Bounded Rationality and Excess

Entrepreneurial Entry

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Abstract

We formalize entrepreneurs' bounded rationality as an information-processing problem using a non-additive information measure based on Tsallis' notion of non-extensive entropy. We then use this formalization to analyze the conditions under which bounded rationality leads to excess market entry.

Keywords: Excess Entry, Bounded Rationality, Non-extensive Entropy

1 Introduction

It is well known (see, for instance, [1], [4], [5], [12]) that most of new businesses fail shortly after inception: up to 60% of start-ups and ventures fail within the first five years ([4]), and about 80% of all entrants in the manufacturing sector exit within ten years ([5]).

This suggests that "too many" entrepreneurs enter in the markets. According to the economists, the high rate of business entrant failures is an unavoidable consequence of entrepreneurs taking rational risks in uncertain situations. According to the psychologists, entry mistakes are a consequence of the various cognitive biases that affect boundedly rational entrepreneurs (e.g. [3], [10]). Both laboratory studies ([3], [2]) and field studies ([10], [13]) have shown how overconfidence might lead to excessive entry.

In this paper we formalize the conditions under which bounded rationality leads to excessive business entry by looking at how entrepreneurs' biases in the process of collecting and processing information. To that end, we formalize bounded rationality as an information-processing problem using a non-additive (generalized) information measure based on Tsallis [14] notion of non-extensive entropy. Within this formalism we provide a representation of bounded rationality by assuming that decision-makers infer the probability distributions of possible future outcomes by solving a maximization problem in which they can make mistakes in both the collecting and the processing of information.

This makes it possible to individuate states that can be used to study the conditions under which entrepreneurs' bounded rationality leads to excessive market entry, and to derive propositions about entrepreneurial behavior and market entry that are consistent with existing psychological and empirical findings.

The paper is organized as follows. Section 2 introduces some basic information theory concepts and explains why the Tsallis entropy is appropriate to formalize bounded rationality. The formal model is presented in section 3. Section 4 presents economic and managerial implications, and section 5 concludes. Proofs of the propositions are provided in the Appendix A.

2 Rational Decision and Information Theory

If it is possible to determine the way decision-makers should attach probabilities to future events is still open. It seems logic to say that a "rational" decision-maker should be evaluates the information at her disposal, without any prejudice.

Obviously, as people suffer from bounded rationality it is possible to distinguish between a "fully rational" decision-maker, that is a rational decision-maker who collects and processes the information in an unbiased way, and a "boundedly rational" decision-maker, that is a rational decision maker who thinks she is collecting and processing the information in an unbiased way, but can be to fails to do so.

The difference between a "fully rational" and a "boundedly rational" decision-maker depends on the way she collects and processes information. Therefore, many of the cognitive biases discussed in the management and behavioral economics literature can be treated as information-processing problems, and thus formalized by relying on the findings of information theory [7].

Information theory is involved with the quantification of information by means of an appropriate information measure. In this regard [9] has introduced four axioms describing the properties that a 'good' information measure should possess. In particular a good information measure is additive and therefore independent of

the way the information is collected.

These axioms are also valid for the well-known entropy measure S that represents the current level of ignorance/uncertainty about the future outcome and can be defined as missing information, *i.e.* as $S = -I$. The entropy which emerges from these axioms is the celebrated Shannon entropy:

$$S_S = - \sum_{i=1}^{n} p_i \, ln \, p_i$$

However, despite its popularity, it is not difficult to think about situations where the order in which information is collected can be very relevant. For example let's consider some of the cognitive biases discussed in the psychology literature: we can consider an entrepreneur in the process of recollecting events from her memory. Clearly the order in which the entrepreneur collects information is very important and leads to different results in the achieved knowledge. Tsallis ([14]) suggested the use of non "ideal" entropy, the so-called Tsallis entropy, that is not additive for independent random variables:

$$S_T = \frac{1}{q-1} \left(1 - \sum_{i=1}^{n} p_i^q \right)$$

where q is an arbitrary real parameter. The Tsallis entropy is, however, a generalization of the Shannon entropy that can be recovered as a special case (for $q \to 1$).

