means of reducing propagation of uncertainty from input to the output. Robustness criteria can be incorporated in the design problem by penalizing the output including a quality cost term [53], based on the offset between the expected value and the nominal one, and on the variability around the expected value. This can be made for instance through penalty functions, such as Taguchi's loss functions (which typically account for nominal-the-best, larger-the-better and smaller-the-better conditions), or by imposing explicit restrictions over robustness metrics of quality related variables such as their variance or other moments [6, 56]. Robust optimization problems are reviewed for instance by Bertsimas et al. [57], Gorissen et al. [58], Dellino et al. [59] and Beyer and Sendhoff [2], while advances in sensitivity analysis are resumed by Borgonovo and Plischke [60].

In case of heat exchangers solution a) is often unfeasible. In fact, although many industrial processes are designed to operate in steady state conditions, as many other processes instead work in intrinsically unsteady conditions owing to periodic or batch operations. Moreover, even processes designed to operate under steady state conditions are subject to random perturbations or occasional changes of production rates, and need to be associated to a control system able to regulate process parameters, limiting their excursion respect target values. Furthermore, auxiliary plants, such as heat recovery systems, are often associated or slaved to process plants, but are not strictly regulated as this could interfere with the operations of the main process. Therefore, it is frequent that such heat recovery systems, utilizing heat exchangers as the means of energy transfer, are fed by sources with a time-varying energy content. Moreover, fluid composition and, consequently, physical properties are subject to changes during operation which are difficult to estimate. From the design point of view equipment designers use empirical correlations to estimate heat transfer and pressure drop coefficients, or fouling rates, which have an intrinsic uncertainty range (often up to ±15% error range), while the designer may not oversee fabrication to ensure that built equipment is consistent with specifications and complies with stated tolerances. Finally, the time trend of equipment degradation during operation, i.e. accumulation of fouling deposits which affect fluid dynamics and heat transfer, is difficult to estimate and influenced by external events such as timing of maintenance cycles.

Moreover, the performance variations of a heat exchanger when the values of inlet process variables change are not always intuitive. For instance, let us assume that the mass flow of the hot fluid increases whereas, for sake of simplicity, the cold fluid has constant mass flow and inlet temperature. In this case, being fixed the equipment architecture, the hot fluid heat transfer increases but, at the same time, the thermal power to be removed increases too. If the hot fluid were the controlling one, the former statement would suggest that the hot stream output temperature will decrease and the latter that it will increase. The coexistence of the two phenomena, with their own influence ad magnitude, may determine a non linear trend in the output of the heat exchanger. Overall, the possible non linear response of the equipment makes it strongly susceptible to uncertainty in process conditions as far as performance estimation is concerned, and small simultaneous variations of process variables may have a relevant amplification effect. This, apart from making the behavior of the equipment not readily intuitive, may prevent from meeting design and operational specifications.

As far as strategy b) is concerned, it is not feasible to build heat exchangers with a modular or changeable structure in order to gain operational flexibility. Therefore, the active uncertainty protection frequently adopted is regulation through installation of a control system [61, 62]. However, while this represents an added cost and an increase of system complexity, the range of allowed stream regulation may be somewhat limited.

For instance, considering that the transferred heat may be expressed as $Q \sim S\ U\ LMTD$, the most frequently applied regulation strategies try to modify the effective log-mean temperature difference $LMTD$ by acting on the flow rate of the controlling stream, or modify the overall heat transfer coefficient U by acting on the flow rate of the controlled stream. Another option is to bypass a portion of the controlled stream flow rate and letting it mix downstream the exchanger with the remaining portion of the stream which passed through the equipment. In case that the controlling stream is a condensing fluid a fourth control method would be to allow partial condensation on the tube bank in order to modify surface area S.

In greater detail, when a service fluid is available and the heat exchange is between a process and that service fluid, without phase change, three following main regulation strategies are possible. However, it must be pointed out that only one of the two outlet temperatures may be controlled. In fact, once the inlet and outlet temperatures and mass flow of one fluid are assigned, the heat exchanger duty is determined.

Let be t_{PI} and t_{PO} respectively the process fluid inlet and outlet temperature, t_{SI} and t_{SO} respectively the service fluid inlet and outlet temperature, and m_P and m_S the corresponding mass flows, being t_{PO} the process variable to be controlled. A first control strategy (Figure 2) is to measure the process fluid outlet temperature by a temperature indicator (TI) and adjust its value by acting on a valve able to manipulate the mass flow of the service fluid.

Modifying m_S, the service fluid convective coefficient and the outlet service fluid temperature change, determining new values for the driving forces and t_{PO}. Even if the control scheme shown in Figure 2 is frequent, when the service fluid convective coefficient is not the controlling one, the change in the overall heat transfer coefficient is very small and the regulation may be not effective.

A possible alternative is the regulation scheme shown in Figure 3, where the measurement on the controlled variable determines a modification of the process fluid mass flow rate. In this case, if an excessively high value of t_{PO} is measured, the control valve is opened. Consequently, a bigger m_P flows through the equipment, and the output temperature reduction is obtained as a net result of the (possible) increment of overall heat transfer coefficient and the reduced heat amount transferred to the unit of mass flowing. Generally, the output temperature change per unit change of mass flow is small and may be not linear.

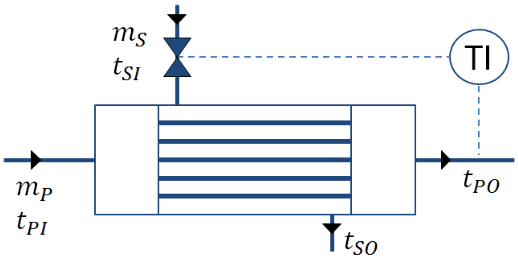

Figure 2 Regulation on service fluid mass flow

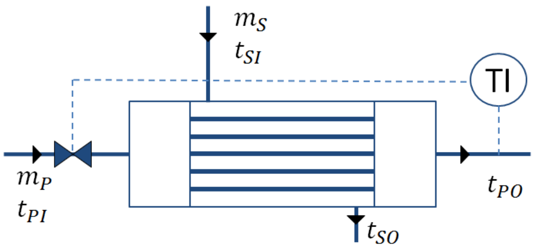

Figure 3 Regulation on process fluid mass flow

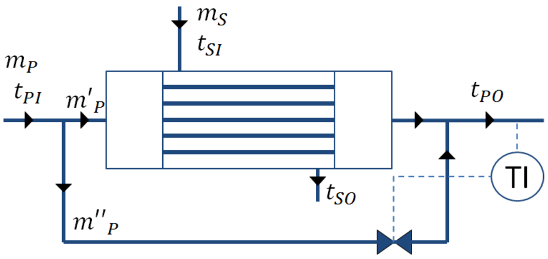

Figure 4 By-pass control scheme

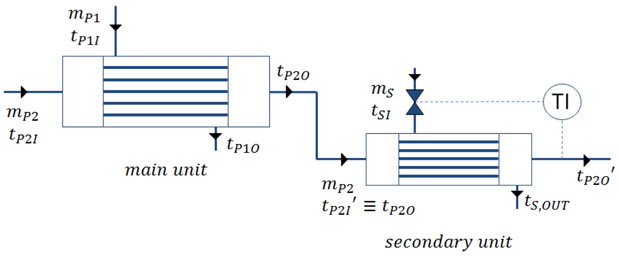

Figure 5 Regulation scheme for process/process heat exchangers

The third control strategy is depicted in Figure 4, where the heat exchanger is overdesigned. At the exit of the equipment the process flow temperature is higher (if the unit has to warm up the process fluid) or lower (if the unit must reduce the temperature of the process fluid) respect the specified value. The prescribed value of t_{PO} is, then, achieved by blending the main stream passing through the heat exchanger and a by-pass stream of the same process fluid.

A different scenario is also possible when the heat exchange is between two process fluids for energy saving purposes. Namely, the first fluid has mass flow rate m_{p1} and the second one m_{p2}, while the inlet temperatures are t_{P1I} and t_{P2I} respectively (see Figure 5).

In this case both outlet temperatures are relevant and the action on one of the two mass flows only is not always possible. This is why in some cases the system is made by two, or more, heat exchangers in series. The first one is the main unit and couples the two process steams obtaining output temperatures which are different from the respective specified values. Secondary units are disposed in series to the main unit in order to reach the specified output temperature resorting to a service stream. In the simple scheme of Figure 5, the main unit is actually without any control system and the temperature of the mass flow rate m_{P2} is tuned resorting to the secondary unit.

As a concluding remark, the actual variations of operating conditions cannot be neglected when designing a heat exchanger expected to operate in non stationary conditions, irrespective from the installation of a process control system.

As far as solution c) is concerned, alternative c1) is most often practiced either by directly increasing the surface area or, indirectly, by increasing the design fouling factors or lowering the adopted heat transfer coefficients. Nevertheless, alternative c2), i.e. design a robust exchanger which can accommodate changes in input streams and uncertainty in internal parameters without penalizing the output performance in excessive manner, would be an attractive solution and possibly superior to the alternative of using extra surface area and allowing for extra pressure loss.

Based on the above discussion it can be stated that the utilized methods for heat exchanger design under uncertainty either fall into one of the following categories.

Method 1) Neglect uncertainty. The exchanger is designed for nominal conditions and uncertainty is neglected. Polley and Pugh [63] in this case suggest to size the equipment according to nominal operating conditions, i.e. referring to a nominal “design point” and then evaluate the responses of the exchanger to changed conditions. In case the obtained performances fall outside an accepted interval, the “design point” is changed in order to obtain an alternative equipment configuration. However, one problem of a design based on average values of input conditions derives from the consideration that excursions of input process parameters above and below the nominal value will not be compensated, as the same percent changes of opposite sign in input variables are likely to determine output effects having different magnitude owing to the possible nonlinear behavior of the equipment. The basic conceptual flaw inherent in designing for average input values is a well known example of the so called Jensen's Law [64], also known as the "Flaw of Averages", which, roughly speaking states that, exception made for linear functions, the average of all the possible outcomes associated with uncertain parameters is generally not equal to the value obtained from using the average value of the parameters" [65]. Finally, it should be also noted that the economic effect of unmet specification in the value of a controlled process parameter may be quite different in case the specification is not met in excess or in defect of the nominal desired value. Furthermore, if changes of input parameters above and below their respective average value have not the same probability of occurrence their impact can be even stronger. For instance, in a heat recovery contexts a heat recovery amount higher than the nominal value could be useless while one lower than the nominal value would shut down the process or require an auxiliary heating. Imagine, for instance, the case where a hot stream should be cooled at least to a threshold temperature level T_{th}. Then exceeding the threshold (i.e. having an output temperature $T < T_{th}$) does not have a significant economic benefit, while not meeting the target cooling level (i.e. having $T > T_{th}$) might imply additional cooling expenses. This means that deviations of the same magnitude but different sign respect the average may have different economic value, so that the economic effects do not compensate by designing the equipment according to average process values.

Method 2) Design for worst case. It is a classic example of “overdesign”, where the reference design point is chosen assuming the (most credible) worst conditions likely to be expected. When the majority of time average operating conditions instead of extreme ones are experienced, this design may prove to be uneconomical and unsatisfying from the thermal performances point of view. It is important to highlight that the worst case definition is not straightforward when several physical quantity are involved and they are interdependent. For example, the growth of process fluid mass flow rate increases the quantity of heat to be transferred but, at the same time, increases the velocity of the fluid and consequently the heat transfer coefficient. Therefore, the worst case definition is difficult under uncertainty.

