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Measuring the Similarity of Concept Maps According to Pedagogical Criteria

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ABSTRACT A concept map provides a graphic hierarchical means of representing how knowledge is structured in a domain. It visually organizes a set of concepts showing their mutual relations. An analysis of the similarities between two concept maps can produce significant results in any fields where Intelligent Knowledge Management is used, such as Healthcare, Policy Development, Energy and Waste Management, Resource Consumption Sustainability, Mobility, Safety, Citizen Empowerment, and, of course, Education. In an educational setting, a concept map conveys the various concepts connected by relations of dependence that a course must cover. However, the similarity between two concept maps for education has to be measured according to criteria that take into consideration the pedagogical properties of the maps, i.e., not only considering the structural aspects of the maps themselves. An automated analysis of the similarity between two concept maps can allow the teacher to reflect on different interpretations of the knowledge domain of a certain course as well as to assess how existing learning material can be implemented in a new course.

Research into this aspect of concept mapping appears to be relatively scarce. This paper proposes criteria to assess the similarity of two concept maps, also based on pedagogical features, with the aim of providing teachers with better support during the course creation process. Each criterion is implemented through a specific measure function. The measures are then shown to be sensitive to their criterion rationale by evaluating them against a collection of random case studies.

INDEX TERMS Concept Maps, Similarity Measure, Technology Enhanced Learning

I. INTRODUCTION

Technology is a major source and motor for change in how human life is organized. Indeed, the development of internet-based infrastructures and services has stimulated unprecedented changes in this respect. Internet based intelligent (or "smart") applications allow us to use knowledge and create knowledge connections to promote informed decision making. The development of such technologies is at the basis of advancements in many fields of human activities such as Healthcare, Policy Development, Energy and Waste Management, Resource Consumption Sustainability, Mobility, Safety, Citizen Empowerment [56], [57], and, of course, Education [1], [25], [36].

Using and creating knowledge connections in intelligent applications entails representing a knowledge domain (KD), and the use of a concept map (CM) is recognized as a powerful way to obtain such representations. Among other things, a CM can illustrate relations between concepts such as dependencies, associations, co-occurrences and correlations [7].

Concept mapping, i.e., the activity of drawing and updating a CM, allows experts and non-experts to express, clarify and establish their own understanding of how the knowledge is structured in a KD. Beside Education, CMs support a variety of crucial human interaction activities where sharing a common understanding of a KD is essential. Among such fields are Social Science [10], [14], [58], Knowledge Management [14], Information Systems Development [21], Collaborative Work [49], Ecological Management [72], and many others.

In the field of Education, concept mapping is already widely used to support learning and teaching. For instance, we find applications in Curriculum Design [20], [64], Concept Organization and Understanding [3], [31], Assessment (examining how learners acquire concepts and organize these concepts in their minds) [2], [11], [24], [67], the description and analysis of teaching strategies [65], and course design and construction by means of gathering available instructional material [12].

This paper focuses on the educational uses of concept mapping. Specifically, we: 1) propose criteria and measures to appraise the pedagogical similarity of two CMs; 2) show examples of cases where such similarity measures can help the teacher design and create their courses; 3) evaluate the measures showing how they each consistently and accurately compute the similarity of two CMs against the pedagogical criterion upon which they are based.

A. RATIONALE AND MOTIVATION

During the process of creating a course, a teacher can refer to a CM as a means of organizing the concepts that need to be covered. In a CM, teachers can link these concepts to semantic relationships in accordance with their didactic strategies. In this study, we focus on just one type of semantic relationship: the *prerequisite* relationship. This choice simplifies our account, while not limiting the generality of our approach to uncovering similarities between CMs. A prerequisite relationship is defined as follows: one concept is a prerequisite of another if the former is learned (i.e., learned according to the teacher's plan) before the latter. The prerequisite relationship is fundamental to the planning and sequencing of concepts delivered within a course.

Where available, the Internet offers several CMs for the same KD. Teachers can refer to these maps in planning their courses, not only in terms of the sequencing of concepts, but also to provide useful input when creating teaching materials. While this is beneficial for improving and sustaining effective teaching, the process of looking for similar CMs on the Internet entails difficulties. On the one hand, seeing the work of others can be beneficial for teachers: they may find inspiration from various pedagogical perspectives, by considering possible changes to their teaching plans (such as replacing, relocating, or adding concepts). Moreover, they might discover learning materials that are suitable for a new course. On the other hand, analyzing the similarities between two CMs can be a lengthy process, and, combined with the amount of potential CMs to assess, the task can become too time consuming and impractical for the teacher. Hence, an instrument capable of measuring the similarity of retrieved CMs with the one proposed by the teacher can be extremely useful in narrowing down the number of CMs to be manually assessed. However, any such automated analysis of similarity needs to be based on a multifaceted analysis of the pedagogical characteristics of the maps.

The degree of similarity between two CMs derives from the placement, position, and mutual relationships of all the concepts, and these factors need to be seen from different pedagogical viewpoints.

In other words:

- the similarity analysis should not only be based on the layout of the concepts or limited to the concepts that the maps have in common. Indeed, it is likely that even concepts that are not common to the maps are significant for gauging the similarity of how the common concepts are used in the maps: common concepts might be used in different local settings in the two maps, reducing the similarity between the maps;
- the similarity analysis should also be polarized according to different criteria, each taking into account the pedagogical significance of the concepts present in the maps.

As we will see in Sec. III, a general framework, or system, allowing to compare CMs from pedagogical points of view, is not currently available. In this paper we intend to contribute to the future development of such systems by defining an initial set of pedagogical criteria, and related measures of CMs similarity. To provide a general framework for evaluating CM similarity, it is crucial to have, and to apply, different criteria of similarity between pairs of CMs. In this study we believe that the following criteria play an important role in running a comprehensive pedagogical similarity measure for two CMs: i) the commonality of relations between concepts; ii) the prerequisite relationships between concepts; iii) the prominence/centrality of the concepts in the CMs.

We propose here the definition of such similarity criteria, as well as the algorithms for their computation, in the form of measures. Each measure provides a value of similarity for two given CMs according to one of the criteria. In defining the criteria, a kind of traditional structural (topological) approach is enhanced by considering additional pedagogical aspects.

The major contribution of this paper is thus in the proposal, implementation, and evaluation of the three criteria of pedagogical similarity outlined above. We will demonstrate that the measures defined in implementing the criteria are sensitive to the rationale of the base criterion of each by evaluating them with reference to a number of case studies.

B. STRUCTURE OF THE PAPER

This subsection acts as a guide for the reader throughout the paper.

In order to focus on the intended use of our similarity criteria and measures, Section II discusses some motivational examples, which evidence how a similarity analysis of CMs could be useful to a teacher. In Sec. III, we present some aspects of related work, ranging from CM principles to graph theory, from which several methods of measuring similarity derive. Sec. IV introduces the three proposed criteria. Sec. V defines the implementation of the three criteria in three related measures, i.e., *JE measure*, *PC measure* and *KC measure*. Following the presentation of criteria and measures,

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Sec. VI evaluates the measures by comparing an initial CM with a sequence of progressively different CMs, obtained by successive transformations applied pseudo-randomly starting from the first CM (or *seed map*). With different seed maps, and applying different transformations, we obtain several case studies, which will allow us to examine the behavior of one measure compared with other measures (those that we devised and a baseline).

The evaluation is deemed to answer the following research question:
Is each of the proposed similarity measures sensitive to the criterion that it is devised to tackle?

Finally, the last section discusses future work and the potential applications of the proposed new measures.

II. EXAMPLES OF THE APPLICATION OF SIMILARITY CRITERIA

A CM represents a KD in the shape of a Directed Acyclic Graph, where the nodes are concepts and the edges are connections between the nodes representing the relations between the concepts. An edge can be labeled by a word or sentence to express the type of relation between the two connected concepts [7].

In this section we describe the behavior, and line of thinking, of teachers when looking for approaches to create a new course or update an old one. We will assume that the teachers have planned their course by way of a concept map. They might now be interested in comparing such a map to other maps relating to existing (and hence already tested) courses. Such comparisons could be advantageous for teachers, leading to significant improvements in their CMs.

For instance, a teacher might be influenced when making a comparison with certain CMs: (s)he can consider different pedagogical viewpoints, eventually adopting one or, on the contrary, strengthening her own. The teacher might then decide to modify the layout of her course: the role and position of certain concepts in the teacher's map could change, and other concepts could be added to enhance the learning process. Moreover, on a more practical note, the retrieved map might already have been used to implement a successful course, and the teacher might appreciate the idea of reusing part of the related learning material that has been tested effectively for that CM. Finally, we also have to consider the fact that teachers would probably prefer to analyze just a small set of relevant CMs. It is essential to provide the teacher with CMs that are pertinent to their teaching domain, which is why the similarity scoring methods for CMs are crucial. The development of a framework that automatically scores and ranks CMs based on the similarity to the teacher's CM is an important step towards incentivizing the reutilization of learning materials, and providing the teacher with additional pedagogical approaches.