3 The Model

Let us assume that the decision-maker collects evidence and evaluates the business opportunity by subjectively assessing: (1) the future level of market performance, and (2) her chances to obtain a given outcome.

She thus faces a maximization problem in which the level of future expected industry profits represents the constraints of the maximization problem, and the probability distribution associated with the possible outcomes emerges as the result this maximization. Note that, the uncertainty here is only represented by the probability distribution associated with the different outcomes.

Formally, let $X \in (0, +\infty)$ be a random variable representing the possible future revenues, $\rho(x)$ its probability density function, $E(X)$ the expected value of the industry revenues, and c the cost of starting the business opportunity. To formalize rationality, assume that it is possible to distinguish between neoclassical

and bounded rational entrepreneurs and that the entrepreneur infers the probability distribution by relying on the maximum entropy principle. We remind that the principle of maximum entropy [7] asserts that, given the information available, the probability distribution which best describes the random variable under scrutiny is the one which leaves the largest remaining uncertainty (*i.e.*, the maximum entropy).

3.1 Neoclassical and boundedly rational entrepreneurs

Neoclassical decision theory assumes that individuals are rational information processors. Then, the decision-makers gather and process information in an unbiased manner. They have complete information about the market constraints (here represented by the future industry revenues), the information is perfectly processed, and the uncertainty is only represented by the probability distribution associated with the different possible outcomes.

Assumption 1

A neoclassical entrepreneur infers the probability distribution of the possible outcomes by maximizing Shannon entropy S_S under the constraint $E[X - c] = \mu_0 - c$, where μ_0 is the unbiased expected value of the industry revenues. Formally, the related maximum problem is:

$$max_\rho \{ S_S = - \int_0^{+\infty} \rho(x) \ln \rho(x) dx \}$$

subject to

$$\int_0^{+\infty} \rho(x) dx = 1$$

and

$$\int_0^{+\infty} x \rho(x) dx = \mu_0 .$$

The well-known solution is (see, for instance [14])

$$\rho(x) = \frac{1}{\mu_0} e^{-\frac{x}{\mu_0}} .$$

Remark 1

The solution simply states that a high level of revenues is associated with an exponentially small probability. However, entrepreneurs are subject to a number of cognitive biases which affect the way they collect and process information. This

implies that, whilst the uncertainty is still only represented by the probability distribution associated with the different outcomes, the entrepreneur does not necessarily have all the relevant information at her disposal and/or process this information in an unbiased way.

Assumption 2

A boundedly rational entrepreneur infers the probability distribution of the possible outcomes by maximizing Tsallis entropy S_T under the constraints $E[X - c] = \mu - c$, where, in general, the expected value of the industry revenues $\mu \neq \mu_0$. Formally, the related maximum problem is:

$$max_\rho \{ S_T = \frac{1}{q-1} \left[1 - \int_0^{+\infty} \rho^q(x)dx \right] \}$$

subject to

$$\int_0^{+\infty} \rho(x)dx = 1$$

and

$$\frac{\int_0^{+\infty} x\rho^q(x)dx}{\int_0^{+\infty} \rho^q(x)dx} = \mu,$$

as it is shown in [14]. In this case the optimal solution of the problem is (see [14] for details)

$$\rho_q(x) = \frac{1}{(2-q)\mu} \left[1 + \frac{q-1}{2-q} \frac{x}{\mu} \right]_+^{\frac{q}{1-q}}$$

for $0 < q < 2$, where

$$[x]_+ = max\{x, 0\}.$$

Remark 2

The solution implies that:

- if $1 < q < 2$, a high level of revenues is associated with a power/Pareto law distribution

$$\rho_q(x) \sim \frac{A}{x^\upsilon}$$

for large x, with an exponent $v = \frac{q}{q-1}$.