Method 3) Accommodate uncertainty through the use of design margins. This alternative overdesign method implies designing for the nominal design point but adding some “design margin”, i.e. addition of extra surface area [63]. A proper way of adopting a safety margin is to obtain the probability density function of the heat transfer area required to meet specifications under the various operating conditions, often assumed to be Gaussian, and then specifying the oversizing level in terms of multiples of the standard deviation in order to obtain a prescribed probability of satisfying the specifications [3, 4]. Nevertheless, in practice this merely means that the heat exchange area computed for nominal design conditions, neglecting uncertainties, is multiplied by a safety factor (from 15% to 100%) chosen relying on designer's experience ([1], page 193). This is rather equivalent to design for worst case conditions, but it may have strong drawbacks under the financial and operational point of view. For instance, in the words of Polley and Pugh [63] “It can give rise to control problems. It can give rise to operability problems. It can promote fouling. It can result in under-performance of heat exchanger networks.”

Method 4) Design to optimize a prescribed performance measure. In this method the output variability is accepted, but equipment is designed in order that, considering the actual performances resulting from off design operation, a specific goal is met. This in general is obtained by optimizing an assigned objective function (i.e. the net economic value including capital and operating costs, value of economic performance *EP* of exchanged or recovered energy and possibly penalties for not meeting specifications).
For instance, such an objective function (OF, €/yr) could be expressed as the net economic result

$$OF = \pm C_I \tau \pm \sum_{j=1}^{N} P_j C_{en} H p_j \pm C_{pe} \pm \sum_{j=1}^{N} C_{TE}\left(t_{jo}\right) \Delta T_j m_j C_p H p_j \quad (2)$$

where the signs of the various terms depend on whether a minimization or maximization of OF is sought and whether the economic value of exchanged heat, represented by the latter term at right hand member, is a revenue or an operating cost. In the above equation C_I is the capital investment, τ the capital recovery factor, j $\in$ (1 to N) the index representing the j-th operating condition, C_{en} (€/kWh) is the electric energy unit cost, P_j the required fluid pumping power in the j-th operating condition, H the yearly operating hours, p_j the probability of the j-th operating condition, C_{pe} a penalty value in case any of the design constraint is violated, $C_{TE}(t_{jo})$ a user-defined function specifying the unit value of exchanged energy (€/kJ) as a function of the actual outlet temperature t_{jo} of the reference stream in the j-th operating state, ΔT_j the difference between t_{jo} and the desired goal temperature for the reference outlet stream in the j-th operating state, m_j the mass flow rate of the reference stream in the j-th operating state, C_p the constant pressure specific heat of the stream.
Adopting this approach, Caputo et al. [42, 43] optimized shell and tube exchangers design considering either deterministic as well random changes in the input streams conditions (flow rate and temperature) and uncertainties in the overall heat transfer coefficient. In both paper a GA optimization algorithm used previously for optimal design in deterministic conditions [66] and to include fouling effects [67] was utilized. This approach proved to be superior to Methods 1, 2, and 3.

Method 5) Design for robustness. This means designing an exchanger with the explicit goal of minimizing the variability of the actual output respect a reference nominal output when factoring in changes in the input streams process conditions and/or internal parameters variations. The first general mathematical formulation of robust design was done by Taguchi introducing a quadratic loss function, giving a quantitative interpretation of the quality cost. Firstly, he remarked the importance of the type of specification (bilateral or unilateral), and then gave three different formulations for the computation of the expected quality loss: nominal is better (eq. 3), more is better and less is better. While the first considers both the standard deviation and the mean of the quantity of interest, the remaining two formulations consider only the distances of the mean from the specification.

$$E(L) = k[(y_m - y_t)^2 + \sigma^2] \quad (3)$$

where E(L) is the expected economic quality loss, k is the quantification of the economic value of quality, y_m is the mean value of the output variable of interest y, y_t is the target value and σ is the standard deviation. Robust design can act on the output performances of interest in three different ways: *a)* reducing the reducible uncertainty, *b)* changing the slope of the transfer function that determine uncertainty propagation from the input to the output and *c)* when the transfer function is non-linear, moving the design point on a flatter portion of the curve (Figure 6).

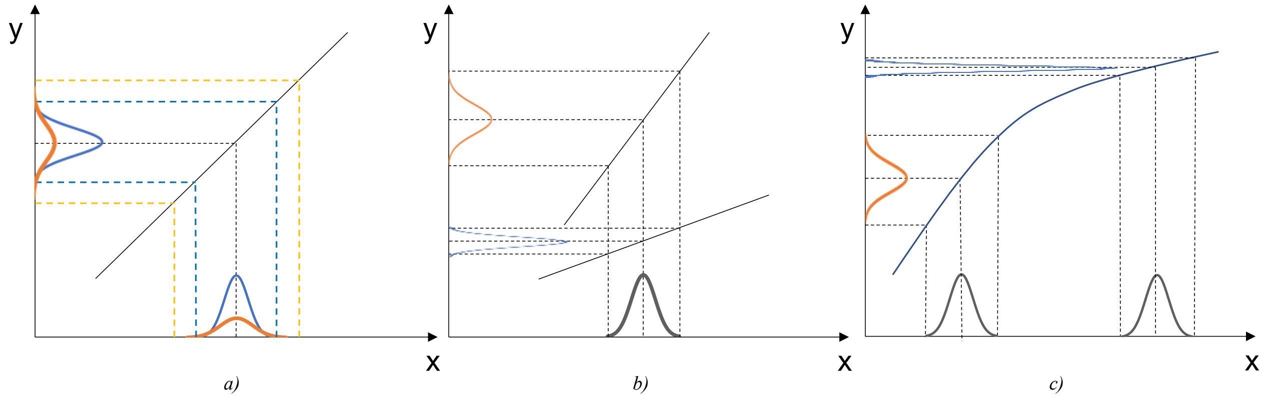

Figure 6 Robust design uncertainty mitigation: a) reducible uncertainty reduction, b) slope changing and c) move on a flatter portion of the curve

For heat exchangers, specification may be assigned in three different ways:

- single value of outlet temperature is used when uncertainty is not considered, and the problem is formulated in deterministic manner. In this case uncertainty is not allowed;
- more/less is better, meaning that the specification is achieve if the outlet temperature is at least equal to a defined value, and the lower (or higher) the best. The uncertainty may be considered;
- nominal is better, meaning that the specification is met when the outlet temperature falls in a specified range around a defined value and the closer the value is on the target the better. The uncertainty may be considered.

In robust design literature the design problem is thus formulated in the following manner [68-70] (eq.4).

$$
\begin{array}{lll}
\text{find} & x & \\
\text{minimising} & \{E(f(x,y)), \sigma(f(x,y))\} & \\
\text{subject to} & E(g_i(x,y)) + \beta_i \sigma(g_i(x,y)) \leq 0 & (i=1,2,\dots,k) \\
& \sigma\left(c_j(x,y)\right) \leq \sigma^{+}{}_j & (j=1,2,\dots,l) \\
& x^{-}{}_a \leq x_a \leq x^{+}{}_a & (a=1,2,\dots,n)
\end{array}
\tag{4}
$$

E(f), that is the expected value of the performance function, and its standard deviation $\sigma(f)$ are both minimised. β_i is a flexibility index for the i-th constraint, that represent the maximum acceptable probability of not satisfying the i-th constraint, and c_j is the structural performances function that imposes an upper limit on standard deviation at $\sigma^{+}{}_j$.

A different formulation which assigns different weights to minimizing the mean and the standard deviation in the objective function is also proposed by Doltsinis et al. [68] (eq.5).

$$
\begin{array}{lll}
\text{find} & x & \\
\text{minimising} & f = (1-\alpha)E(f(x,y))/\bar{x} + \alpha\sigma(f(x,y))/\sigma^{*} & \\
\text{subject to} & E(g_i(x,y)) + \beta_i \sigma(g_i(x,y)) \leq 0 & (i=1,2,\dots,k) \\
& \sigma\left(c_j(x,y)\right) \leq \sigma^{+}{}_j & (j=1,2,\dots,l) \\
& x^{-}{}_a \leq x_a \leq x^{+}{}_a & (a=1,2,\dots,n) \\
& 0 \leq \alpha \leq 1 &
\end{array}
\tag{5}
$$

However, no research explicitly focused on a formally robust exchanger seems to be available.

4. COMPARISON OF ALTERNATIVE APPROACHES FOR HEAT EXCHANGER DESIGN UNDER UNCERTAINTY

In order to compare in a consistent manner the above five design methods they will be applied to a common case study. However, two different problem instances will be considered to focus on both random and epistemic uncertainty. Case study 1 will examine the circumstance where uncertainty exists in the input streams conditions (i.e. external random variability), while Case 2 will examine the impact of uncertainties in the design correlations (i.e. internal epistemic uncertainty).

In Case 1 we assume that a countercurrent heat exchanger has to be designed to cool a process fluid, and that nominal flow rates, temperatures and thermophysical properties of both fluids are as shown in Table 1.

Table 1 Nominal conditions and thermophysical properties of the two fluids for the Case 1.

Fluid	m (kg/s)	t_i (°C)	t_o (°C)	ρ (kg/m^3)	Cp (kJ/kgK)	μ (Pa s)	λ (W/m K)	Rf (m^2 K/W)
methanol	25	95	40	750	2.84	0.00034	0.19	0.0002
sea water	74.38	27.5	40	995	4.2	0.0008	0.59	0.00033

Although in literature it is frequent the use of an economic objective function, here we will only consider the thermal performance as objective, while economic evaluation will be used only to compare the obtained alternative design solutions.

For sake of cost comparison, the total discounted heat exchanger cost is computed as sum of investment cost (Eq.6) and present worth of pumping cost (DOC, Eq.7). Economic values and parameters for cases study are shown in Table 2. For symbols meaning please refer to the nomenclature at the end of the paper.

$$IC = a_1 + a_2 S^{a3} \tag{6}$$

$$DOC = \sum_{n=1}^{Ny} \frac{\left[\left(\frac{1}{\eta}\right)\cdot\frac{\left(\frac{m_T}{\rho_T}\right)\cdot\Delta P_T+\left(\frac{m_S}{\rho_S}\right)\cdot\Delta P_T}{1000}\right]\cdot Cen\cdot H}{(1+I)^n} \tag{7}$$

Table 2: Parameters used to assess the investment and operating costs.

a_1 (€)	a_2 (€/$m^{2\cdot a_3}$)	a_3	η	H (h/y)	I (%/y)	Ny (y)	Cen (€/kWh)
8000	259.2	0.91	0.7	7000	10	10	0.12

In order to evaluate the impact of aleatory uncertainty on equipment design, both *more is better* (MisB) and *nominal is better* (NisB) specifications are compared. Aleatory uncertainty of inputs has been modeled as shown in Table 3, assuming that all variables have a normal probability density distribution

Table 3 Probability distributions of flow rates and temperatures of inlet fluids.

	t_{hi} (°C)	m_h (kg/s)	t_{ci} (°C)	m_c(kg/s)
E[x]	95	25	27.5	75
σ	2.14	0.83	1.25	0.83
CV	0.02	0.03	0.05	0.01

where the operator E[x] is the expected value of each *x-th* process parameter labelled in the columns, σ is the standard deviation, and CV = σ/E[x] is the coefficient of variation.

