



## A preliminary approach based on numerical simulations for the design of a PWV-Varying arterial simulator

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### ABSTRACT

Nowadays, in both scientific and medical communities, Pulse Wave Velocity (PWV) is commonly considered as a predicting factor of cardiovascular diseases. Such issue leads to the necessity of accurate PWV measurements as PWV non-local assessment is currently affected by low accuracy distance measurements. Despite the great interest on such predictor, a reference standard for PWV assessment is still awaited. In sight of the development of a reference standard able to reproduce PWV variations, the necessity arises of a mathematical model involving the main physical quantities influencing PWV. Therefore, the present study aims at giving a contribution in the development of such model as well as in the implementation of numerical simulations for the design of a sensorized PWV-varying arterial simulator as regards its metrological and physical characteristics. Despite the promising results, further developments could include a greater system parametrization, a system miniaturization as well as the investigation of other metrological characteristics.

### 1. Introduction

The Pulse Wave Velocity (PWV) has been defined as the propagation velocity of the pulse wave traveling along the arterial tree due to heart contraction [1]. In the current scientific literature, the strict relationship between such physical quantity and arterial stiffness is commonly accepted, therefore PWV assumes a key role in the early diagnosis and therapy of cardiovascular diseases worldwide [2,3]. To date, PWV is assessed as the ratio between the distance between two recording sites and the time delay a pulse wave takes to travel from one site to another (i.e., transit time) [4]. Despite the great interest around PWV on behalf of the scientific and medical community, *in vivo* PWV assessment over a long arterial portion is still affected by high distance measurement uncertainty [5]. Such issue could be overcome by designing and implementing measurement methods where PWV is locally assessed. This leads to the necessity of developing *in vitro* experimental set-up (e.g., arterial simulator) able to reproduce PWV variations, therefore constituting a reference standard for PWV assessment, which nowadays is still awaited. In the current state of the art, many studies focused their attention on *ad hoc* experimental set-up to investigate hemodynamic phenomena as well as the relationships subsisting between pressure, flow and the biomechanical and/or geometrical characteristics of the tube under analysis without any attempt to assess a reference PWV standard [6,7]. Furthermore, from a metrological point of view, the major part of the proposed solutions lacks a metrological characterization of the employed measurement chain.

The goal of the present study is to develop a mathematical model for an arterial simulator design and its measurement equipment, involving all the geometrical and mechanical characteristics influencing pulse wave velocity, starting from the most simple and consolidated ones (i.e., commonly accepted in the current scientific literature) [8,9]. Such a

model has been implemented in MATLAB environment to numerically simulate PWV variations according to the parametrization of the main physical quantities involved (i.e., geometrical and mechanical). From such simulations, the transducers to be included in the design and development of a novel arterial simulator as well as the corresponding measurement chain, have been selected and their main metrological characteristics (e.g., measurement range, accuracy and bandwidth) have been evaluated.

### 2. Materials and methods

The numerical simulation presented in this preliminary study has been implemented starting from the mathematical model already proposed in [10]. By considering an elastic tube of given Young Modulus  $E$ , under inner and outer pressures,  $p_i$  and  $p_o$ , respectively, the following relationship for the static radial displacement  $u$  has been obtained:

$$u(r) = \frac{r r_o^2}{E(r_o^2 - r_i^2)} \left[ (1 - \nu) \left( p_i \frac{r_i^2}{r_o^2} - p_o \right) + (1 + \nu) \frac{r_i^2}{r^2} (p_i - p_o) \right] \quad (1)$$

where  $r_i$  and  $r_o$  are the inner and outer tube radii (with  $r_o - r_i \ll r_o$ ) respectively, and  $\nu$  the Poisson ratio. When an over-pressure  $P$  is applied to the tube, the total inner pressure  $p_{i,tot}$  is given by the sum  $p_i + P$  therefore causing a maximum radial displacement  $u_{max}$  whose expression can be retrieved from (1) for  $r = r_o$ :

$$u_{max} = \frac{2r_o r_i^2 (p_{i,tot} - p_o)}{E(r_o^2 - r_i^2)} \quad (2)$$

Through the Bramwell-Hill formula [8], the relationship between the PWV, the variation of cross-sectional lumen area  $\Delta A$  and the pressure variation  $\Delta p = p_{max} - p_{min}$  can be derived as follows:

$$PWV = \sqrt{\frac{A_{min} \Delta P}{\rho \Delta A}} \quad (3)$$

where  $\rho$  is the inner fluid density and  $A_{min} = \pi (r_o + u)^2$  is the cross-sectional lumen area before the over-pressure  $P$  is applied. In particular, the variation of cross-sectional lumen area  $\Delta A$  can be expressed as the difference between the maximum cross-sectional lumen area  $A_{max} = \pi (r_o + u + \Delta u_{max})^2$  after the over-pressure application and  $A_{min}$  as follows:

$$\Delta A = A_{max} - A_{min} = \pi \Delta u_{max} (\Delta u_{max} + 2r_o + 2u) \quad (4)$$

where  $\Delta u_{max}$  is the maximum radial displacement variation given by

$$\Delta u_{max} = u_{max} - u(r_o) \quad (5)$$

Therefore (3) can be rewritten as a function of both  $P$  and  $\Delta u_{max}$ :

$$PWV = \sqrt{\frac{P}{\rho \Delta u_{max} (\Delta u_{max} + 2r_o + 2u)}} \quad (6)$$