Several factors contribute in determining whether two given CMs are generally similar. Concepts that are in one map might not appear in the other, influencing the way that a given concept might be learned. Moreover, the same concepts might be laid out differently or they could have different prerequisites in the various maps. The same concepts might also be vested with a different prominence in the maps (e.g., being the learning aim of one portion of a map or intermediate elements in a learning path).

However, let us return to our teacher, and propose that (s)he has defined a CM (CM1) on fractions, as illustrated in Fig. 1. Dozens of other maps could have been retrieved in the same knowledge domain, but for the sake of space, let us assume that two maps have been retrieved: CM2 (Figure 2) and CM3 (Figure 3).

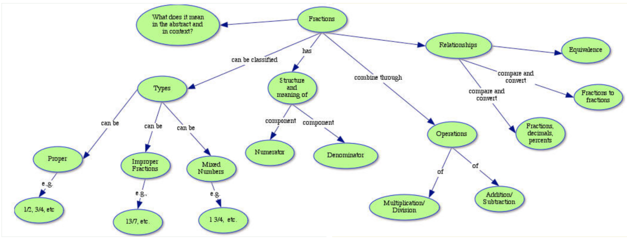

FIGURE 1. CM1: a teacher's concept map on fractions (source: https://www.mathmatik.com/concept-map-for-fractions-based-on-our-work.html).

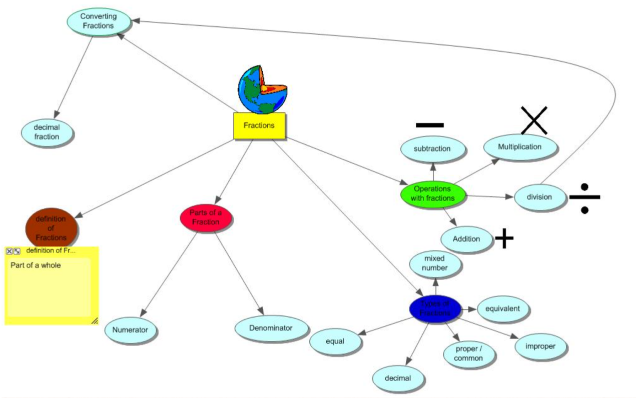

FIGURE 2. CM2: a map found on the Web (source: https://johnanthoney75.wordpress.com/2013/11/19/concept-map/).

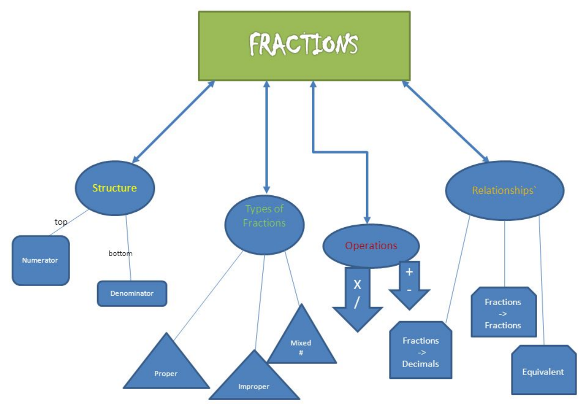

FIGURE 3. CM3: another map from the Web (source: https://slideplayer.com/slide/10218714/).

Let us also assume that only one map – the one most similar to CM1 – should have the teacher's attention. The maps used in this example are authentic (as evidenced by the sources provided).

In the maps there are synonyms (the same concept under different names), but let us also assume that such synonyms (which are not the focus of this paper) collapse into homogeneous names . This assumption makes the comparison simpler, although selecting the map that is most similar to CM1 would still be fairly laborious: beside the synonyms, the layout of the maps can mislead the teacher. A major problem here is that similarity has to be measured with respect to a criterion, or perhaps criteria, rather than by a one-fits-all method.

As mentioned in the Introduction, this paper proposes three criteria of similarity based on pedagogical considerations. Tab. 1) provides three similarity scores that result from analyzing the maps according to the similarity criteria proposed in this study. The analysis of such data shows that CM3 is more similar to CM1 than CM2 against two criteria: in particular, the concepts and associated relations in CM1 "overlap" with CM3 far better (0.778 out of 1) than with CM2 (0.4). This means that the arrangement of the concepts in CM1 is markedly more compatible with CM3 than it is with CM2. Furthermore, if we also consider the factor of the overall centrality (importance) given to the concepts, CM3 is more similar to CM1 than to CM2. Regarding the concepts that are prerequisite in CM1, both CM2 and CM3 are basically equally compatible with CM1.

In conclusion, the teacher might well consider CM3 more suitable than CM2 for further comparison.

When designing an intelligent system to support teachers in the retrieval of CMs, a corner stone is the automatic scoring of similarity between CMs based on pedagogical criteria. Such a system can suggest CMs that contain valuable pedagogical stimuli for teachers in terms of learning materials and teaching plans.

Comparison	JE	PC	KC
CM1 - CM2	0.4	0.979	0.514
CM1 - CM3	0.778	1.0	0.751

TABLE 1. Comparison between the concept map proposed by the teacher and the two maps taken from the Web. JE, PC and KC are the proposed measures.

III. RELATED WORK

As stated earlier, CMs are graphical renderings of structured and interconnected knowledge [8]. Advances in the fundamentals of CM similarities date back to 1980s / early 90s. This is when Novak presented CMs as a means of studying the development – carried out over more than a decade – of children's understanding of scientific knowledge [47], [48], [50], [51]. From the perspective of cognitive psychology, Novak's work was grounded in the learning theories of Ausubel [4], where: 1) learning takes place by assimilating new concepts; 2) such assimilation occurs by integrating and harmonizing the new concepts in the cognitive structure of the concepts the learner already possesses.

A CM is effective in representing the learner's organization of concepts regarding a subject matter, and so it is also effective in supporting the integration of new concepts in such a cognitive structure, allowing the learner to achieve meaningful learning (as opposed to rote learning) [7], [48], [49].

A. USE OF CMS

CMs can be used for a wide range of tasks in (science) education [41], [47]. CMs can foster critical thinking in an educational setting and generally support the social collaborative development of knowledge [29], [61], [71]. They can be used to support the development and representation of learning strategies (improving learner autonomy) as well as instructional strategies (empowering the teacher with a suitable instrument for managing the teaching and learning flow). CM usage has also been shown to help learner interaction and peer learning [69].

Other educational applications of concept mapping relate to assessment and performance prediction: essentially, an analysis of CMs derived or directly drawn from learners can help in evaluating their comprehension (and miscomprehension) of scientific concepts [2], in interpreting student response where knowledge is lacking [39], and exploiting previous assessments to predict future performance [52].

CMs have been seen as a way of empowering students, offering them negotiating and "idea exchanging" skills that can be used both inside and outside the classroom [38]. In [12] the use of CMs ("simple diagrams of instructional concepts") supports the organization of class materials, and helps the teacher to reduce the occurrence of rote knowledge. In particular, Cliburn's paper deals with an example of the sequencing of learning material concerning the nervous system. In the 1990s, we find research that validates the CM approach to education and its advantages in assessment [41]. This tackled: 1) the reliability of CM assessment (by comparing six different assessment methods); 2) its validity (i.e., the accuracy of the conclusions that can be drawn from the assessment); 3) the practical applicability of CMs to the classroom situation. Regarding reliability, it was concluded that scoring methods do have an effect on assessment, but that they should be structured so as to favor neither a fully holistic scoring approach (where everything is based on the response of the individual rater) nor a totally structured scoring approach (painstakingly extracting hierarchies and cross-links from the learner's CM). In contrast, an intermediate approach is to be preferred. A comparison of the learner's CM with a master CM (i.e., teacher's CM) appeared to be the most suitable assessment method.

Adding a final observation concerning validity, in [41] the authors concluded that the scores given to learners' CMs correlate to the measure of similarity that they have to the master

CM: this observation provides an effective introduction to the notion of grading a learner's map simply by computing a similarity measure. Nevertheless, analysis of CM similarity is not a frequently discussed topic in the literature.

B. PEDAGOGICAL SIMILARITY

A study that explicitly relies on graph comparison for educational purposes is presented in [24], where the Pathfinder Scaling Algorithm [59] computes the "closeness" between graphs of concepts (nodes) connected by relations (edges). Closeness is computed by the measure *C*, based on neighborhood sets [23]. The following quotation describes *C* and the idea of closeness as similarity [24]:

The C measure is a set-theoretic method of quantifying the configural similarity between two networks having a common set of nodes. Very briefly, C examines the degree to which the same node in two graphs is surrounded by a similar neighborhood of nodes. This neighborhood comparison is performed for each node in the two graphs and the results averaged across the nodes to compute an overall index of similarity.

The values of C range from zero (for complementary graphs) to one (for identical graphs).