The design campaign has been carried out as follows:

- *1. nominal design* is done mimicking the traditional design process manually changing the following design variables, outer diameter of tube, fluids side pass, shell diameter, baffle spacing, number of tube side passes and tube pattern, until all constrains are satisfied and suitable cost reduction is obtained;
- *2. worst case design* scenario is carried out considering the lowest logarithmic mean temperatures difference and the higher value of duty;
- *3. safety factor design* is carried out considering a safety factor with a value of 1.5. The value has been chosen to reach a probability approximately 100% of meeting specification for more is better design. However, in the following even the values of 1.25, and 1.75 will be considered in order to perform a sensitivity analysis.
  The effect of safety factor use is an increase of exchanger's length and consequently of heat exchange surface;
- *4. objective function optimized design*. In this case two alternatives are considered depending on *NisB* or *MisB* specifications. For NisB case, the objective function OBJ is to maximize the probability of obtaining a temperature in a range ±2 °C across the specification (Eq.8). For the MisB case, the adopted objective function is to maximize the probability to have an exit temperature lower than the specification (Eq.9).

$$OBJ = \max\ \mathrm{P}\left[(t_o - 2) < Tout_i < (t_o + 2)\right] \quad (8)$$

$$OBJ = \max\ \mathrm{P}\left[Tout_i < t_o\right] \quad (9)$$

where P[..] is the probability of obtaining the wanted result, $Tout_i$ is the actual outlet temperature when scenario *i* is under analysis and t_o is the specification.

5. robust design. Regardless of traditional formulations of robust design, which usually include in the objective both the average and standard deviation of the performance measure, here the objective is pursued minimizing the expression in Eq.10, i.e. centering the outlet temperatures distribution on the specification and reducing its standard deviation. Moreover, although usual formulation for robust design resorts to economic penalties to asses a solution, here no economic penalty is included in the objective function to guarantee more consistency with the other design methods. In fact, here we are more interested in minimizing the distance from the specification regardless of economic consequence of unmet specification.

The proposed objective function (OBJ) simultaneously minimizes the difference between the mean of outlet temperature and the specification and its standard deviation

$$OBJ = \min \sqrt{\sum_{i=1}^{ns} (Tout_i - t_o)^2 \cdot p_i} \quad (10)$$

where ns is the number of considered scenarios, and p_i is the probability of the *i-th* scenario.
This novel formulation differs from the traditional Taguchi formulation which separately minimizes deviation and standard deviation resorting to penalty cost factors. As compared to other objective functions which can be found in the literature, the rather simple structure of the chosen function is justified considering that the goal of this design approach is to reduce uncertainty propagation, which merely implies to minimize output variability in the face of input variability.

In order to globally evaluate the behavior of obtained designs, each configuration has been assessed in a set of possible operating conditions including all possible permutations of discretized probability density functions of each aleatory process quantity (m_h, t_{hi}, m_c, t_{ci}).
To build the design scenarios a six-sigma truncated Gaussian distribution is used and five equally spaced samples are taken thus generating 625 combinations. The pdf of the mass flow rates is included to correctly size the pumping system and avoid excessive pressure drops.

It should be noted that design methods 1), 2), 3) do not utilize any optimization procedure. Therefore, obtained solutions are not necessarily the best, while methods 4) and 5) seek optimization of the above stated objective functions.

The adopted optimal design procedure for methods 4) and 5), based on a GA, is shown in Figure 7.

Procedure starts with specification definition (outlet temperature of one of the two fluids) and data collection (fluids chemical composition, computation of thermophysical properties μ, λ, ρ, Cp at mean temperature, *Rf*, HE type – fixed tube sheet, U-tube, split ring floating head and others - and choice of probability density function for process variables m_c, m_h, t_{ci} and t_{hi}). Second step of adopted procedure builds scenarios containing expected operating conditions. Each scenario is obtained by permuting discretized pdf, previously identified. Next step is managed by optimization algorithm (GA) which selects initial values of Independent Design Variables (IDV, namely D_s, B, d_o, N_{tp}, *Tube side* (fluid that flow through tubes), *Pattern* and a fictitious value for reference process condition to be used for design purposes m_c, t_{hi}, t_{ci}). Such values are supplied to a design routine using Kern relations [9] to determine Dependent Design Variables (DDV, namely fictitious m_h and Q, v_s, v_t, h_S, h_T, U_{dirt}, d_i, Pt, Cl, N_{tt}, L_{tt}, S). “Fictious” means that temperatures and mass flow rates values selected by GA are used for design purposes but do not correspond to nominal values of process parameters. The use of a fictitious design point is required because it is not assured, giving the variability of actual streams conditions and the Jensen’s inequality, that designing the equipment with a deterministic algorithm (Kern method) for the nominal design point delivers an equipment configuration which optimizes the selected objective function [42]. In fact, Jensen inequality suggests that mean values of temperatures and mass flow rates are not always the best choice for equipment design. The use of a GA assigning fictitious values to the input process conditions, to be used as the reference nominal operating point for design purposes, frees the designer from assuming an average design point or choosing arbitrary design point in case of process conditions variability.

The obtained equipment configuration is evaluated in Performance Evaluation step over the entire range of actual operating conditions described by the assumed scenarios, determining the expected value of pressure drop $E[\Delta P]$, operating cost $E[OC]$ and discounted operating cost $E[OCD]$, expected total cost $E[TC]$ and fluids outlet temperatures probability density functions. Based on the obtained equipment configuration and the scenario values of the input process variables, the corresponding output temperature values are computed resorting to the ε-NTU method.

During Constraint Violation Analysis a penalty cost is considered if some constraint is violated (e.g. use of a correlation out of its validity range), rejecting unsuitable equipment. Each feasible design found is saved. GA acts iteratively modifying IDV and supplying them to design routine looking for alternative designs. Adopted optimization tool, provided by Matlab, uses traditional genetic algorithm tools – elite count, crossover, mutation and migration. Procedure stops when the best current value is maintained for a defined number of iterations or when a maximum number of runs is reached.

No explicit validation of the design procedure and obtained results is provided given that the adopted method iteratively utilizes an established and well proven design algorithm from the literature (Kern and efficiency method). The numerical implementation was validated in previous works of same authors utilizing the same code. The optimization algorithm was taken from widely utilized MATLAB library.

Expected values of outlet temperatures, for each solution, are computed resorting to ε-NTU method across all scenarios. Before comparing a solution with others found by GA, possible constraint violation is checked. Solutions not respecting constraints are disregarded. Considered constraints are: maximum allowed pressure drop of 70 kPa for both shell and tube side, Reynolds number > 300 shell side or 10.000 tube side, and tube length to shell diameter ratio in the range 3 to 15. Reynolds number is computed according to Kern's method [9] using the equivalent hydraulic diameter corresponding the adopted tubes pattern and using the flow velocity evaluated referring to the shell free pass area along diametral plane.

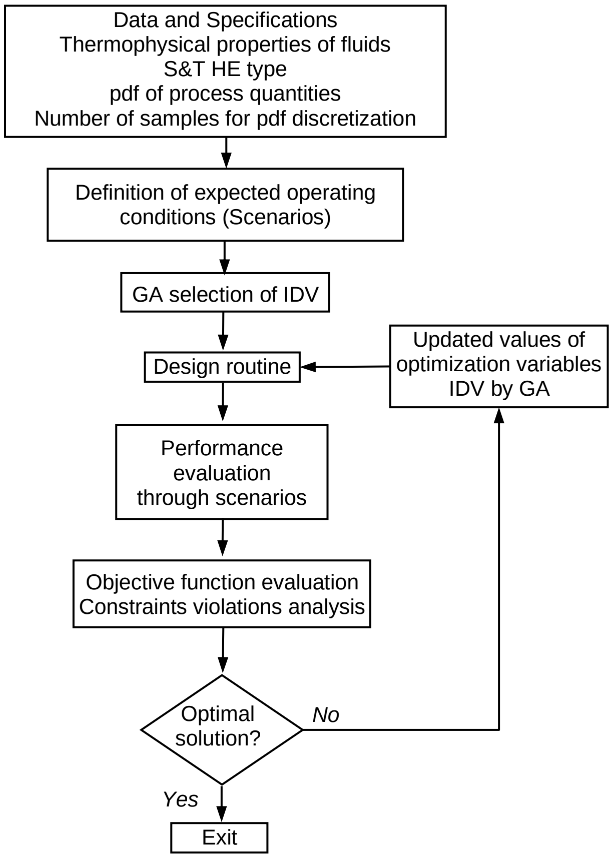

Figure 7 Optimized design procedure considering operating variable conditions.

In Case study 2 the influence of epistemic uncertainty on the performance of the equipment is instead investigated. Input variables are fixed at their nominal values while uncertainty of heat transfer coefficients correlations and fouling resistances values are accounted for. In particular, nominal values of heat transfer coefficients are computed with deterministic correlations (Eq.11, 12) [9],whereas fouling resistances resorting to common used value [9].

$$h_S = 0.36(\lambda_S/D_e)({Re_S}^{0.55}{Pr_S}^{1/3})(\mu_S/\mu_{wS})^{0.14} \tag{11}$$

$$h_T = 0.027(\lambda_T/d_i)({Re_T}^{0.8}{Pr_T}^{1/3})(\mu_T/\mu_{wT})^{0.14} \tag{12}$$

Then to model epistemic uncertainty and to evaluate the effect of low and high level of uncertainty their actual values are changed in the range ±20% and ±40% of the respective nominal value assuming a uniform distribution. For each variable the range is divided into eight equally spaced intervals. All the 4096 admissible permutations are considered.

5. RESULTS DISCUSSION

5.1 Comparison of design solutions under Aleatory Uncertainty

Design results for the NisB (Case 1A) and MisB (Case 1B) for each of the five design methods are shown in the appendix (Table A1 and A2). Please note that in the Tables results related to method 1,2, 3 and 5 are the same for both NisB and MisB case, because the procedure is unchanged, and the only difference is in the probability of meeting the specification. Results for method 4 instead are different due to the change in the objective function. Relevant geometric characteristics and outputs of interest are printed in bold typeface.

Geometrical differences between the optimized equipment (Methods 4 and 5) and the non-optimized ones reside mainly in the larger length of the firsts and in the number of tube passes, that are doubled. Significant differences in baffle spacing and shell diameter are not detectable. The outer diameter of tube doubles in Method 5 as compared to the others. The hot fluid pass in the optimized design is tube side. This choice can be explained by the greater simplicity in reaching the goal for the algorithm thanks to the larger number of degrees of freedom offered by tube side pass. Being the probability density function of the inlet temperature centred on the specification, the nominal design (Method 1) has high performances, comparable with those of optimized equipment, but this does not happen for worst case design (Method 2) and safety factor (Method 3) because the higher heat exchange area increases the difference between the expected value of the outlet temperature and the specification. Robust design (Method 5) has the best performances in this case, achieving the lowest standard deviation of the output and the same probability to meet the specification of the Method 4 design. Expected total cost is computed resorting to the probability density function of pumping costs, as affected by the pressure drops, and the investment cost, but does not consider any cost or revenue coming from meeting or not the specification. From this point of view Robust design is the most expensive while the nominal design the least one, but comparable with the optimized design obtained from Method 4.