- if $0 < q < 1$, after a certain threshold, $x_c = \mu\frac{2-q}{1-q}$, the probabilities associated
with high level $(x > x_c)$ of revenues are exactly null.

Remark 3

The parameter q can be interpreted as a measure of optimism/pessimism: if $1 < q < 2$ the decision maker is optimist and the degree of optimism increases with q; if $0 < q < 1$ the decision maker is pessimist and the degree of pessimism decreases with q.

3.2 Entry level and excess entry

Let us determine the expected level of entry according to the two cases from above. In particular, let us assume that $\pi(\mu_0)$ is the expected level of profits of the neoclassical decision-maker and that $\pi_q(\mu)$ is the expected level of profits of the boundedly rational decision-maker.

Assumption 3

The number n of entrepreneurs entering the market is a positive and monotonically increasing function f of the expected level of profits, with $f(0)=0$. The number of entrants in the case of boundedly rational entrepreneurs is equal to:

$$n(\mu) = f(\pi_q(\mu))$$

where

$$\pi_q(\mu) = E_q[X|X > c]^1$$

and

$$E_q[g(X)] = \frac{\int_0^{+\infty} g(x)\rho^q(x)dx}{\int_0^{+\infty} \rho^q(x)dx}.$$

The number of entrants in the case of neoclassical entrepreneurs is equal to:

$$n(\mu_0) = f(\pi(\mu_0)),$$

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[1] The term c represents the cost entry and we assume $c < \mu$.

where $\pi(\mu_0) = \pi_{q=1}(\mu = \mu_0)$.

In our model we define excess entry by looking at the difference between the number of entry in the ideal case of neoclassical entrepreneurs and that of boundedly rational entrepreneurs. Therefore, the excess entry might well be positive as well as negative, the latter case referring to deficient entry.

Assumption 4

The excess entry is assumed to be

$$\varepsilon(q, \mu) = \frac{n_q(\mu) - n(\mu_0)}{n(\mu_0)}$$

4 Results

We can now use the model to analyze how the level of entrepreneurial entry is affected by entrepreneurs' external and internal considerations, and the interplay between them.

Proposition 1

Excess entry $\varepsilon(q, \mu)$ increases as q increases: $\dfrac{\partial \varepsilon(q, \mu)}{\partial q} > 0$.

This result shows that, other things equal, the level of excess entry increases as the level of optimism increases or the level of pessimism decreases (see Remark 2 and 3). This of course is in line with the findings of the psychological literature reviewed above in that entrepreneurs' entry increases with the increasing of the chances of being successful. Note that this is true whatever is the sign and size of the distortion.

Proposition 2

The excess entry $\varepsilon(q, \mu)$ increases as μ increases: $\dfrac{\partial \varepsilon(q, \mu)}{\partial \mu} > 0$.

This result is consistent with the usual assumption that if the level of expected profits in a certain industry increases, the number of entries increases.

Proposition 3

The excess entry $\varepsilon(q, \mu)$ is positive if the entrepreneur is optimist $(1 < q < 2)$ and $\mu > \mu_0$.

This result shows that the level of excess entry is positive if the entrepreneurs are optimist and assume that the industry is characterized by supernormal profits. This is the case of overconfidence, where bounded rationality affects both the ways entrepreneurs collect and process information and therefore implies a number of entries which is higher than in the neoclassical case.

Proposition 4

The excess entry $\varepsilon(q,\mu)$ is negative if the entrepreneur is pessimist $0 < q < 1$ and $\mu < \mu_0$.

This result shows that the level of excess entry is negative if the entrepreneurs are pessimist and assume that the industry is characterized by subnormal profits. This corresponds to the case of underconfidence, where bounded rationality affects both the ways entrepreneurs collect and process information and therefore leads to a number of entries which is lower than in the neoclassical case.