A comparison of CMs is involved in [33], which proposes a way of suggesting additional concepts and learning materials to the instructional designer during the creation of the CM. The method presented is based on detecting and evaluating concept similarities between the CM that is being developed and other CMs that are used for comparison. Topological similarity between CMs is computed by a usefulness measure, which evaluates the structural (i.e., contextual) correspondence between the CMs, and assesses the viability of the connections between the concepts (i.e., between the related learning materials) in the maps. In [40] Marshall et. al. present a method for matching elements or parts of CMs, based on a similarity flooding algorithm with the aim of supporting the comparison and merging of maps. Another research area that is very close to the present study regards the scoring methods for the similarity/dissimilarity of ontologies. CMs are very similar to ontologies as they both describe a domain of concepts (classes) and relationships. Moreover, ontologies are more expressive than CMs, so it is possible to obtain a CM from an ontology [26], and it is also reasonable to modify some of the similarity checks used for ontologies and to apply them to CM similarity. For ontologies, a similarity measure based on a rough set/concept lattice is presented in [74]. Furthermore, [32] describes a graph-based similarity measure using similarity graphs to represent domain ontologies, while [55] proposes a feature-based approach to ontology similarity and also discusses a classification method.

C. GRAPH MATCHING PROBLEMS

As is evident from the above, the issue of CM similarity relates to the family of graph matching problems. Graph matching has applications in many fields of research, including computer vision [34], handwriting and fingerprint image recognition [63] as well as applications in medicine [28] and chemistry [62]. We find several algorithmic techniques for scoring graph similarity such as subgraph isomorphism [68], computation of the Maximum Common Subgraph (MCS) through maximum clique [35], min-max [30] and Expectation Maximization [16]. The distance between two graphs is usually computed as 1 minus the ratio of a similarity measure (such as the MCS cardinality, or the number of nodes in the larger graph, or in the union of the graphs [70]). Unfortunately, both subgraph isomorphism and MCS problems are known to be NP-complete [13], [22]. However, it is still possible to compute the MCS when graphs have a small number of nodes and edges. Given this assumption, and with some further requirements, it has been proven that MCS-based distances can enjoy the properties of metrics. Other proposals based on MCS have combined it with the minimum common supergraph, the former being obtainable from the latter and vice-versa. This allows us to take into account both redundant and missing structural information when measuring the similarity between graphs as, for instance, in the case of attributed relational graphs that represent objects [19]. Since MCS determination is a NP-complete problem, different classes of matching problems have been defined under the name of inexact or error-tolerant graph matching [37]. Studies in the literature include the RASCAL algorithm [54], which consists of a two-step procedure: first it is determined whether the two graphs may actually be similar; then, and only if a threshold of similarity is exceeded, the MCS is computed.

Other methods focus on the common nodes of two graphs obtaining a so-called *matching* graph. One such method is the similarity flooding algorithm, used to verify the validity of XML or RDF files against schemas [43]. This is also applied to the computation of similarity for labelled graphs representing cases in Case-Based Reasoning [9].

An alternative way of dealing with graph similarity is computing an edit distance. An edit distance evaluates the cost of performing edit operations, such as the insertion and deletion of nodes and edges, to make the structure of a graph exactly the same as the other graph. Each edit operation may have a different cost. When a sequence of operations is defined, all possible permutations should be explored in order to find the minimal edit cost, which is eventually what makes two graphs different [6]. On the problem of distance between graphs, recent studies indicate the need to consider nodes and edges locally rather than globally [60], [73]. This finding makes it possible to achieve a sub-optimal distance with a computation cost that is polynomial rather than exponential. The efficacy of the final function depends on the technique used for discovering locally dominant sets of nodes and edges [60].

The presentation of literature in this section shows that research work on similarity between CMs is active and well established, yet it still only superficially meets the needs of teachers. In our opinion, the chance to compare CMs according to specific pedagogical criteria would significantly help teachers, allowing them to focus on aspects of interest, and eventually to use their time more efficiently. With these considerations in mind, the proposals for similarity criteria and measures are presented in the sections that follow.

IV. THE SIMILARITY CRITERIA

Similarities in teaching expressed by two CMs can be determined by: i) looking at the arrangement of the common edges; ii) the prerequisite knowledge required by the common concepts; iii) the key concepts expressed by the maps. Consequently, we propose the following three criteria to evaluate the similarity between two CMs with respect to their pedagogical features: 1) Overlapping Edges Degree (JE), 2) Prerequisite Constraints (PC), 3) Knowledge Commonality (KC). The three criteria focus on different aspects of the teaching approaches that are highlighted in the CMs being compared.

JE compares the layouts of edges (relations between nodes) in the maps. Evidently, the occurrence of common nodes in the maps indicates similarity, but the occurrence of common edges (i.e., of nodes common to both maps and connected by the same relations) strengthens this similarity also from the prerequisite point of view.

Furthermore, given a set of common concepts between two CMs, the two maps might present very different learning paths that students might follow in acquiring the same concepts. These are the "learning paths of prerequisites" for the given concepts and PC analyses of how close they are: the closer they are, the more similarly the concepts are reached in the CMs, and the more pedagogically similar the CMs are.

The third criterion, KC, deals more generally with the cognitive load borne by the CMs by considering the extent to which the same concepts are central ("important") in the CM learning paths.

A. THE OVERLAPPING EDGES DEGREE (JE) CRITERION

Given two CMs, say CM_1 and CM_2, a means for assessing their similarity is provided by the Jaccard measure, which is directly proportional to the number of common nodes [27].

This measure is widely used to compute similarity between graphs. It may also be used for concept maps and can indicate to what extent two CMs offer the same concepts. However, it is not able to deal with more in-depth pedagogical approaches, such as how the common concepts are related (possibly in different ways) in the CMs. From a didactic viewpoint this deficiency is a limitation as learning paths built on the same set of concepts can provide very different learning experiences, and so the learning materials that implement such paths are likely to be very different.

To define the Overlapping Edges Degree criterion we suggest revising the Jaccard measure (JE) so that we can take into account the ratio of common edges against the total number of edges in the two CMs. Note that implicitly JE does take into consideration the nodes in common. This is because an edge can be common in the maps *if* it links the same concepts. JE captures the co-presence of the learning contents (the concepts) in the maps together with the learning paths along which the student will follow these concepts.

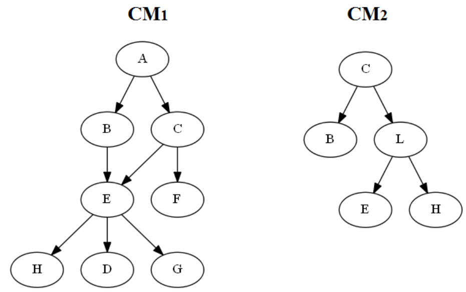

FIGURE 4. Two sample CMs.

For example, Fig. 4 shows two CMs with the following common nodes set CN: $CN \equiv \{B, C, E, H\}$ without any common edge. In other words, the CMs have the same concepts, but they are present in completely different learning paths.

B. THE PREREQUISITE CONSTRAINTS (PC) CRITERION

The decision regarding where to place a concept c in a CM is part of the didactic strategy that the teacher establishes. When a learner reaches c, (s)he has been through specific concepts along the learning path: these concepts are prerequisites for c in CM. The learning path defined in a CM is part of the teacher's learning plan: while learning c, the learner is meant to have already acquired the prerequisite concepts for c.

Let us assume that the same concept c is taught in two CMs, and the learning path to c in one map is different to what it is in the other. Common concepts in the two CMs can be positioned very differently in the two maps as well as there being non-common concepts in the learning paths . The more the learning paths to common concepts vary, the more the CMs reflect the teaching of those concepts differently. We believe that such a disparity leads to different teaching methods on the part of the teachers who designed the two CMs

For example, Fig. 4 shows maps CM_1 and CM_2 with the following common concepts set: {B, C, E, H}. In particular, B has A as a prerequisite concept in CM_1, whereas it has C as a prerequisite in CM_2. From a learning perspective, CM_1 suggests that the knowledge acquired before B is different from what it would be in CM_2. To acquire concept B in CM_1, it would be assumed in the learning material that the learner is already familiar with concept A. In contrast, the material in CM_2 addresses concept B referring to concept C. Consequently, the learning paths elicited by the two maps may suggest a different arrangement of the learning material. This consideration also aligns with *Pedagogical Content Knowledge* in teaching, where teachers plan and organize the learning of a concept according to their students' prior knowledge [44].

C. THE KNOWLEDGE COMMONALITY (KC) CRITERION

A CM is a graph representing a possible arrangement of concepts to be learned. In such an arrangement the position of a concept can indicate something about its prominence in the CM, which is essentially the *importance* of the concept itself. The Knowledge Commonality Criterion takes into account the extent to which the CM is directly or indirectly connected to the concept C. In defining the predecessors of node C as all the nodes that are part of a path to C, the larger the set of the predecessors of C is, the more important C is in the CM. We consider the importance of a concept C in a CM as the size of the cognitive load required by the map in order to learn C.

This criterion can be effective in revealing the different pedagogical prominence of the concepts being compared in the CMs, and it allows us to consider the CMs by adopting other criteria from those described earlier. In Sec. V-C, we will provide an operational definition of the KC-based measure by means of the *Spread Activation* algorithm [15].