It can be noticed that in the MisB case, method 3 (safety factor) performs as good as the optimization method 4 because it directly acts on the surface area. Conversely, method 4 indirectly tries to correct the *LMTD* by changing the input reference conditions (i.e. fluid mass flow rates and temperatures) but always within the variability range imposed by the assumed probability distribution functions. This constraint is included to prevent the optimization algorithm from selecting fictitious conditions leading to excessive pressure drops or oversizing. However, the safety factor method should be used with caution. In fact, while it can ensure total satisfaction of the specification, this may be obtained at the expense of excessive oversizing whereas other methods car reach more effective compromise design. For instance, method 4 reduces by 5% the probability of meeting the specifications but allows a 30% reduction of heat exchange surface, although, in this case study, suffering a pressure drop increase.

The poor performance of robust design in MisB case, similar to method 1 design, is justified considering that reducing the variability of the output is not the objective of this kind of problem.

In Figure 8 and Figure 9 the relevant results, namely the probability of satisfying the specification and the obtained value of the objective function (Eq.10), are shown. The lower is the value of the objective function, and the higher the probability of meeting the specification, the better is the obtained exchanger. A low value of the objective function means outlet temperature distributions with low standard deviation and centered on the specification, while high probability to meet the specification means the correct sizing of the equipment. The usage of safety factor and of the worst-case scenario is useful when the assigned specification is MisB, but leads to inferior solutions when the problem is NisB as the distribution of the outlet temperature is not centered on the specification. For the perfect centering robust design results the best solution.

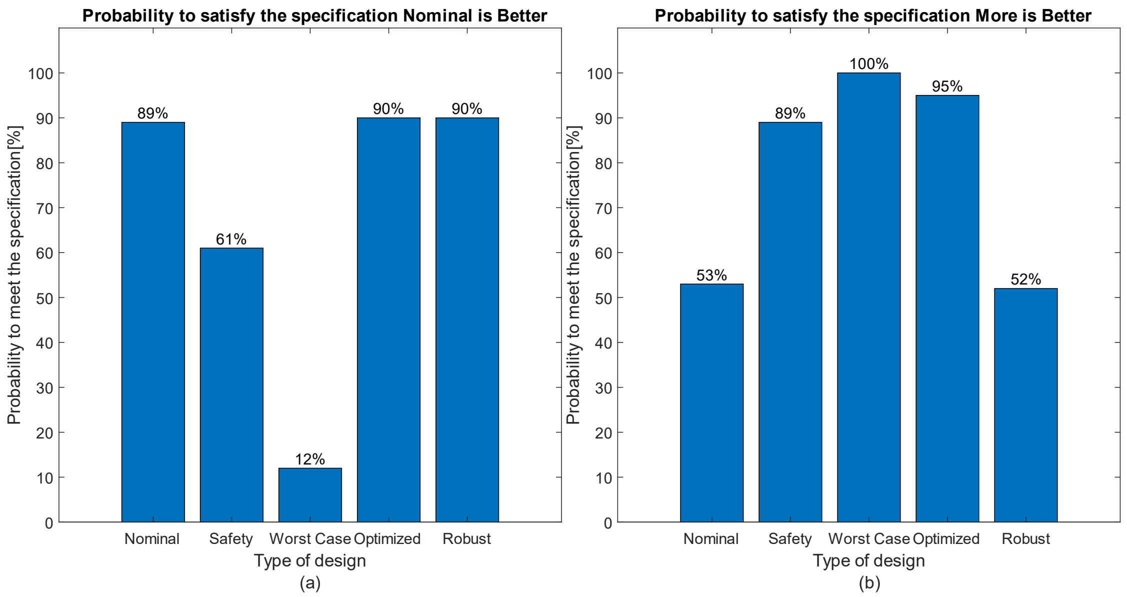

Figure 8 Probability to satisfy the specification under NisB (a) and MisB (b) for different design methods.

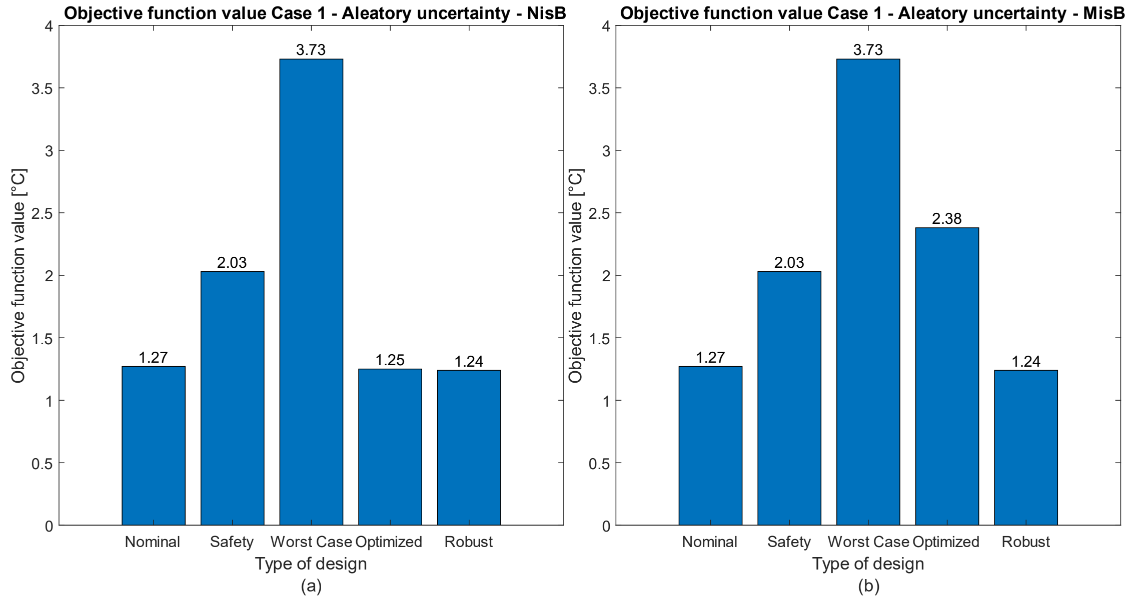

Figure 9 Objective function value under NisB (a) and MisB (b) specification fo different design methods.

In order to find dominating solutions, it is useful to represent the obtained equipment configurations in terms of total cost and probability to meet the specification as made in Figure 10.

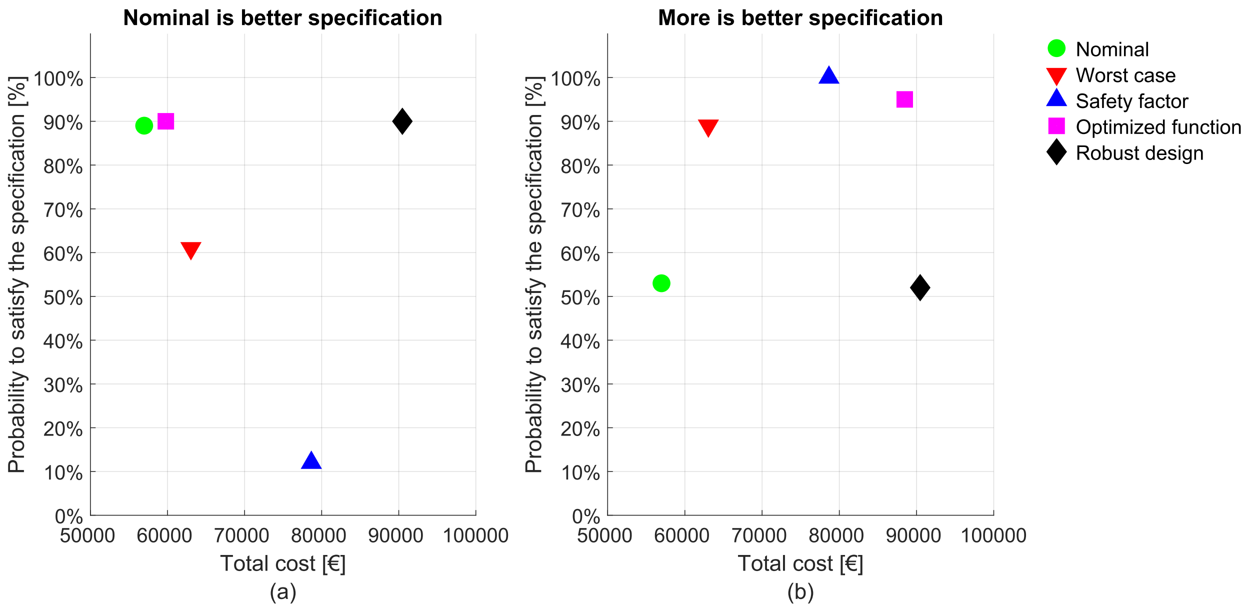

Figure 10 Probability to meet the specification and expected total cost of designs in Case 1A (a) and 1B (b).

In case of NisB specification, and when the standard deviation of the output is neglected, then Method 1 provides the lowest cost and a probability of satisfying the specification comparable with the optimized approaches (Methods 4 and 5).

However, it should be borne in mind that the cost shown neglects the economic effect of failing to meet the specification. When such effects are accounted for, on a case-specific basis, the solutions comparison could lead to significantly different results.

5.2 Sensitivity analysis on Safety factor effect

To show the impact of the safety factor (SF), three different values, namely 1.25, 1.5, 1.75, are assumed, and the resulting equipment configurations are shown in Table A3, while in Figure 11 ~~Figure~~ 11 Comparison of probability to meet the specification NisB (a) and MisB (b) and expected total cost of designs obtained with different safety factor values is shown.

As it can be seen in Figure 11, in NisB case is not useful to adopt the safety factor method because the total cost rises and the probability of meeting specification lowers when the value of SF increases. This happens because in method 3 at first the equipment is designed according to method 1 procedure and then the surface area is increased by safety factor. Thus, the pdf of the outlet temperature, which is centered on the nominal specification in method 1, when applying the surface extension is shifted towards lower temperatures.

Instead, when the specification is MisB the growth of SF increases the expected total cost but also improves the performances. Caution must be applied in choosing SF value because it has decreasing marginal return. In fact, when it grows from 1.25 to 1.5 gain of only 3% is obtained in the probability to meet the specification, and there is no gain rising SF from 1.5 to 1.75.