5 Conclusion

In this paper we have explained the high rate of business failure by assuming that boundedly rational entrepreneurs make mistakes in the process of collecting and processing information. To formalise bounded rationality, we have used a non-additive information measure based on Tsallis' notion of non-extensive entropy. In accordance with previous empirical studies, our model has shown (propositions 1-4) how entrepreneurs' cognitive biases might lead to excessive market entry.

Appendix A

In this Appendix the proofs of Propositions 1-4 are provided.

Let $\rho_q(X) = \frac{1}{\mu(2-q)}\left(1 + \frac{q-1}{2-q}\frac{x}{\mu}\right)^{\frac{q}{1-q}}$ with $0 < q < 2$ and $(X)_+ = max\{X,0\}$ be the probability density of X. The profit $\pi_q = \mathrm{E_q}[(X - c)_+]$ is given by

$$\pi_q = \mu\left(1 + \frac{q-1}{2-q}\frac{c}{\mu}\right)^{\frac{2-q}{1-q}}_{+} \qquad 0 < q < 2$$

Proof of Proposition 1

Excess entry $\varepsilon(q,\mu)$ increases as q increases: $\frac{\partial \varepsilon(q,\mu)}{\partial q} > 0.$

If $1 < q < 2$ we have:

$$\frac{\partial \varepsilon(q,\mu)}{\partial q} = \mu \frac{f'\left(\pi_q(\mu)\right)}{f(\pi_1(\mu_0))} \left(\frac{2-q}{q-1}\right)^2 (1+t)^{\frac{2-q}{1-q}} \left[ln(1+t) - \frac{t}{1+t}\right] \quad \text{, with} \quad t = \frac{q-1}{2-q}\frac{c}{\mu} > 0$$

$$\Rightarrow \frac{\partial \varepsilon(q,\mu)}{\partial q} > 0 \quad \Leftrightarrow \quad ln(1+t) - \frac{t}{1+t} > 0 \quad \forall t > 0.$$

Let $h(t) = ln(1+t) - \frac{t}{1+t}$. Since $h(0) = 0$ and $h'(t) = \frac{t}{(1+t)^2} > 0$ it follows that $h(t) > 0 \Rightarrow \frac{\partial \varepsilon(q,\mu)}{\partial q} > 0.$

For $0 < q < 1$ we have:

$$\frac{\partial \varepsilon(q,\mu)}{\partial q} = \mu \frac{f'\left(\pi_q(\mu)\right)}{f(\pi_1(\mu_0))} \left(\frac{2-q}{q-1}\right)^2 (1-t)^{\frac{2-q}{1-q}} \left[ln(1-t) + \frac{t}{t-1}\right] \quad \text{with } t = \frac{1-q}{2-q}\frac{c}{\mu} \in (0,1).$$

Hence

$$\frac{\partial \varepsilon(q,\mu)}{\partial q} > 0 \Leftrightarrow ln(1-t) + \frac{t}{t-1} > 0 \; \forall t \in (0,1).$$

Let

$h(t) = ln(1-t) + \frac{t}{1-t}$. Since $h(0) = 0$ and $h'(t) = \frac{t}{(1-t)^2} > 0 \; \forall t \in (0,1)$ it follows that $h(t) > 0 \Rightarrow \frac{\partial \varepsilon(q,\mu)}{\partial q} > 0.$

Proof of Proposition 2

The excess entry $\varepsilon(q,\mu)$ increases as μ increases.