V. SIMILARITY MEASURES

In this section we present the measures used to implement the similarity criteria. Before launching into the implementation of the measures, we will present some definitions that are essential to the measures in question. In the following definitions CM, CM_1, and CM_2 are generic CMs represented as graphs, i.e., a set of nodes and edges, where the nodes are concepts and the edges between nodes indicate a *prerequisite of* relationship between the concepts. In particular, the edges are *oriented*, so that if we have two nodes, c and $\overline{c}$, the former is called a *direct predecessor* of the latter if there is an edge from c to $\overline{c}$. A *path* between nodes c and c' is a set of nodes in the graph, $n_1, n_2, ..., n_{k-1}, n_k$, where $n_1 = c$, $n_k = c'$, and for all $i \in [1, k-1]$ n_i is a direct predecessor of n_{i+1}.

Definition 1. *Given CM_1 and CM_2, the set of their common nodes is defined as:*

$$CN = CM_1 \cap CM_2 \tag{1}$$

For instance, the CMs in Fig. 4 have $CN = \{B, C, E, H\}$.

Definition 2. *The set of predecessors of node c in CM is $Predecessors(c, CM) = \{c_j \in CM$ s.t. $\exists$ at least one path between c_j and $c\}$.*

In the example in Fig. 4, we have: $Predecessors(E, CM_1) = \{A, B, C\}$.

We can now proceed by defining the measures that implement the similarity criteria discussed in Sec. IV. These similarity measures will be defined as functions over CMs: JE(CM_1, CM_2), PC(CM_1, CM_2), and KC(CM_1, CM_2). Such functions will be shown to fulfil the "measure properties" [55] given in definition 3:

Definition 3. *A function $dist()$, over pairs of CMs, is a measure if it verifies the following properties (where CM_1 and CM_2 are two generic CMs):*

i) $dist(CM_1, CM_2) \geq 0$ (positiveness)
ii) $dist(CM_1, CM_1) = 0$ (minimality)
iii) $dist(CM_1, CM_2) = dist(CM_2, CM_1)$ (symmetry)

We define a distance for each of our measures as $dist_{JE}(CM_1, CM_2)$, $dist_{PC}(CM_1, CM_2)$ and $dist_{KC}(CM_1, CM_2)$ by

$$dist_{measure}(CM_1, CM_2) = 1 - measure(CM_1, CM_2) \tag{2}$$

with *measure* ranging over $\{JE, PC, KC\}$.

Given two CMs, the higher their similarity measure is, the *lower* the distance is.

A. THE JE MEASURE

This measure implements the *Overlapping Edges Degree* criterion described in Sec. IV-A.

Definition 4. *Given two CMs, CM_1 and CM_2, with $|CM_1| \neq 1$ and $|CM_2| \neq 1$, and given their respective set of edges, TE_1 and TE_2*

$$JE(CM_1, CM_2) = \frac{|TE_1 \cap TE_2|}{|TE_1 \cup TE_2|} \tag{3}$$

Note that $|S|$ is the size of set S, and that two edges are the same when they connect the same nodes in the same direction (a CM is a directed graph).

The range of this measure is $[0, 1]$. $JE(CM_1, CM_2)$ is 0 when the maps have no common edges and 1 when the two maps are identical.

The similarity distance based on this measure is:

$$dist_{JE}(CM_1, CM_2) = 1 - JE(CM_1, CM_2) \tag{4}$$

Such a formulation enjoys the properties of *Positiveness*, *Minimality* and *Symmetry* and this is why we can call it a measure.

Theorem 1. *$dist_{JE}$ fulfils the properties of Def. 3.*

Proof. (*Positiveness*). From formula (4) we have that $0 \leq dist_{JE} \leq 1$ and consequently, following Def. 4, the property is verified because the JE measure is in the range of $JE \in [0, 1]$.

(*Minimality*). From Def. 4, it follows that when two CMs are identical, the ratio is equal to 1 and consequently, following Def. 4, this property is satisfied because the distance measure is 0.

(Symmetry). All the operators of the JE measure are symmetric and, consequently, it follows that: $dist_{JE}(CM_1, CM_2) = dist_{JE}(CM_2, CM_1)$ □

The next section discusses whether or not the JE measure is sensitive to changes in the map according to its relative criterion. Note that this measure is not applicable if the two CMs have only one node.

B. THE PREREQUISITE CONSTRAINTS MEASURE

This measure is designed to reveal the similarity of two CMs with regard to teaching by means of the prerequisite knowledge needed for the common concepts. Given two CMs, namely CM_1 and CM_2, we define the following two sets:

Definition 5. $P_1 = Predecessors(c, CM_1) \cup c$

Definition 6. $P_2 = Predecessors(c, CM_2) \cup c$

For example, looking at concept E in the CMs in Figure 4 we have: $P_1 = Predecessors(E, CM_1) \cup E = \{A, B, C, E\}$ and $P_2 = Predecessors(E, CM_2) \cup E = \{C, E, L\}$ (refer to Def. 2).

To fulfil the PC criterion, the new measure has to take into account three main aspects of the prerequisite knowledge for each node in CN: i) the number of prerequisite concepts required by both CMs; ii) the number of prerequisite concepts not required by the two CMs belonging to set CN (some prerequisite knowledge not available in one of the two maps); iii) the extent of the required knowledge, shared or not, by the two CMs.

Therefore, we propose the Prerequisite Constraints similarity measure (PC) as a combination of the following three factors computed for each common concept C:

$$a = \frac{|P_1 \cap P_2|}{|P_1 \cup P_2|} \tag{5}$$

a is the ratio between the common predecessors and the total number of predecessors of concept C.

$$b = \frac{|CN \cap (P_1 \cup P_2)|}{|P_1 \cup P_2|} \tag{6}$$

b is the number of predecessors of the node C that are available in the two CMs, even if they do not appear as prerequisites in one of the two.

$$c = \frac{min\{|P_1|, |P_2|\}}{max\{|P_1|, |P_2|\}} \tag{7}$$

c tends to zero if, for a given node C, there is a large difference in the number of predecessors.

Given maps CM_1 and CM_2, based on the related P_1, P_2, a, b, c, and CN (set of common nodes of the CMs), we define the PC measure as follows:

Definition 7.

$$\mathrm{PC}(CM_1, CM_2) = \frac{1}{|CN|} \sum_{\forall c_i \in CN} \frac{a_{c_i} + b_{c_i} + c_{c_i}}{3} \tag{8}$$

For each common concept, this measure represents the amount of common knowledge expressed by its predecessors. The associated distance function is

$$dist_{PC}(CM_1, CM_2) = 1 - \mathrm{PC}(CM_1, CM_2) \tag{9}$$

which fulfils the three properties required by a measure.

Theorem 2. *$dist_{PC}$ fulfils the properties of Def. 3.*

Proof. (*Positiveness*). PC is a ratio of absolute values so: $0 \le PC \le 1$.

As $PC(CM_1, CM_2) = 1 - dist_{PC}(CM_1, CM_2)$ we have: $0 \le dist_{PC} \le 1$.

(*Minimality*). Let n be the number of nodes of CM_1. $a_k = n/n$, $b_k = n/n$ and $c = 1$, so $PC(CM, CM) = \frac{1}{n}\sum_{k=1}^{n} 1 = n/n = 1 \Longrightarrow dist_{PC}(CM_1, CM_1) = 0$.

(*Symmetry*). All the operators are symmetric. □

For the example in Fig. 4 we have: $\mathrm{PC}(CM_1, CM_2) = 0.571$ This measure is not applicable in cases where both CMs have only one node.

C. THE KNOWLEDGE COMMONALITY MEASURE

According to Sec. IV-C, the importance of a concept in a CM should reflect how much effort the CM requires for it to be learnt. Such effort can be defined by the number of concepts leading to the concept in question. The KC measure is based on the *Spread Activation* algorithm, a widely used technique with applications in information retrieval and social recommendations [15], [46], as well as to highlight important concepts in ontologies [66]. Basically, all concepts leading to a given concept c are said to activate c, and spread some flux of information to c. c, in turn, spreads some of the received flux over its successors. This means that if c has many predecessors, it will be activated multiple times, and so it retains much flux. In contrast, if c does not have many predecessors, it will not be activated frequently, and so it will end up with a low amount of flux. From the amount of flux retained by c, we can determine the importance of c: the more flux remaining in c, the greater importance c has in the CM.

The spread activation algorithm proposed in this study is described in greater detail in Fig. 5. In the first step of the algorithm, each concept $c \in CM$ is activated, receiving a flux amount of 1. Each concept then retains θ amount of the incoming flux and the rest of the flux is equally spread to its child concepts. By such "spreading", each node receives flux from its direct predecessors as well as from the direct activation that occurs in the previous step. Every time there is incoming flux to concept c, the same logic applies: part of the flux is retained (up to θ), and the rest is spread to the child concepts equally. Following [17], we allow c to retain a maximum of $\theta = 0.3$ units of received flux. (The exception is for the leaves – nodes without successors – which retain *all* the incoming flux). The flux remaining in c, $\varphi(c, CM)$ is called *knowledge-load*: in short, it represents the amount of knowledge required to learn c.