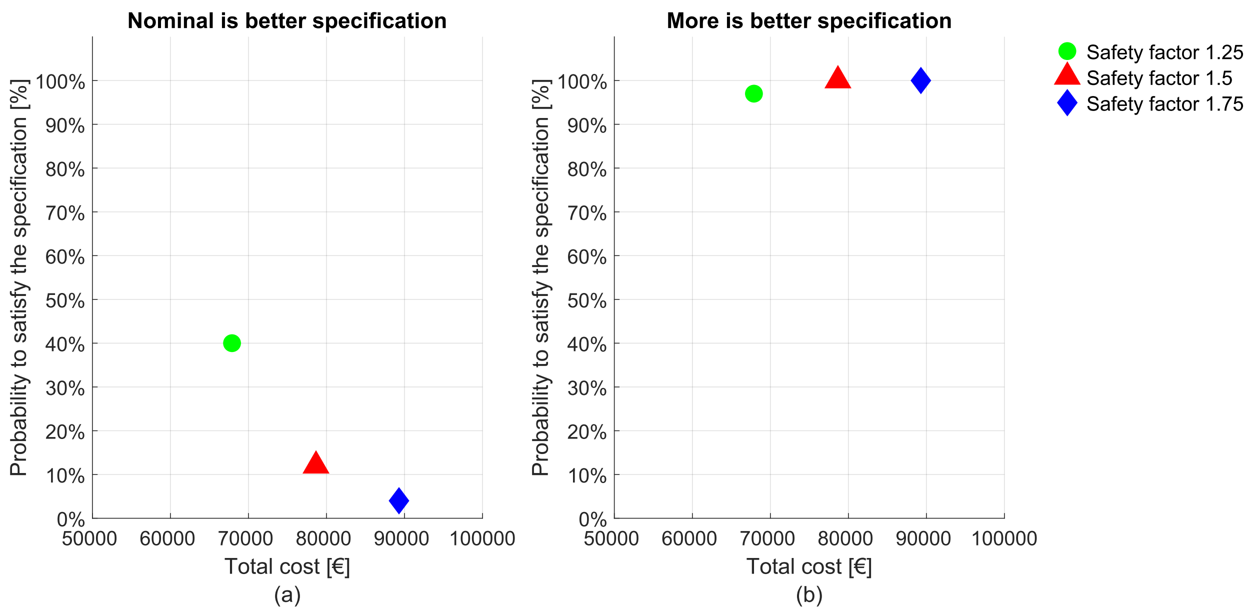

Figure 11 Comparison of probability to meet the specification NisB (a) and MisB (b) and expected total cost of designs obtained with different safety factor values.

5.3 Comparison of design solutions under Epistemic Uncertainty

As previously stated, in this section attention is focused on the impact of epistemic uncertainty only. This refers to “internal” variability imputable to unperfect knowledge of heat transfer and fouling phenomena through empirical correlations.

In this section an exchanger designed for nominal operating conditions (method 1) will be used as a reference case, and compared to robust design (method 5) configuration observing the resulting output variability.

In practice, the shell side and tube side heat transfer coefficients (Eq. 11 and 12) and fouling factors will be computed resorting to widely accepted literature correlations in nominal operating conditions and the obtained values will be then changed in the +/- 40% range with a more detailed analysis focused on +/- 20% variations using uniform distributions.

In particular, possible variation of heat transfer coefficients over such wide range may determine the shift of controlling film from tube side to shell or vice-versa. Moreover, the change of the heat transfer coefficients when the heat exchange area was assigned would modify the outlet temperatures thus shifting their probability distribution.

However, heat transfer coefficients variation is here caused only by epistemic uncertainty instead of by changes in fluids flow rates or thermophysical properties.

Figure 12 shows both the probability of satisfying the specifications (outlet hot fluid temperature of 40 +/-2 °C) and the weighted average dispersion of outlet temperature around the specification (i.e. the objective function value, Eq. 10).

In general, an increase of objective function value, as well as a decrease of the probability of satisfying the specification is observed when the uncertainty in heat transfer coefficient calculation and fouling factor increase. This is caused by the structure of input/output temperature transfer function as shown below. However, resorting to robust design (method 5) better results can be obtained compared to nominal design (method 1), with gains increasing when the uncertainty level increases. In fact, when the uncertainty range increases from +/- 20% to +/-40% the output temperature variability reduction in method 5 compared to method 1 grows from 7% to 11%.

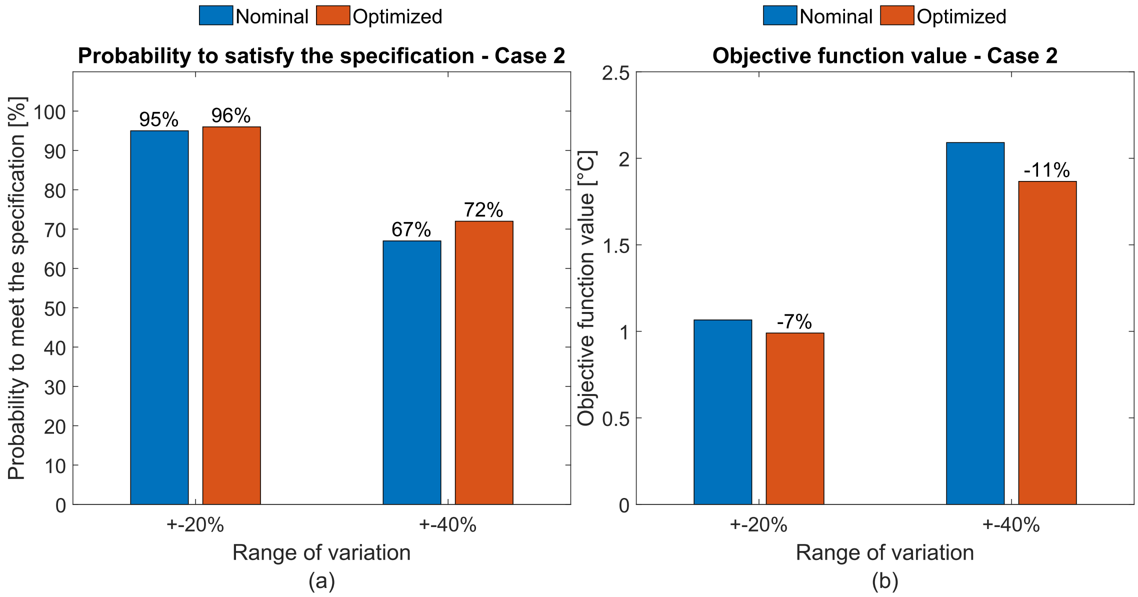

Figure 12 Probability to satisfy the specification NisB (a) and Objective Function values (b) under epistemic uncertainty effect.

Obtained design configurations and their performances are shown in Table A4. The optimized designs (method 5) are more compact that the equipment designed according to Method 1. In particular the outer diameter of tubes and of the shell decrease, while the length is similar. The number of tube passages increases, and the final result is that the fluid velocity both in the shell and tube side increases. Results suggest that robust design can be effective to counter epistemic uncertainty although to minor extent.

5.4 Comparison of aleatory and epistemic uncertainty impact

In order to compare the relative impact of epistemic and aleatory uncertainty the standard deviation of outlet temperature σ_y is evaluated resorting to the Taylor expansion formula for the moments of functions of random variables.

$$\sigma_y = \sqrt{\sum_{i=1}^{N} \left(\frac{dy}{dx_i}\sigma_{xi}\right)^2} \tag{13}$$

In eq. 13 N is the number of variables affected by uncertainty (i.e. mass flow rates and temperatures when aleatory uncertainty is under investigation, and the heat transfer coefficients and fouling resistances when epistemic one is considered), being σ_{xi} is the standard deviation of the *i-th* input variable *x*.

In Figure 13 the standard deviation of the hot stream outlet temperature is shown when uncertainty is propagated in an optimized design (Method 5) and a nominally designed (Method 1) exchanger. This result is computed assuming for each *i-th* variable $\sigma_i = z \cdot E[x_i] \cdot CV_i$, where z is a scaling factor, *E[x]* and *CV* are respectively expected value and coefficient of variation of the assigned variables distribution. The same factor *z* (0.5, 1, 1.5, 2) is used to scale each process variable in the same manner.

Overall, the outlet temperature variability increases linearly with the inlet temperature variability but less than proportionally with the mass flow rates variability. This linear growth of output also means that the possible compensation effects occurring when homogeneous variables assume opposite values is not able to significantly reduce the outlet variability.

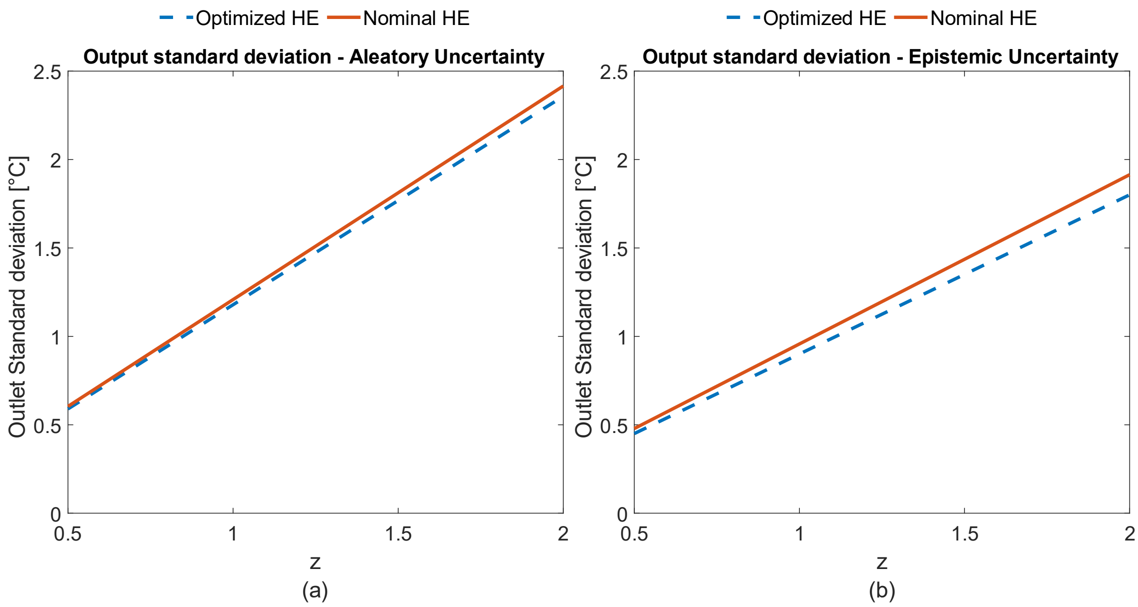

Figure 13 Output standard deviation under aleatory (a) and epistemic(b) uncertainty computed with uncertainty propagation formula applied on a robust optimized and a non-optimized HE.

5.5 Findings and results discussion

Although the above results were obtained referring to a single specific numerical experiment, there are some findings which appear to be somewhat generalizable in order to provide some guidelines for designers as summarized below.

1. The input uncertainty is linearly propagated in output uncertainty. The linear correlation implicates that if inlet uncertainty increases the difference of performance between the robust heat exchanger (method 5) and an equipment designed for nominal conditions (method 1) become more significant.
2. Aleatory uncertainty has a higher impact than epistemic one on performances of the exchanger. In fact, mass flow rates and temperatures variations influence the log mean temperature difference and even the heat transfer coefficients, while epistemic uncertainty only affects the latter.
3. A symmetric change in homogeneous variables do not necessarily lend itself to effective variability compensation.
4. As far as random variations of external process variables are concerned, optimization method, including robust design generally leads to more compact equipment in terms of surface area, although with smaller number of longer and larger diameter tubes and more tube passes. In case of NisB specification a nominal point design provides performances comparable to

optimal design while safety factor and worst-case design deliver poor performances as the distribution of the outlet temperature is not centered on the specification. This suggests that in NisB cases with symmetrical variations of process parameters around their average, a simple design based on average conditions neglecting uncertainty can be acceptable. Worst-case design and safety factor method is not effective with symmetric variations of process parameters because it leads to higher heat exchange area increasing the gap between expected performance and the design specification, also lowering the probability of meeting the specification.

In MisB situations the latter method can ensure satisfaction of specifications over a wide range of operating conditions but at the expense of excessive oversizing. Caution must be applied in choosing safety factor value as it has decreasing marginal return. Robust design performs as poorly as nominal design point method as reducing the variability of the output is not the design goal in this type of application.