If $1 < q < 2$ we have

$$\frac{\partial \varepsilon(q,\mu)}{\partial \mu} = \frac{f'\left(\pi_q(\mu)\right)}{f(\pi_1(\mu_0))} \left[(1+t)^{\frac{2-q}{1-q}} + \frac{c}{\mu}(1+t)^{\frac{1}{1-q}}\right] > 0, \text{with} \quad t = \frac{q-1}{2-q}\frac{c}{\mu} > 0$$

If $0 < q < 1$ we have

$$\frac{\partial \varepsilon(q,\mu)}{\partial \mu} = \frac{f'\left(\pi_q(\mu)\right)}{f(\pi_1(\mu_0))} \left[(1-t)^{\frac{2-q}{1-q}} + \frac{c}{\mu}(1-t)^{\frac{1}{1-q}}\right] > 0, \text{with } t = \frac{1-q}{2-q}\frac{c}{\mu} \in (0,1).$$

Proof of Proposition 3

The excess entry $\varepsilon(q,\mu) > 0$ if $1 < q < 2$ and $\mu > \mu_0$.
We have

$$\varepsilon(q,\mu) > 0 \iff \pi_q(\mu) - \pi_1(\mu_0) > 0 \iff h(q,\mu) > 0, \quad \text{with} \quad h(q,\mu) = \pi_q(\mu) - \pi_1(\mu_0).$$

It results

$h(1,\mu) = \mu e^{-\frac{c}{\mu}} - \mu_0 e^{-\frac{c}{\mu_0}} > 0 \quad \forall \mu > \mu_0 \quad$ since $\ g(\mu) = \mu e^{-\frac{c}{\mu}}\ $ is an increasing function. We have:

$$\frac{\partial \varepsilon(q,\mu)}{\partial q} = \frac{f'\big(\pi_q(\mu)\big)}{f\big(\pi_1(\mu_0)\big)} \frac{\partial \pi_q(\mu)}{\partial q} = \frac{f'\big(\pi_q(\mu)\big)}{f\big(\pi_1(\mu_0)\big)} \frac{\partial h(q,\mu)}{\partial q} \Rightarrow$$

$$\frac{\partial h(q,\mu)}{\partial q} = \frac{f\big(\pi_1(\mu_0)\big)}{f'\big(\pi_q(\mu)\big)} \frac{\partial \varepsilon(q,\mu)}{\partial q} > 0,$$

since $\frac{\partial \varepsilon}{\partial q} > 0$ by Proposition 1.

Since $h(1,\mu) > 0$ and $\frac{\partial h(q,\mu)}{\partial q} > 0$ it follows that $h(q,\mu) > 0$.

Proof of Proposition 4

The excess entry $\varepsilon(q,\mu) < 0$ if $0 < q < 1$ and $0 < \mu < \mu_0$.
We have

$$\varepsilon(q,\mu) < 0 \iff \pi_q(\mu) - \pi_1(\mu_0) < 0 \iff h(q,\mu) < 0, \quad \text{with} \quad h(q,\mu) = \pi_q(\mu) - \pi_1(\mu_0).$$

It results

$h(1,\mu) = \mu e^{-\frac{c}{\mu}} - \mu_0 e^{-\frac{c}{\mu_0}} < 0 \quad 0 < \mu < \mu_0\ $ since $g(\mu) = \mu e^{-\frac{c}{\mu}}$ is an increasing function.

We have:

$$\frac{\partial \varepsilon(q,\mu)}{\partial q} = \frac{f'\big(\pi_q(\mu)\big)}{f\big(\pi_1(\mu_0)\big)} \frac{\partial \pi_q(\mu)}{\partial q} = \frac{f'\big(\pi_q(\mu)\big)}{f\big(\pi_1(\mu_0)\big)} \frac{\partial h(q,\mu)}{\partial q} \Rightarrow$$

$$\frac{\partial h(q,\mu)}{\partial q} = \frac{f\big(\pi_1(\mu_0)\big)}{f'\big(\pi_q(\mu)\big)} \frac{\partial \varepsilon(q,\mu)}{\partial q} > 0$$

since $\frac{\partial \varepsilon}{\partial q} > 0$ by Proposition 1.

Since $h(1,\mu) < 0$ and $\frac{\partial h(q,\mu)}{\partial q} > 0$ it follows that $h(q,\mu) < 0$ for any $0 < q < 1$

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