Given two CMs, we define the *Knowledge Commonality* measure KC as the quantity computed taking into account all

```
function SPREAD-ACTIVATION(concept, concept2child, fluxIn, concept2flux) ▷ return void
    children ← concept2child[c]
    currentFlux ← concept2flux[c]
    if children is empty then
        currentFlux ← currentFlux + fluxIn
    else
        if fluxIn ≥ θ then ▷ Each concept retains θ = 0.3 amount of flux at most. The exceeding flux is spread to the child concepts.
            currentFlux ← flux + θ
            fluxIn ← fluxIn − θ
        else
            currentFlux ← currentFlux + fluxIn
            fluxIn ← 0.0
        end if
    end if
    concept2flux[c] ← currentFlux
    if fluxIn > 0.0 then                                  ▷ There is some remaining flux
        fluxIn ← fluxIn/SIZE(children)
        for i = 0toSIZE(children) do
            SPREAD-ACTIVATION(children[i], concept2child, fluxIn, concept2flux)
        end for
    end if
end function

function START-ACTIVATION(CM) ▷ return concept2flux, the array with flux accumulated on each concept of CM at the end of the algorithm.
    concept2flux ← INIT-FLUX(CM)    ▷ Creates a map concept-flux with 0.0 values for all the entries
    concepts ← CONCEPTS(CM)                          ▷ Returns all the concepts of CM
    concept2child ← GET-MAP-CONCEPT2CHILD(CM)        ▷ Returns a map with key the concepts of CM and values the set of child concepts
    for Concept c ∈ SIZE(concepts) do
        SPREAD-ACTIVATION(c, concept2child, 1.0, concept2flux)    ▷ Activate the concept c with one unit of flux and spread the activation
    end for
end function
```

FIGURE 5. The *Spreading Activation* algorithm of the flux running through a concept map CM. Each concept is activated once with 1 unit of flux. The incoming flux is then spread to the child concepts if it exceeds θ, which, in our case, has been set at $\theta = 0.3$.

the differences between the *knowledge-load* of all the nodes in the CMs in question:

$$KC(CM_1, CM_2) = \\ = 1 - \frac{\sum_{c \in CM_1 \cup CM_2} abs(\varphi(c, CM_1) - \varphi(c, CM_2))}{|CM_1| + |CM_2|} \tag{10}$$

Note that if $c \notin CM_i, \varphi(c, CM_i) = 0$ (i=1,2). Moreover, it is $0 \leq \varphi(c, CM) \leq 1, \forall c \in CM$, so we have $0 \leq KC \leq 1$. The associated distance function is

$$dist_{KC}(CM_1, CM_2) = 1 - KC(CM_1, CM_2) \tag{11}$$

which fulfils the three properties required by a measure.

Theorem 3. *$dist_{KC}$ fulfils the properties of Def. 3.*

Proof. (*Positiveness*). From Def. (10), the minuend is always less or equal to 1. In fact, it is always positive and the numerator can be 1 at most. In cases where the two CMs are disjoint graphs, i.e., $CM_1 \cap CM_2 = \emptyset$, each concept C either belongs to CM_1 or to CM_2. In this case the numerator is equal to $|CM_1| + |CM_2|$ and the minuend is equal to 1. Consequently, we have: $dist_{KC}(CM_1, CM_2) = 1$.

(*Minimality*). If the two CMs are exactly the same, i.e., $CM_1 \equiv CM_2$, we have: $\varphi(c, CM) = 0, \forall c \in CM$ and the minuend is equal to 0. In this case, we have: $dist_{KC}(CM_1, CM_2) = 0$.

(*Symmetry*). In Def. 10 the symmetry is provided by the union operator. □

The KC value for the sample CMs shown in Fig. 4 is $dist_{KC}(CM_1, CM_2) = 0.5$.

VI. CASE STUDIES

Evaluating similarity measures between CMs is not a simple task due to the lack of benchmarks and standard test sets. The literature proposes various studies on similar topics. In the case of ontology matching measures, the evaluation of new similarity measures is based on piloted changes to a *seed* ontology and comparing it vs. a set of piloted modifications [53]. In this way, differences between measures are highlighted. Following this approach, we present three ad-hoc groups of case studies where a *seed* CM is incrementally modified by changing its structure step by step. The incremental modifications proceed by applying sequences of atomic changes selected from the following: 1) removing an edge; 2) adding an edge; 3) removing a node; 4) adding a node; 5) swapping two nodes.

Each case study is dedicated to one of the measures we propose: starting from a seed map, several new maps are derived by progressive atomic changes and compared with the seed one. To analyze better whether our measures accurately capture the changes relating to the pedagogical criterion they refer to, we use the Jaccard measure as a baseline. We expect our measures to be more significant than Jaccard when changes to the seed CM refer to the pedagogical criterion behind the measure.

Our experimental goal is to run three case studies, one for each criterion (and thus measure). For each case study, we need to generate pseudo-random variations of the seed CM to reflect pedagogical changes within the scope of a specific criterion. To run such experimentation, we produced a web application[1] ([18]) to generate the pseudo-random CMs and compare them with the seed CM. The web application allows us to import a seed map, select options relating to the number of new maps to generate from the seed CM, define the atomic changes to be used, and select the measures to experiment with. It then pseudo randomly applies the changes to the i-th map to produce the next one. The first map is the seed CM. For each new map, the application automatically evaluates the distance between the newly generated CM and the seed CM by computing the selected similarity measures.

A. CREATING THE SEED CM

The construction of the seed map, i.e., the case study starting point, is a crucial step in our process. We created it based on the following didactic and operational criteria: (i) simplicity: it has to consist of a small yet sufficient number of nodes and edges; (ii) didactic significance: it should derive from an authentic educational context.

Fig. 6 shows an example of the application in use. As a seed CM we selected the map in Fig. 1: it consists of 19 concepts renamed with integer labels for easier reference for the present discussion. In Fig. 6 we see the seed map, the selected measures and the atomic changes to apply. For instance, "Add source" instructs the system to create a new map by adding prerequisite nodes, rather than, say, swap two nodes, or add leaves. The figure also shows that we requested a sequence of four maps to be produced by means of adding two concepts each time. Since the addition of a node involves the addition of a new edge, the "Number of Operations" is four. Fig. 7 shows the four randomly generated CMs.

Fig. 6 also depicts the variations of the measures computed when comparing the seed CM with the new maps. In this case, the figure highlights that the PC measure is very much more reactive than the others, clearly demonstrating the reduction in similarity between the maps. If the teacher is interested in the similarity of maps from the prerequisite viewpoint, then the PC measure seems to be more relevant than the Jaccard baseline as it captures the changes more

[1]http://18.198.178.195/ConceptExp/

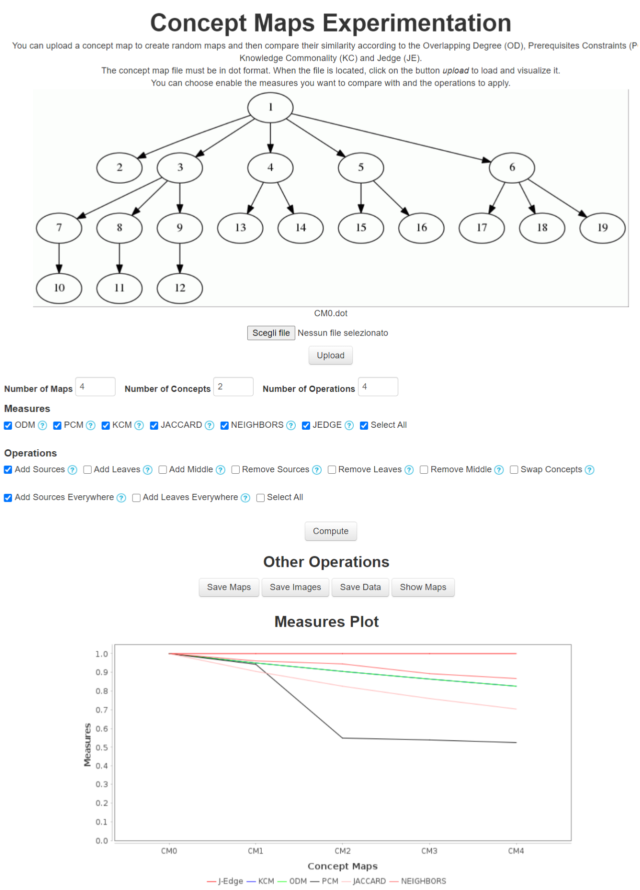

FIGURE 6. The web application ConceptMapExperimentation.

substantially than the latter. Fig. 7 shows that there are eight concepts of difference between the seed CM and the last generated map, which, as we would expect, causes the similarity score to decrease more with PC than with Jaccard.

B. THE FIRST CASE STUDY: JE

Here the seed map is $CM0$, as shown in Fig. 6. To evaluate the JE measure, we instructed a new map to be created by swapping pairs of nodes from the previous map. This was to highlight how the role of the same – and derived – concepts changed in the maps.