5. In case of epistemic uncertainty of internal parameters the robust design method can be slightly superior to nominal point design and also provide a more compact equipment.
6. Robust design appears to be effective in compensating both aleatory and epistemic uncertainty although in a minor manner. In fact, with a given inlet uncertainty some configurations amplify the uncertainty while others dampen it, so it is in theory possible to optimize the HE and obtain a more robust design, but in practice, the behavior difference is not really significant. Moreover, a heat exchanger is characterized by a geometrically fixed physical structure making the equipment hardly adaptable to changing operating conditions.

Based on the above findings, Table 4 can be built providing a suggestion for choosing the preferable design method to address uncertainty. This table is to be intended as a starting reference only. We do not make a claim of its absolute validity, and the designer is encouraged to explore alternative methods as it is hard to predefine which is the best method in a specific instance.

Table 4 Suggested STHE design approaches for addressing uncertainty.

Type of uncertainty	Bilateral specifications (Nominal is Better)	Unilateral specifications (More is Better)
Random variation of external process conditions	I, IV, V	II, III, IV
Epistemic uncertainty of internal parameters	V	

Legend: I) design for nominal reference condition; II) worst case design; III) use of safety margins; IV) objective function optimization; V) robust design.

6. CONCLUSIONS

The purpose of this article was to discuss the existing methods to cope with uncertainty in shell and tube heat exchangers design, understand the influence of uncertainty on the thermal performances of the equipment and explore the potential for optimal robust design of exchangers under uncertainty.

This is the main novelty of this contribution as a direct and consistent comparison of alternative design methods under uncertainty was not performed earlier, and no detailed analysis of their effectiveness is available. Moreover, to the best of our knowledge, this is the first time that robust design of heat exchanger has been practically considered as a tool to address uncertainty.

Results show that different methods may give different results and they are not exchangeable. There is not a preferred method to deal with the uncertainty, but obviously the best solution is not to neglect it. In fact, variations in mass flow rates could lead to error in pumping system sizing and to unexpectedly high pressure drop values, with correspondingly high pumping costs, while changes in inlet temperatures could reduce dramatically the driving force. The application of the uncertainty propagation formula through Taylor expansions shows that in shell and tube heat exchangers uncertainty in the input propagates to the output in linear manner. Different geometric configurations of the equipment could reduce the proportionality constant, but not in a significant manner. Aleatory uncertainty has the major impact on the output. In fact, it influences with mass flow rates heat transfer coefficient and with inlet temperatures the log-mean temperature difference, while epistemic one influences only the first one. As a practical advice, at first it is necessary to understand the nature of the problem. In fact, when NisB specifications apply, results suggest avoiding Worst Case scenario and Safety factor methods and to prefer Objective function optimization and Robust optimization. In MisB case, the Safety factor method with the correct selection of safety factor value, as shown, can lead to superior performances, while Robust optimization has no benefit.

The main limitation of this work is that a single objective function is considered, that economic consequences of failing to meet specifications are not considered, ad that results refer to a single, although extensive, numerical experiment. Generality of findings must be verified resorting to a more extensive experimental campaign.

In future works the use of Method 4 will be examined in greater detail by exploring the impact of different objective functions including both thermal and economic parameters and the value of exchanged heat. Moreover, the combined effect of both aleatory and epistemic uncertainty will be explored.

Overall, based on the findings of this work, designers of shell and tube heat exchangers can gain greater awareness of when and why uncertainty should be accounted for and can be assisted in choosing with greater confidence the proper design approach for heat exchangers operating under uncertainty.

NOMENCLATURE

Symbol	Unit	Caption	Symbol	Unit	Caption
B	[m]	Baffle spacing	*Max ΔP*	[Pa]	Maximum value of pressure drop
Cen	[€/kWh]	Energy cost	*m*	[kg/s]	Mass flow rate
Cp	[kJ/kgK]	Specific heat	*Nb*	[-]	Baffle number
CV	[-]	Coefficient of variation	*Pr*	[-]	Prandlt number
do	[m]	Tube outer diameter	*Re*	[-]	Reynolds number
d_i	[m]	Tube inner diameter	N_{tp}	[-]	Number of tube passages
DOC	[€]	Discounted operating cost	N_{tt}	[-]	Tube number
D_e	[m]	Shell side equivalent diameter	*Ny*	[y]	Equipment life
D_s	[m]	Shell inner diameter	*OC*	[€/y]	Operating cost
E[x]		Expected value of *x*	*Pattern*	[-]	Tube pattern
F	[-]	Temperature difference corrective factor	*Q*	[W]	Duty
H	[h/yr]	Annual operating time	*Rf*	[m^2 K/W]	Conductive fouling resistance
HE type	[-]	Heat exchanger type	*S*	[m^2]	Heat exchanger surface
h	[W/m^{2}K]	Heat transfer coefficient	*TC*	[€]	Total cost
I	[%/y]	Annual discount rate	*t*	[°C]	Fluid temperature
IC	[€]	Investement cost	*Tube Side*	[-]	Fluid tube side passage
L/D	[-]	Length diameter ratio	U_{dirt}	[W/m^{2}K]	Global heat transfer coefficient
LMTD	[°C]	Logarithmic mean temperature difference	*v*	[m/s]	Fluid velocity
L_{tt}	[m]	Tube length			
Subscripts			***Greek symbols***		
c	Cold fluid		ΔP	[Pa]	Pressure drop
h	Hot fluid		λ	[W/mK]	Thermal conductivity
i	Inlet fluid		η	[-]	Overall pumping efficiency
o	Outlet fluid		μ	(Pa s)	Viscosity
S	Shell side fluid		ρ	[kg/m^3]	Density
T	Tube side fluid		σ		Standard deviation

Funding

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

References

[1] K. Thulukkanam, Heat Exchanger Design Handbook, CRC Press, Boca Raton, FL, USA, 2013.
[2] H.-G. Beyer, B. Sendhoff, Robust optimization–a comprehensive survey, Computer Methods in Applied Mechanics and Engineering, 196 (2007) 3190-3218.

[3] P.S. Buckley, Statistical methods in process design, Chemical Engineering, September (1950) 112-114.
[4] S.M. Cho, Uncertainty analysis of heat exchanger thermal-hydraulic designs, Heat Transfer Engineering, 8 (1987) 63-74.
[5] A.K.M. Mahbub Uddin, K.J. Bell, Effect of uncertainties on the design and operation of systems of heat exchangers, in: Heat Transfer Equipment Design (R. K. Shah, E. C. Subbarao, and R. A. Mashelkar, eds.), Hemisphere, Washington, DC, 1988, pp. 39–47.
[6] C. Zang, M. Friswell, J. Mottershead, A review of robust optimal design and its application in dynamics, Computers & structures, 83 (2005) 315-326.
[7] R. De Neufville, O. De Weck, D. Frey, D. Hastings, R. Larson, D. Simchi-Levi, K. Oye, A. Weigel, R. Welsch, Uncertainty management for engineering systems planning and design, in: Engineering Systems Symposium, MIT, Cambridge, MA, 2004.
[8] A.E. Jones, Thermal design of the shell-and-tube: exchanger sizing takes a trial and error approach. Use these guidelines to center your aim.(Cover Story), Chemical Engineering, 109 (2002) 60-66.
[9] D.Q. Kern, Process heat transfer, McGraw-Hill, New York, USA, 1950.
[10] G.F. Hewitt, Heat Exchanger Design Handbook, Begell House, New York, USA, 1998.
[11] R. Mukherjee, Effectively design shell-and-tube heat exchangers, Chemical Engineering Progress, 94 (1998) 21-37.
[12] S. Kacak, H. Liu, A. Pramuanjaroenkij, Heat Exchangers. Selection, Rating and Thermal Design, CRC Press, Boca Raton, Florida, USA, (2012).
[13] M. Nitsche, R.O. Gbadamosi, Heat exchanger design guide: a practical guide for planning, selecting and designing of shell and tube exchangers, Butterworth-Heinemann, Oxford, UK, 2015.
[14] A.C. Caputo, P.M. Pelagagge, P. Salini, Heat exchanger optimized design compared with installed industrial solutions, Applied Thermal Engineering, 87 (2015) 371-380.
[15] W.H. Saldanha, F.R.P. Arrieta, G.L. Soares, State-of-the-art of research on optimization of shell and tube heat exchangers by methods of evolutionary computation, Archives of Computational Methods in Engineering, 28 (2021) 2761-2783.
[16] R. Venkata Rao, A. Saroj, P. Oclòn, J. Taler, Design Optimization of Heat Exchangers with Advanced Optimization Techniques: A Review, Archives of Computational Methods in Engineering, 27 (2020) 517-548.
[17] A.C. Caputo, P.M. Pelagagge, P. Salini, Manufacturing cost model for heat exchangers optimization, Applied Thermal Engineering, 94 (2016) 513-533.
[18] K.J. Bell, On the pessimization of heat exchangers, Heat Transfer Engineering, 21 (2000) 1-2.
[19] I.E. Grossmann, R. Sargent, Optimum design of chemical plants with uncertain parameters, AIChE Journal, 24 (1978) 1021-1028.
[20] N.V. Sahinidis, Optimization under uncertainty: state-of-the-art and opportunities, Computers & Chemical Engineering, 28 (2004) 971-983.
[21] I. Grossmann, R. Apap, B. Calfa, P. Garcia-Herreros, Q. Zhang, Mathematical Programming Techniques for Optimization under Uncertainty and Their Application in Process Systems Engineering, Theoretical Foundations of Chemical Engineering, 51 (2017).
[22] A.L. Costa, M.J. Bagajewicz, 110th Anniversary: On the departure from heuristics and simplified models toward globally optimal design of process equipment, Industrial & Engineering Chemistry Research, 58 (2019) 18684-18702.
[23] C.d.O. Gonçalves, A.L. Costa, M.J. Bagajewicz, Shell and tube heat exchanger design using mixed-integer linear programming, AIChE Journal, 63 (2017) 1907-1922.
[24] C.d.O. Gonçalves, A.L. Costa, M.J. Bagajewicz, Alternative mixed-integer linear programming formulations for shell and tube heat exchanger optimal design, Industrial & Engineering Chemistry Research, 56 (2017) 5970-5979.
[25] A. Al Zakri, K.J. Bell, Estimating performance when uncertainties exist, Chemical Engineering Progress, 77 (1981) 39–49.