Fig. 8 shows the diagrams with the distance (y axis) of each generated map (x axes) and the seed map.

Note that the Jaccard baseline measure is always 1, since the number of nodes never changes. The graph shows a marked tendency towards zero for the value of $JE(CM0, CMi)$, with $i \in [1, 30]$, together with the values from the other measures. The JE curve then becomes more irregular, which can be explained by the fact that the edges that disappeared previously (due to a swap) may well reappear if the same nodes are swapped again later.

In particular, the similarity between the first and the second map decreases abruptly. In fact, if we look at the second map, as illustrated in Fig. 9, we note that the root of the map has

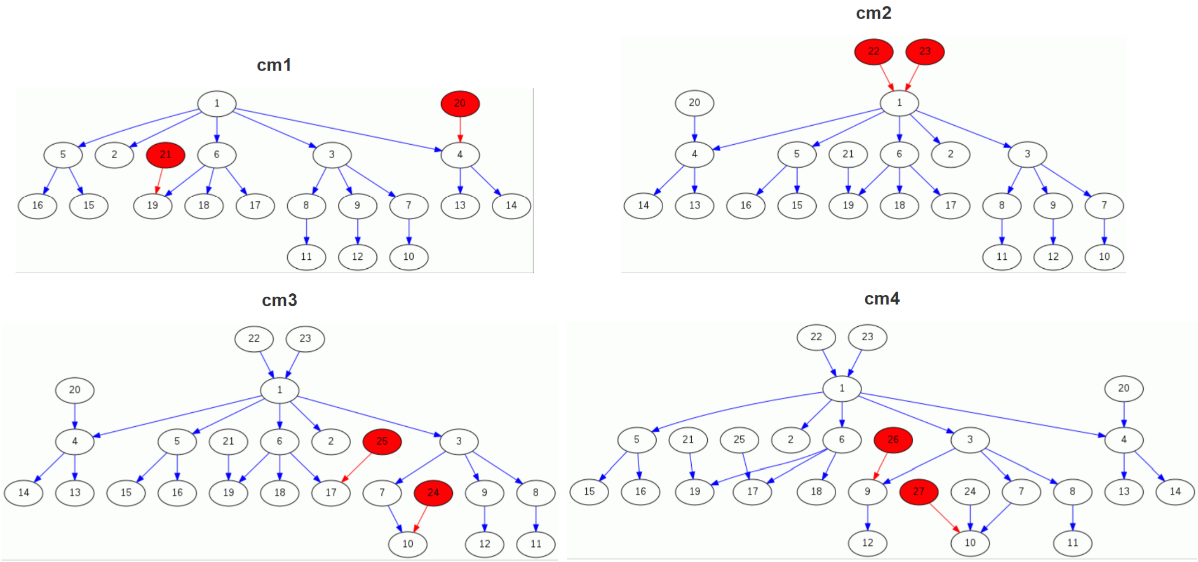

FIGURE 7. The randomly generated maps. Each map was generated by adding the highlighted nodes to the previous one.

changed. Concept 1 has been swapped with concept 12 and concept 11 with 18. Common edges are highlighted in yellow (ten edges), and the cardinality of the union of all the edges of CM0 and CM1 is 24. We thus have: $JE = 10/24 = 0.42$.

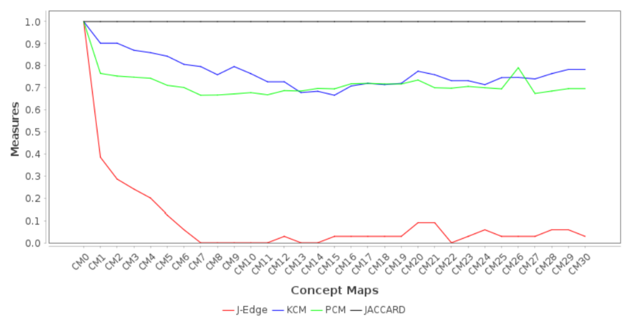

FIGURE 8. Generation of 30 pseudo-random maps: each new map is created by swapping two pair of nodes from the previous map.

In this case study we see that while a measure based solely on the nodes present in a map fails to pick up on the changes in the relationships between the nodes, JE is reactive according to the Overlapping Edges Degree criterion and produces reliable results. After as many as twelve, or more, node swaps performed on a seed map of 19 nodes, it is to be expected that the similarity would be reduced as Fig. 8 shows.

C. SECOND CASE STUDY: PC

The PC measure captures the similarity between two CMs based on an analysis of the prerequisite knowledge required by the common nodes.

In the second case study, a sequence of 30 maps is created from the same seed map as above ($CM0$), by adding two nodes to each new map. These are positioned as new roots (nodes without predecessors) in the graph. The addition of roots is meant to maximize the effect of the modifications (a new root will be a prerequisite for all the nodes in the map that can be reached along a path from the new root). There are four elementary operations here, as adding two nodes implies adding two edges and connecting them to the map.

Fig. 10 shows the similarity values computed for $PC(CM0, CMi)$, with $i \in [1, 30]$ together with the values from the other measures. In this case, we observe that Jaccard, JE, and KC measures decrease monotonically since Jaccard and JE, which are essentially a ratio between the intersection of nodes (or edges) with respect to the union of nodes (or edges) of the maps, maintain the same numerator while the denominator increases regularly by two nodes (or edges). KC also decreases regularly, since the amount of knowledge that has to be acquired before learning a given concept increases regularly with the addition of sources. The PC measure clearly highlights a different behavior with reference to the other measures. In particular, we can observe that while the similarity between CM0 and CM1 rapidly decreases, the similarity between CM1 and CM2 does not change greatly compared to the previous one . In fact, we see that while the nodes are added to the root of CM0 in CM1, so that all the nodes change their prerequisites, the nodes are added closer to the leaves in CM2 and this addition influences a few descendant nodes. Fig. 11 illustrates the CM1 and CM2 maps generated by the application.

D. THIRD CASE STUDY: KC

The KC measure captures the similarity of two CMs based on the amount of *knowledge load* implied by the same concepts in the maps. We represent the knowledge load through the flux computed by a *spread activation algorithm*, as illustrated in Sec. IV-C. From Fig. 12 we note that the PC

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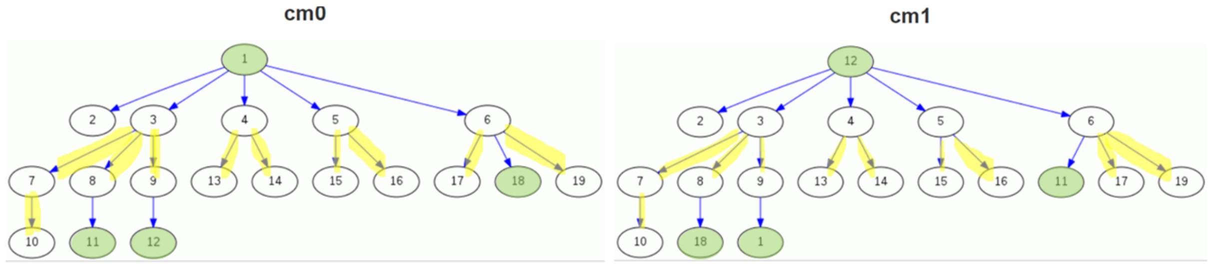

FIGURE 9. The difference between the first and the second generated maps.

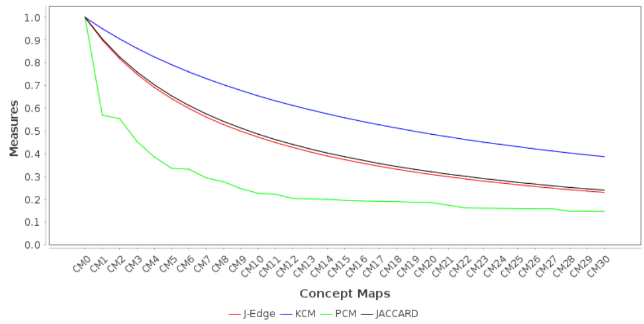

FIGURE 10. Generation of 30 pseudo-random maps obtained by adding two nodes and two edges as source nodes.

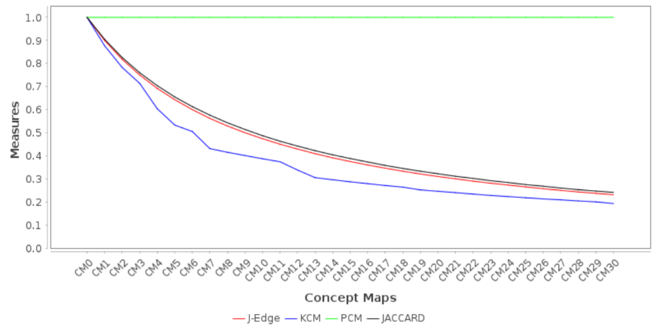

FIGURE 12. 30 pseudo-random maps obtained by adding leaves everywhere in the map.