[26] L. Haseler, R. Owen, R. Sardesai, The sensitivity of heat exchanger calculations to uncertainties in the physical properties of the process fluids, Proceedings of the Institution of Mechanical Engineers, Part A: Power and Process Engineering, 197 (1983) 171-178.
[27] D. Clarke, V. Vasquez, W. Whiting, M. Greiner, Sensitivity and uncertainty analysis of heat-exchanger designs to physical properties estimation, Applied Thermal Engineering, 21 (2001) 993-1017.
[28] C.A. James, R.P. Taylor, B. Hodge, The application of uncertainty analysis to cross-flow heat exchanger performance predictions, Heat Transfer Engineering, 16 (1995) 50-62.
[29] M.A. Badar, S.M. Zubair, A.K. Sheikh, Uncertainty analysis of heat-exchanger thermal designs using the Monte Carlo simulation technique, Energy, 18 (1993) 859-866.
[30] R. Prasad, K. Bharadwaj, Stochastic modeling of heat exchanger response to data uncertainties, Applied Mathematical Modelling, 26 (2002) 715-726.
[31] N. Kayansayan, Thermal behavior of heat exchangers in off-design conditions, Heat Recovery Systems and CHP, 9 (1989) 265-273.
[32] T. Knetsch, U. Hauptmanns, Integration of stochastic effects and data uncertainties into the design of process equipment, Risk Analysis: An International Journal, 25 (2005) 189-198.
[33] J. Lambert, L. Gosselin, Sensitivity analysis of heat exchanger design to uncertainties of correlations, Applied Thermal Engineering, 136 (2018) 531-540.
[34] R.P. Taylor, B. Hodge, C.A. James, Estimating uncertainty in thermal systems analysis and design, Applied Thermal Engineering, 19 (1999) 51-73.
[35] O. Abdelaziz, R. Radermacher, Modeling heat exchangers under consideration of manufacturing tolerances and uncertain flow distribution, International Journal of Refrigeration, 33 (2010) 815-828.
[36] J.R. Khan, S.M. Zubair, A risk based performance evaluation of plate-and-frame heat exchangers, Heat and mass transfer, 39 (2003) 327-336.
[37] J.U.R. Khan, S.M. Zubair, A risk based performance analysis of plate-and-frame heat exchangers subject to fouling: Economics of heat exchanger cleaning, Heat Transfer Engineering, 26 (2004) 87-100.
[38] S.M. Zubair, B.A. Qureshi, A probabilistic cost mode for plate-and-frame heat exchangers, International Journal of Energy Research, 30 (2006) 1-17.
[39] B. Yeap, G. Polley, S. Pugh, D. Wilson, Retrofitting crude oil refinery heat exchanger networks to minimize fouling while maximizing heat recovery, Heat Transfer Engineering, 26 (2005) 23-34.
[40] J.C. Lemos, A.L. Costa, M.J. Bagajewicz, Linear method for the design of shell and tube heat exchangers including fouling modeling, Applied Thermal Engineering, 125 (2017) 1345-1353.
[41] J.C. Lemos, A.L. Costa, M.J. Bagajewicz, Globally optimal linear approach to the design of heat exchangers using threshold fouling modeling, AIChE Journal, 64 (2018) 2089-2102.
[42] A.C. Caputo, P.M. Pelagagge, P. Salini, Heat exchanger design optimization under stochastic operating conditions, in: Proc. 13th Brazilian Congress of Thermal Sciences and Engineering, ENCIT 2010, Uberlandia, MG, Brazil, 2010.
[43] A.C. Caputo, P.M. Pelagagge, P. Salini, Robust approach for shell and tube exchangers optimization under uncertain heat transfer estimation., in: Proceedings of ENCIT 2012 14th Brazilian Congress of Thermal Sciences and Engineering, Rio de Janeiro, Brazil, 2012.
[44] W.H. Saldanha, F.R.P. Arrieta, P.I. Ekel, T.M. Machado-Coelho, G.L. Soares, Multi-criteria decision-making under uncertainty conditions of a shell-and-tube heat exchanger, International Journal of Heat and Mass Transfer, 155 (2020) 119716.
[45] R.L. Shilling, M.P. Rudy, T.M. Rudy, Risk-Based Design Margin Selection for Heat Exchangers, Heat Transfer Engineering, 32 (2011) 307-313.
[46] M. Arner, Statistical robust design: An industrial perspective, John Wiley & Sons, Chichester, UK, 2014.
[47] K. Zheng, H.H. Lou, J. Wang, F. Cheng, A Method for Flexible Heat Exchanger Network Design under Severe Operation Uncertainty, Chemical Engineering & Technology, 36 (2013) 757-765.

[48] F. Al Khulaifi, E. Al Mutairi, Synthesis of Heat Exchangers Network (HEN) and Optimization under Variability and Uncertainty, in: Computer Aided Chemical Engineering, Vol. 38, Elsevier, 2016, pp. 1063-1068.
[49] P. Floquet, G. Hétreux, R. Hétreux, L. Payet, Analysis of operational heat exchanger network robustness via interval arithmetic, in: Computer Aided Chemical Engineering, Vol. 38, Elsevier, 2016, pp. 1401-1406.
[50] E. de Rocquigny, N. Devictor, S. Tarantola, Uncertainty in industrial practice: a guide to quantitative uncertainty management, John Wiley & Sons, Chichester, UK, 2008.
[51] R. Ghanem, D. Higdon, H. Owhadi, Handbook of uncertainty quantification, Springer, Cham, Switzerland, 2017.
[52] M. Zimmermann, S. Königs, C. Niemeyer, J. Fender, C. Zeherbauer, R. Vitale, M. Wahle, On the design of large systems subject to uncertainty, Journal of Engineering Design, 28 (2017) 233-254.
[53] F.P. Bernardo, E.N. Pistikopoulos, P.M. Saraiva, Quality costs and robustness criteria in chemical process design optimization, Computers & Chemical Engineering, 25 (2001) 27-40.
[54] M.J. Chalupnik, D.C. Wynn, P.J. Clarkson, Comparison of ilities for protection against uncertainty in system design, Journal of Engineering Design, 24 (2013) 814-829.
[55] J.H. Saleh, G. Mark, N.C. Jordan, Flexibility: a multi-disciplinary literature review and a research agenda for designing flexible engineering systems, Journal of Engineering Design, 20 (2009) 307-323.
[56] M.S. Phadke, Quality engineerig using robust design, Prentice Hall, Englewood Cliffs, new Jersey, 1989.
[57] D. Bertsimas, D.B. Brown, C. Caramanis, Theory and applications of robust optimization, SIAM Review, 53 (2011) 464-501.
[58] B.L. Gorissen, İ. Yanıkoğlu, D. den Hertog, A practical guide to robust optimization, Omega, 53 (2015) 124-137.
[59] G. Dellino, J.P. Kleijnen, C. Meloni, Robust optimization in simulation: Taguchi and response surface methodology, International Journal of Production Economics, 125 (2010) 52-59.
[60] E. Borgonovo, E. Plischke, Sensitivity analysis: a review of recent advances, European Journal of Operational Research, 248 (2016) 869-887.
[61] J. Love, Process automation handbook: a guide to theory and practice, Springer Science & Business Media, London, UK, 2007.
[62] F. Shinskey, Process Control Systems: Application, Design, and Tuning, McGraw-Hill, New York, USA, 1996.
[63] G. Polley, S. Pugh, Dealing with Uncertainties During Heat Exchanger Design, in: National industrial energy technology conference, Vol. 23, Texas A&M University, 1999, 2001, pp. 123-128.
[64] J.L.W.V. Jensen, Sur les fonctions convexes et les inégalités entre les valeurs moyennes, Acta mathematica, 30 (1906) 175-193.
[65] R. De Neufville, S. Scholtes, Flexibility in engineering design, MIT Press, Cambridge, Massachussets, USA, 2011.
[66] A.C. Caputo, P.M. Pelagagge, P. Salini, Heat exchanger design based on economic optimisation, Applied Thermal Engineering, 28 (2008) 1151-1159.
[67] A.C. Caputo, P.M. Pelagagge, P. Salini, Joint economic optimization of heat exchanger design and maintenance policy, Applied Thermal Engineering, 31 (2011) 1381-1392.
[68] I. Doltsinis, Z. Kang, Robust design of structures using optimization methods, Computer Methods in Applied Mechanics and Engineering, 193 (2004) 2221-2237.
[69] K.-H. Lee, G.-J. Park, Robust optimization considering tolerances of design variables, Computers & structures, 79 (2001) 77-86.
[70] D. Lönn, Robust design: Accounting for uncertainties in engineering, in: Linköping University Electronic Press, 2008.
[71] E. de Rocquigny, Modelling Under Risk and Uncertainty: An Introduction to Statistical, Phenomenological and Computational Methods, John Wiley & Sons, Chichester, Sussex, UK, 2012.

APPENDIX

Table A1 Different design obtained by different design mode and performances evaluation for nominal is better specification.

Name	***[-]***			***Case 1A***		
Design type	[-]	Nominal	Worst Case	Safety factor	Objective function	Robust design
t_{hi}	[°C]	95.0	101.4	95.0	96.3	94.6
t_{ho}	[°C]	40.0	40.0	40.0	40.0	40.0
m_h	[kg/s]	25.0	27.5	25.0	25.9	27.3
t_{ci}	[°C]	27.5	31.3	27.5	25.9	25.2
t_{co}	[°C]	40.0	40.0	40.0	40.0	40.0
m_c	[kg/s]	74.4	130.6	74.4	70.1	68.1
Q	[kW]	3905	4798	3905	4136	4222
LMTD	[°C]	28.7	27.0	28.7	30.4	30.4
F	[-]	0.81	0.82	0.81	0.81	0.81
HE type	[-]	SRFH	SRFH	SRFH	SRFH	SRFH
Tube Side	**[-]**	**Cold fluid**	**Cold fluid**	**Cold fluid**	**Hot fluid**	**Hot fluid**
D_s	**[m]**	**0.9**	**1.0**	**0.9**	**1.0**	**1.0**
B	**[m]**	**0.4**	**0.3**	**0.4**	**0.8**	**0.5**
do	**[m]**	**0.02**	**0.02**	**0.02**	**0.0318**	**0.04**
N_{tp}	**[-]**	**2**	**2**	**2**	**4**	**4**
N_{tt}	[-]	918	1191	918	345	258
L_{tt}	**[m]**	**4.2**	**4.0**	**6.4**	**8.1**	**8.7**
v_s	[m/s]	0.52	0.53	0.52	0.42	0.72
v_t	[m/s]	0.81	1.10	0.81	0.79	0.70
Re_S	[-]	16701	17200	16701	16391	25956
Pr_S	[-]	5.1	5.1	5.1	5.7	5.7
Re_T	[-]	16109	21813	16109	44163	49453
Pr_T	[-]	5.7	5.7	5.7	5.1	5.1
h_S	[W/m² K]	1709	1737	1709	2507	3513
h_T	[W/m² K]	4126	5259	4126	1803	1569
U_{dirt}	**[W/m² K]**	**684**	**720**	**684**	**598**	**602**
S	**[m²]**	**244**	**299**	**367**	**279**	**284**
L/D	[-]	4.74	4.00	7.10	7.94	8.36
ΔP_T	[Pa]	25331	29610	37996	9371	56752
ΔP_S	[Pa]	7373.7	12421	9757	10092	7168
Eq. 10 Value	**[°C]**	**1.27**	**2.03**	**3.73**	**1.25**	**1.24**
$E[\Delta P_T]$	[Pa]	7487	4493	9905	9492	6045
$E[\Delta P_S]$	[Pa]	25355	24840	38032	10631	66712
Pattern	[-]	Triang	Triang	Triang	Square	Triang
Eq. 10 Value norm	[-]	3.17%	5.08%	9.33%	3.12%	3.09%
$MAX[\Delta P_T]$	[Pa]	7857	4714	10388	10939	6973
$MAX[\Delta P_S]$	[Pa]	29214	28620	43821	11161	70040
IC	[€]	46618	54459	63851	51590	52213
OC	[€/y]	1675	2299	2395	1244	5378
E[OC]	[€/y]	1686	1397	2410	1333	6226

E[DOC]	[€]	10359	8582	14807	8193	38257
DOC	[€]	10290	14124	14717	7646	33043
TC	[€]	56908	68583	78567	59236	85257
E[TC]	[€]	56976	63040	78658	59783	90470
Probability to satisfy the specificication NisB	**[-]**	**89%**	**61%**	**12%**	**90%**	**90%**
σT_{hout}	[°C]	1.27	1.26	1.24	1.25	1.24
$\mathbf{E[T_{hout}]}$	**[°C]**	**39.9**	**38.4**	**36.5**	***40.0***	**40.0**
CV	[-]	3.17%	3.27%	3.40%	3.12%	3.09%
Eq. 10 Value norm $\mathbf{E[T_{hout}]}$	**[-]**	**3.18%**	**5.29%**	**10.24%**	**3.12%**	**3.09%**

Table A2 Different design obtained by different design mode and performances evaluation for more is better specification.