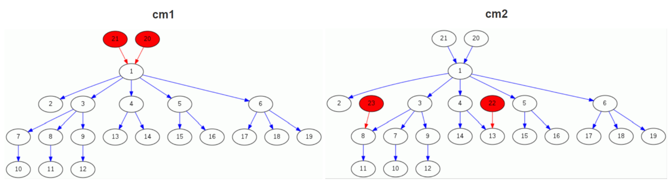

FIGURE 11. The first two maps obtained by adding two nodes and two edges as roots in the maps.

measure does not change, as adding leaves does not affect the prerequisite relations of the existing nodes. On the contrary, Jaccard and JE decrease monotonically, because at each step the denominator of the ratio increases by a fixed quantity corresponding to the two new nodes (and edges).

In a fairly similarly way, the KC measure decreases to the baseline, so in this case we cannot claim that there is a dramatic difference: the measure seems reliable, at least as the baseline. However, as with the other two measures proposed in this paper, the use of a polarized measure, such as KC, is beneficial for the teacher as it allows the decrease in similarity to be identified as a distinct pedagogical criterion rather than a general observation.

This third case study reveals the nature of the proposed KC measure: it comprises the effort needed to learn a concept as a function in the learning path that a learner is required to follow. We can say for this measure too that the addition of leaves close to the root of the map does not greatly affect the workflow, while adding nodes in the deepest branch of the map decreases the measure considerably.

E. DISCUSSION

The experiment, carried out by making the comparison between a seed map and its progressive pseudo-random transformations, confirmed the usefulness of the proposed similarity measures. The case studies showed that each one of the three measures was more responsive to its own inspiring criterion than the other two and the baseline. The JE measure, was much more sensitive than the others to the changes made on the edges and on the swapping of two concepts. The PC measure, was found to be much more sensitive than the others to changes induced on the learning paths by the modification of prerequisite relationships. The KC measure, showed to outperform the others in addressing the growth of learning paths (and of the related cognitive load) due to progressive addition of leaf concepts.

VII. CONCLUSIONS AND FUTURE WORK

In this paper we have presented a family of criteria devised to assess the similarity of two CMs from different viewpoints. The common ground on which we worked in defining the criteria was accomplished with the aim of taking into account the pedagogical characteristics of the maps in addition to the more usual structural/topological concerns. Being interested in the didactic strategies represented by CMs, we considered the advances made in the literature regarding ontology similarity, and eventually defined three criteria.

The first (JE) takes into account the common knowledge shared by two CMs through their common concepts and relationships. The second (PC) analyses the prerequisite relationships occurring among concepts. The third (KC) qualifies the similarity of two CMs by considering how close is the weight (in terms of knowledge flow) of the same concepts in the two maps.

In order to implement the above criteria, we have created three corresponding similarity measures, and tested them alongside three case studies, each one emphasizing the effectiveness of the related criteria.

In terms of future work, we aim to make use of the distance functions devised here in order to offer teachers support when they need to create a new course. In the system we intend to develop a CM retrieval process would automatically collect several CMs, rank them differently, according to our pedagogical distance measures and thus enable the teacher to potentially make use of already existing CMs.

In studying the proposed measures and their educational value, we did not consider in this first phase the semantic issues relating to the possible occurrence of the same concept in the maps through synonymy. Future work will enhance these measures with semantic comparison metrics. This will allow for the semantic similarities and educational aspects of the CMs to be discerned simultaneously in order to minimize the need for human intervention in their classification and use.

REFERENCES

[1] V. Albino, U. Berardi, and R.M. Dangelico. Smart cities: Definitions, dimensions, performance, and initiatives. Journal of Urban Technology, 22(1):3–21, 2015.

[2] A. Anohina-Naumeca, J. Grundspenkis, and M. Strautmane. The concept map-based assessment system: functional capabilities, evolution, and experimental results. International Journal of Continuing Engineering Education and Life-Long Learning, 21(4):308–327, 2011.

[3] G. Asiksoy. Computer-based concept mapping as a method for enhancing the effectiveness of concept learning in technology-enhanced learning. Sustainability, 11, 2019.

[4] D.P. Ausubel, J.D. Novak, and H. Hanesian. Educational Psychology: A Cognitive View (2nd ed.). Holt, Rinehart and Winston, 1978.

[5] J.T. Barksdale, E.D. Ragan, and D.s. McCrickard. Easing team politics in agile usability - a concept mapping approach. In Proc. Agile Conference, IEEE, pages 19–25, 2009.

[6] Horst Bunke and Abraham Kandel. Mean and maximum common subgraph of two graphs. Pattern recognition letters, 21(2):163–168, 2000.

[7] A.J. Cañas, J.W. Coffey, M.J. Carnot, P. Feltovich, R.R. Hoffman, J. Feltovich, and J.D. Novak. A summary of literature pertaining to the use of concept mapping techniques and technologies for education and performance support. Technical report, IHMC, Tech. Rep. Chief of Naval Education and Training, East Lansing, Michigan, 2003.

[8] Alberto J Cañas, Alejandro Valerio, Juan Lalinde-Pulido, Marco Carvalho, and Marco Arguedas. Using wordnet for word sense disambiguation to support concept map construction. In String Processing and Information Retrieval, pages 350–359. Springer, 2003, 2003.

[9] Pierre-Antoine Champin and Christine Solnon. Measuring the similarity of labeled graphs. In Case-Based Reasoning Research and Development, pages 80–95. Springer, 2003.

[10] K. Chan, S. Evans, Y. Ng, M. Chiu, and P. Huxley. A concept mapping study on social inclusion in hong kong. Social Indicators Research, 119(1):121–137, 2014.

[11] Hsi-Cheng Chang and HSU Chiun-Chieh. Using topic keyword clusters for automatic document clustering. IEICE TRANSACTIONS on Information and Systems, 88(8):1852–1860, 2005.

[12] J.W.Jr Cliburn. Using concept maps to sequence instructional materials. Journal of College Science Teaching, 15(4):377–79, 1986.

[13] Diane J Cook and Lawrence B Holder. Mining graph data. John Wiley & Sons, 2006.

[14] P.R.M. Correia. The use of concept maps for knowledge management: from classrooms to research labs. Analytical and Bioanalytical Chemistry, 402, 2012.

[15] Fabio Crestani. Application of spreading activation techniques in information retrieval. Artificial Intelligence Review, 11(6):453–482, 1997.

[16] Andrew DJ Cross and Edwin R Hancock. Graph matching with a dual-step em algorithm. Pattern Analysis and Machine Intelligence, IEEE Transactions on, 20(11):1236–1253, 1998.

[17] Alan Dix, Akrivi Katifori, Giorgos Lepouras, Costas Vassilakis, and Nadeem Shabir. Spreading activation over ontology-based resources: from personal context to web scale reasoning. International Journal of Semantic Computing, 4(01):59–102, 2010.

[18] O. Elsayed, C. Limongelli, F. Sciarrone, M. Lombardi, A. Marani, and M. Temperini. An on-line framework for experimenting with concept maps. In 2019 18th International Conference on Information Technology Based Higher Education and Training (ITHET), pages 1–8, 2019.

[19] Mirtha-Lina Fernández and Gabriel Valiente. A graph distance metric combining maximum common subgraph and minimum common supergraph. Pattern Recognition Letters, 22(6):753–758, 2001.

[20] B. Ferry, J. Hedberg, and B. Harper. How do preserve teachers use concept maps to organize their curriculum content knowledge. Journal of Interactive Learning Research, (9):83—-104, 1997.

[21] L.A. Freeman. The effects of concept maps on requirements elicitation and system models during information systems development. In First Int. Conference on Concept Mapping, Pamplona, Spain, pages 257–264, 2004.

[22] Michael R Garey and David S Johnson. Computers and intractability: a guide to the theory of np-completeness. 1979. San Francisco, LA: Freeman, 1979.

[23] T.E. Goldsmith and D.M. Davenport. Assessing structural similarity of graphs. In R.W. Schvaneveldt, editor, Pathfinder associative networks: Studies in knowledge organization, pages 75—87. Ablex, Norwood, NJ, USA, 1990.

[24] T.E. Goldsmith, P.J. Johnson, and W.H. Acton. Assessing structural knowledge. J. of Educational Psychology, 1(83):88–96, 1991.

[25] E. Gomede, F.H. Gaffo, G.U. Briganó, R.M. De Barros, and L. De Souza Mendes. Application of computational intelligence to improve education in smart cities. Sensors, 18(1), 2018.

[26] V. Graudina and J. Grundspenkis. Concept map generation from owl ontologies. In Third Int. Conf. on Concept Mapping, Tallinn, Estonia and Helsinki, Finland, 2008, 2008.

[27] David J. Hand, Heikki Mannila, and Padhraic Smyth. Principles of Data Mining. MIT Press, 2001.

[28] Kostas Haris, Serafim N Efstratiadis, Nicos Maglaveras, Costas Pappas, John Gourassas, and George Louridas. Model-based morphological segmentation and labeling of coronary angiograms. Medical Imaging, IEEE Transactions on, 18(10):1003–1015, 1999.