Name	***[-]***			***Case 1B***		
Design type	[-]	Nominal	Worst Case	Safety factor	Objective function	Robust design
t_{hi}	[°C]	95.0	101.4	95.0	101.3	94.6
t_{ho}	[°C]	40.0	40.0	40.0	40.0	40.0
m_h	[kg/s]	25.0	27.5	25.0	27.5	27.3
t_{ci}	[°C]	27.5	31.3	27.5	31.3	25.2
t_{co}	[°C]	40.0	40.0	40.0	40.0	40.0
m_c	[kg/s]	74.4	130.6	74.4	130.4	68.1
Q	[kW]	3905	4798	3905	4791	4222
LMTD	[°C]	28.7	27.0	28.7	27.0	30.4
F	[-]	0.81	0.82	0.81	0.82	0.81
HE type	[-]	SRFH	SRFH	SRFH	SRFH	SRFH
Tube Side	**[-]**	**Cold fluid**	**Cold fluid**	**Cold fluid**	**Hot fluid**	**Hot fluid**
$\mathbf{D_s}$	**[m]**	**0.9**	**1.0**	**0.9**	**0.8**	**1.0**
B	**[m]**	**0.4**	**0.3**	**0.4**	**0.5**	**0.5**
do	**[m]**	**0.02**	**0.02**	**0.02**	**0.0127**	**0.04**
$\mathbf{N_{tp}}$	**[-]**	**2**	**2**	**2**	**6**	**4**
N_{tt}	[-]	918	1191	918	2151	258
$\mathbf{L_{tt}}$	**[m]**	**4.2**	**4.0**	**6.4**	**2.7**	**8.7**
v_s	[m/s]	0.52	0.53	0.52	1.50	0.72
v_t	[m/s]	0.81	1.10	0.81	1.26	0.70
Re_S	[-]	16701	17200	16701	17094	25956
Pr_S	[-]	5.1	5.1	5.1	5.7	5.7
Re_T	[-]	16109	21813	16109	28285	49453
Pr_T	[-]	5.7	5.7	5.7	5.1	5.1
h_S	[W/m^2 K]	1709	1737	1709	8794	3513
h_T	[W/m^2 K]	4126	5259	4126	3161	1569
$\mathbf{U_{dirt}}$	**[W/m^2 K]**	**684**	**720**	**684**	**918**	**602**
S	**[m^2]**	**244**	**299**	**367**	**235**	**284**
L/D	[-]	4.74	4.00	7.10	3.25	8.36
ΔP_T	[Pa]	25331	29610	37996	179170	56752

ΔP_S	[Pa]	7373.7	12421	9757	37451	7168
Eq. 10 Value	**[°C]**	**1.27**	**2.03**	**3.73**	**2.38**	**1.24**
$E[\Delta P_T]$	[Pa]	7487	4493	9905	31427	6045
$E[\Delta P_S]$	[Pa]	25355	24840	38032	64428	66712
Pattern	[-]	Triang	Triang	Triang	Triang	Triang
Eq. 10 Value norm	[-]	3.17%	5.08%	9.33%	5.94%	3.09%
$MAX[\Delta P_T]$	[Pa]	7857	4714	10388	36201	6973
$MAX[\Delta P_S]$	[Pa]	29214	28620	43821	67641	70040
IC	[€]	46618	54459	63851	45236	52213
OC	[€/y]	1675	2299	2395	17570	5378
E[OC]	[€/y]	1686	1397	2410	7037	6226
E[DOC]	[€]	10359	8582	14807	43237	38257
DOC	[€]	10290	14124	14717	107962	33043
TC	[€]	56908	68583	78567	153199	85257
E[TC]	[€]	56976	63040	78658	88473	90470
Probability to satisfy the specificication MisB	**[-]**	**53%**	**89%**	**100%**	**95%**	**52%**
σT_{hout}	[°C]	1.27	1.26	1.24	1.24	1.24
$\boldsymbol{E[T_{hout}]}$	**[°C]**	**39.9**	**38.4**	**36.5**	**38.0**	**40.0**
CV	[-]	3.17%	3.27%	3.40%	3.27%	3.09%
Eq. 10 Value norm $\boldsymbol{E[T_{hout}]}$	**[-]**	**3.18%**	**5.29%**	**10.24%**	**6.26%**	**3.09%**

Table A3 Safety factor selection effect on performances and designs configuration.

Name	***[-]***	***Safety factor effect***		
Safety factor value	[-]	1.25	1.5	1.75
t_{hi}	[°C]	95.0	95.0	95.0
t_{ho}	[°C]	40.0	40.0	40.0
m_h	[kg/s]	25.0	25.0	25.0
t_{ci}	[°C]	27.5	27.5	27.5
t_{co}	[°C]	40.0	40.0	40.0
m_c	[kg/s]	74.4	74.4	74.4
Q	[kW]	3905	3905	3905
LMTD	[°C]	28.7	28.7	28.7
F	[-]	0.81	0.81	0.81
HE type	[-]	SRFH	SRFH	SRFH
Tube Side	**[-]**	**Cold fluid**	**Cold fluid**	**Cold fluid**
$\boldsymbol{D_s}$	**[m]**	**0.9**	**0.9**	**0.9**
B	**[m]**	**0.4**	**0.4**	**0.4**
do	**[m]**	**0.02**	**0.02**	**0.02**
$\boldsymbol{N_{tp}}$	**[-]**	**2**	**2**	**2**
N_{tt}	[-]	918	918	918
$\boldsymbol{L_{tt}}$	**[m]**	**5.3**	**6.4**	**7.4**
v_s	[m/s]	0.52	0.52	0.52
v_t	[m/s]	0.81	0.81	0.81

Re_S	[-]	16701	16701	16701
Pr_S	[-]	5.1	5.1	5.1
Re_T	[-]	16109	16109	16109
Pr_T	[-]	5.7	5.7	5.7
h_S	[W/m^2 K]	1709	1709	1709
h_T	[W/m^2 K]	4126	4126	4126
U_{dirt}	**[W/m^2 K]**	**684**	**684**	**684**
S	**[m^2]**	**305**	**367**	**428**
L/D	[-]	5.92	7.10	8.29
ΔP_T	[Pa]	31663	37996	44328
ΔP_S	[Pa]	8565	9757	10948
Eq. 10 Value	**[°C]**	**2.62**	**3.73**	**4.36**
$E[\Delta P_T]$	[Pa]	8696	9905	11114
$E[\Delta P_S]$	[Pa]	31694	38032	44371
Pattern	[-]	Triang	Triang	Triang
Eq. 10 Value norm	[-]	6.55%	9.33%	10.91%
DELTA	[-]	-	-	-
$MAX[\Delta P_T]$	[Pa]	9122	10388	11653
$MAX[\Delta P_S]$	[Pa]	36518	43821	51125
IC	[€]	55312	63851	72261
OC	[€/y]	2035	2395	2755
E[OC]	[€/y]	2048	2410	2772
E[DOC]	[€]	12583	14807	17032
DOC	[€]	12503	14717	16930
TC	[€]	67815	78567	89191
E[TC]	[€]	67895	78658	89293
Probability to satisfy the specificication MisB	**[-]**	**97%**	**100%**	**100%**
Probability to satisfy the specificication NisB	**[-]**	**40%**	**12%**	**4%**
σT_{hout}	[°C]	1.25	1.24	1.23
$E[T_{hout}]$	**[°C]**	37.7	36.5	35.8
CV	[-]	3.32%	3.40%	3.44%
Eq. 10 Value norm $E[T_{hout}]$	**[-]**	**6.95%**	**10.24%**	**12.18%**

Table A4 Nominal design evaluation and optimized heat exchanger design under epistemic uncertainty effect.

Name	***[-]***	***Nominal***		***Optimized***	
Design type	[-]	-0.2	-0.4	-0.2	-0.4
t_{hi}	[°C]	95.0	95.0	95.0	95.0
t_{ho}	[°C]	40.0	40.0	40.0	40.0
m_h	[kg/s]	25.0	25.0	25.0	25.0
t_{ci}	[°C]	27.5	27.5	27.5	27.5

t_{co}	[°C]	40.0	40.0	40.0	40.0
m_c	[kg/s]	74.4	74.4	74.4	74.4
Q	[kW]	3905	3905	3905	3905
LMTD	[°C]	28.7	28.7	28.7	28.7
F	[-]	0.81	0.81	0.81	0.81
HE type	[-]	SRFH	SRFH	SRFH	SRFH
Tube Side	[-]	Cold fluid	Cold fluid	Hot fluid	Hot fluid
D_s	**[m]**	**0.9**	**0.9**	**0.8**	**0.6**
B	**[m]**	**0.4**	**0.4**	**0.8**	**0.6**
do	**[m]**	**0.02**	**0.02**	**0.018**	**0.012**
N_{tp}	**[-]**	**2**	**2**	**6**	**4**
N_{tt}	[-]	918	918	726	1026
L_{tt}	**[m]**	**4.2**	**4.2**	**4.8**	**4.5**
v_s	[m/s]	0.52	0.52	0.55	0.91
v_t	[m/s]	0.81	0.81	1.69	1.80
Re_S	[-]	16701	16701	12088	13425
Pr_S	[-]	5.1	5.1	5.7	5.7
Re_T	[-]	16109	16109	53716	38030
Pr_T	[-]	5.7	5.7	5.1	5.1
h_S	[W/m^2 K]	1709	1709	3747	5954
h_T	[W/m^2 K]	4126	4126	3726	4239
U_{dirt}	**[W/m^2 K]**	**684**	**684**	**846**	**959**
S	**[m^2]**	**244**	**244**	**198**	**174**
L/D	[-]	4.74	4.74	5.81	7.03
ΔP_T	[Pa]	25331	25331	14068	53992
ΔP_S	[Pa]	7374	7374	69992	69986
Eq. 10 Value	**[°C]**	**1.07**	**2.09**	**0.99**	**1.87**
Pattern	[-]	Triang	Triang	Square	Square
Eq. 10 Value norm	[-]	2.66%	5.23%	2.48%	4.67%
IC	[€]	46618	46618	39816	36379
OC	[€/y]	1675	1675	4062	7643
DOC	[€]	10290	10290	24957	46961
TC	[€]	56908	56908	64773	83340
Probability to satisfy the specificication NisB	**[-]**	**95%**	**67%**	**96%**	**72%**
σT_{hout}	[°C]	1.05	1.98	0.98	1.80
$E[T_{hout}]$	**[°C]**	**40.2**	**40.7**	**40.2**	**40.5**
CV	[-]	2.61%	4.86%	2.43%	4.43%
Eq. 10 Value norm $E[T_{hout}]$	**[-]**	**2.65%**	**5.14%**	**2.46%**	**4.61%**