[29] C.M. Harris and Z. Shenghua. Concept mapping: A critical thinking technique. Education, 134(2):207—-211, 2013.

[30] Lawrence Hubert. Min and max hierarchical clustering using asymmetric similarity measures. Psychometrika, 38(1):63–72, 1973.

[31] Gloria Yi-Ming Kao, Kuan-Chieh Chen, and Chuen-Tsai Sun. Using an e-learning system with integrated concept maps to improve conceptual understanding. Int. J. of Instructional Media, 37(2):151–162, 2010.

[32] JuHum Kwon, O-Hoon Choi, Chang-Joo Moon, Soo-Hyun Park, and Doo-Kwon Baik. Deriving similarity for semantic web using similarity graph. Journal of Intelligent Information Systems, 26(2):149–166, 2006.

[33] D.B. Leake, A. Maguitman, and A. Cañas. Assessing conceptual similarity to support concept mapping. In Fifteenth Int. Florida Art. Int. Research Conf. (2002), 2002.

[34] Marius Leordeanu, Rahul Sukthankar, and Martial Hebert. Unsupervised learning for graph matching. International journal of computer vision, 96(1):28–45, 2012.

[35] Giorgio Levi. A note on the derivation of maximal common subgraphs of two directed or undirected graphs. Calcolo, 9(4):341–352, 1973.

[36] D. Liu, R. Huang, and M. Wosinski. 1 - development of smart cities: Educational perspective. In D. Liu, R. Huang, and M. Wosinski, editors, Smart Learning in Smart Cities, Lecture Notes in Educational Technology. Springer, 2017.

[37] Lorenzo Livi and Antonello Rizzi. The graph matching problem. Pattern Analysis and Applications, 16(3):253–283, 2013.

[38] G.R. MacKinnon and M. Keppell. Concept mapping: A unique means for negotiating meaning in professional studies. Journal of Educational Multimedia and Hypermedia, 14(3):291—-315, 2015.

[39] N. Maharjan and V. Rus. A concept map based assessment of free student answers in tutorial dialogues. In Proc. of Artificial Intelligence in Education, AIED 2019, number 11625 in LNCS. Springer, 2019.

[40] Byron Marshall, Hsinchun Chen, and Therani Madhusudan. Matching knowledge elements in concept maps using a similarity flooding algorithm. Decision Support Systems, 42(3):1290–1306, 2006.

[41] J.R. McClure, B. Sonak, and H.K. Suen. Concept map assessment of classroom learning: Reliability, validity, and logistical practicality. J. of Research in Science Teaching, 4(36):475—-492, 1999.

[42] D. McLinden. And then the internet happened: Thoughts on the future of concept mapping. Eval Program Plann., 60:293–300, 2015.

[43] Sergey Melnik, Hector Garcia-Molina, and Erhard Rahm. Similarity flooding: A versatile graph matching algorithm and its application to schema matching. In Data Engineering, 2002. Proceedings. 18th International Conference on, pages 117–128. IEEE, 2002.

[44] Punya Mishra and Matthew Koehler. Technological pedagogical content knowledge: A framework for teacher knowledge. The Teachers College Record, 108(6):1017–1054, 2006.

[45] B. Moon, R.R. Hoffman, J. Novak, and A. Canas. Applied Concept Mapping: Capturing, Analysing, and Organizing Knowledge. CRC Press, 2011.

[46] Ana Régia De M Neves, Álvaro Marcos G Carvalho, and Célia G Ralha. Agent-based architecture for context-aware and personalized event recommendation. Expert Systems with Applications, 41(2):563–573, 2014.

[47] J.D. Novak. Concept mapping: A useful tool for science education. Journal of Research in Science Teaching, (10):923—-949, 1990.

[48] J.D. Novak. Meaningful learning: The essential factor for conceptual change in limited or inappropriate propositional hierarchies leading to empowerment of learners. Science Education, 5(86):548–571, 2002.

[49] J.D. Novak and A.J. Cañas. The theory underlying concept maps and how to construct and use them. In IADIS Int. Conf. e-Learning 2007, pages 443–450, 2007.

[50] J.D. Novak, D.B. Gowin, and G. Johansen. The use of concept mapping and knowledge vee mapping with junior high school science students. Science Education, 5(67):625–645, 1983.

[51] Joseph D Novak. Learning how to learn. Cambridge University Press, 1984.

[52] Uzma Omer, Muhammad Shoaib Farooq, and Adnan Abid. Cognitive learning analytics using assessment data and concept map: A framework-based approach for sustainability of programming courses. Sustainability, 12(17), 2020.

[53] Lorena Otero-Cerdeira, Francisco J Rodríguez-Martínez, and Alma Gómez-Rodríguez. Ontology matching: A literature review. Expert Systems with Applications, 42(2):949–971, 2015.

[54] John W Raymond, Eleanor J Gardiner, and Peter Willett. Rascal: Calculation of graph similarity using maximum common edge subgraphs. The Computer Journal, 45(6):631–644, 2002.

[55] David Sánchez, Montserrat Batet, David Isern, and Aida Valls. Ontology-based semantic similarity: A new feature-based approach. Expert Systems with Applications, 39(9):7718–7728, 2012.

[56] H. Schaffers, N. Komninos, and M. Pallot. Smart cities and innovation ecosystems sustained by the future internet, white paper, 2012. Accessed 2020.11.30 www.fireball4smartcities.eu.

[57] H. Schaffers, C. Ratti, and N. Komninos. Special issue on smart applications for smart cities - new approaches to innovation: Guest editors' introduction. J. theor. appl. electron. commer. res., 7(3):923—-949, 2012.

[58] R. Schipper and G. Silvius. Opportunities for the circular economy in smart cities: The role of digital technology. International Journal of Information Systems and Social Change, 10(4), 2019.

[59] R.W. Schvaneveldt, F.T. Durso, and D.W. Dearholt. Network structures in proximity data. In G.H. Bower, editor, The psychology of learning and motivation (Vol. 24), pages 249–284. Academic Press, 1989.

[60] Francesc Serratosa and Xavier Cortés. Graph edit distance: moving from global to local structure to solve the graph-matching problem. Pattern Recognition Letters, 65:204–210, 2015.

[61] Y-A Shih and B. Chang. Empirical study on the effects of social network–supported group concept mapping. Research and Practice in Technology Enhanced Learning, 15, 2020.

[62] Mariya I Skvortsova, Igor I Baskin, Ivan V Stankevich, Vladimir A Palyulin, and Nikolai S Zefirov. Molecular similarity. 1. analytical description of the set of graph similarity measures. Journal of chemical information and computer sciences, 38(5):785–790, 1998.

[63] Sébastien Sorlin and Christine Solnon. Reactive tabu search for measuring graph similarity. In Graph-Based Representations in Pattern Recognition, pages 172–182. Springer, 2005.

[64] M.L. Starr and J.S. Krajcik. Concept maps as heuristic for science curriculum development: Toward improvement in process and product. Journal of Research in Science Teaching, (27):987–1000, 1990.

[65] M. Strautmane. Concept map-based knowledge assessment tasks and their scoring criteria: an overview. In Concept Maps: Theory, Methodology, Technology. Proc. of the Fifth Int. Conference on Concept Mapping, 2012, 2012.

[66] Shengtao Sun, Jibing Gong, Jijun He, and Siwei Peng. A spreading activation algorithm of spatial big data retrieval based on the spatial ontology model. Cluster Computing, 18(2):563–575, 2015.

[67] Jennifer Turns, Cynthia J Atman, and Robin Adams. Concept maps for engineering education: A cognitively motivated tool supporting varied assessment functions. Education, IEEE Transactions on, 43(2):164–173, 2000.

[68] Julian R Ullmann. An algorithm for subgraph isomorphism. Journal of the ACM (JACM), 23(1):31–42, 1976.

[69] C. van Boxtel, J. van der Linden, E Roelofs, and G. Erkens. Collaborative concept mapping: Provoking and supporting meaningful discourse. Theory Into Practice, 41(1):40—-46, 2002.

[70] Walter D Wallis, Peter Shoubridge, M Kraetz, and D Ray. Graph distances using graph union. Pattern Recognition Letters, 22(6):701–704, 2001.

[71] M. Wang, B. Cheng, J. Chen, N. Mercer, and P.A. Kirschner. The use of web-based collaborative concept mapping to support group learning and interaction in an online environment. The Internet and Higher Education, 34:28—-40, 2017.

[72] A. White. The use of concept mapping in ecological management: A case study involving grassland ecosystem in victoria, australia. In B. Moon, R.R. Hoffman, J. Novak, and A. Canas, editors, Applied Concept Mapping: Capturing, Analysing, and Organizing Knowledge. CRC Press, 2011.

[73] Laura A Zager and George C Verghese. Graph similarity scoring and matching. Applied mathematics letters, 21(1):86–94, 2008.

[74] Yi Zhao and Wolfgang Halang. Rough concept lattice based ontology similarity measure. In 1st Int. Conf. on Scalable Information Systems, page 15. ACM, 2006.

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