Nowadays PWV is determined both in *in vitro* and *in vivo* experiments as the ratio of the distance between two different sensors  $\Delta s$  and the transit time  $\Delta t$ :

$$PWV = \frac{\Delta s}{\Delta t} \quad (7)$$

From (7), it is possible to retrieve the maximum frequency (i.e., the minimum sensor bandwidth) of the pulse wave propagation phenomenon, when the distance  $\Delta s$  between the two sensors is fixed, as follows:

$$f_{max} = \left( \frac{\Delta s_{min}}{PWV_{max}} \right)^{-1} \quad (8)$$

On the basis of (1)-(8), a custom-written algorithm has been developed in MATLAB environment in order to carry out the numerical simulations in sight of the arterial simulator design. In such simulations different components should be included: the elastic element which has been chosen with geometrical characteristics (i.e., inner and outer tube radii) as similar as possible to the ones of an adult healthy subject's aorta [11], whereas the mechanical characteristics (i.e., Poisson ratio and Young Modulus) have been retrieved from the ones of natural rubber [12]. Furthermore, in the numerical simulation, the elastic element is supposed to be straight and infinitely long, to neglect pulse wave reflections as well as sharp travel direction changes. As regards the inner fluid flowing in the simulated tube, it has been supposed to be distilled water. The parameters describing the abovementioned simulated tube as well as the inner fluid characteristics have been reported in Table 1 and they have been kept constant throughout all the simulation steps.

### 3. Results

The ideal arterial simulator has been designed according to the following three steps for its metrological characteristics determination. In all cases, the over-pressure  $P$  has been kept constant to 5.3 kPa ( $\sim 40$  mmHg), by supposing a physiological over-pressure due to heart pumping action between systolic and diastolic phases.

#### 3.1. Inner pressure evaluation

In the first step, the tube inner pressure  $p_i$  has been evaluated for a

**Table 1**  
Geometrical and mechanical characteristics of the simulated tube.

Parameter	Description	Value
$r_i$ (mm)	Inner radius of the elastic tube	12.5
$r_o$ (mm)	Outer radius of the elastic tube	14.0
$\rho$ (kg·m <sup>-3</sup> )	Inner fluid density	1000
$\nu$	Poisson's ratio	0.5
$E$ (MPa)	Tube Young Modulus	1.5

pathological PWV value of 15 m s<sup>-1</sup> [13,14], by imposing an outer pressure  $p_o = 0.05 \cdot p_i$  on the basis of *in-vitro* experiments carried out in [10]. The computation of  $p_i$  value has been carried out by inverting (6) after applying (1) and (5). The obtained  $p_i$  value has been reported in Table 2.

#### 3.2. Outer pressure evaluation

In the second step, from the  $p_i$  value computed above, it has been possible to retrieve  $p_{o,j}$  ( $j = 1, \dots, 6$ ) values corresponding to PWV ranging from 10 to 15 m s<sup>-1</sup> with steps of 1 m s<sup>-1</sup>. Such range has been chosen to simulate a progressive tube stiffening which is related to PWV values reflecting human pathological conditions [13,14]. The  $p_{o,j}$  values have been computed by inverting (6) after applying (1) and (5). Such step allows to simulate both an increment of PWV as well as the consequent decrement of  $\Delta u_{max}$  due to a stiffening process analogous to the one occurring in human physiology. The obtained  $p_{o,j}$  values have been reported in Table 2.

#### 3.3. Monte Carlo simulation

In the third step, PWV and  $\Delta u_{max}$  measurement accuracies have been retrieved through the implementation of a series of Monte Carlo Simulations (MCSs). In fact, in the current scientific literature, MCS is a widespread and robust tool, employed in many different studies [15–20] for the uncertainty evaluation of software-derived measurements. Six MCSs have been carried out for all the  $p_{o,j}$  values with a number of iterations of 10<sup>5</sup> each, by assigning a distribution to the quantities involved in (5) and (6). The distributions and their characteristics have been reported in Table 2. The inner and outer radii standard deviations (SDs) have been retrieved from [21]. The inner and outer pressures SDs have been chosen according to the measurement accuracy reported in digital manometers datasheets as 0.1% with respect to a full scale of 1 MPa, by supposing a 95% confidence level. The SD of the over-pressure  $P$  has been set to 0.7 kPa ( $\sim 5$  mmHg). The standard deviation of the fluid density  $\rho$  has been set at 1 kg m<sup>-3</sup> considering the density variation for temperature changes of 5 °C. Finally, the Young Modulus SD has been chosen to 6% with respect to its mean value accordingly to the percentage errors previously determined in [21].

#### 3.4. Discussion

The PWV and  $\Delta u_{max}$  results according to the variation of  $p_o$  together with the PWV percentage error  $\varepsilon_{PWV\%}$  have been reported in Table 3. As shown in Fig. 1, PWV increases for decreasing outer pressure values. Moreover, the obtained PWV percentage error for all the  $p_o$  values is  $\leq 5\%$ . The PWV standard deviation values computed with MCS suggest that an arterial simulator designed with the abovementioned characteristics could be able to reproduce PWV values where it is possible to distinguish about the tenth of m·s<sup>-1</sup>, in theory.

The outcomes show a decreasing trend of  $\Delta u_{max}$  in correspondence of

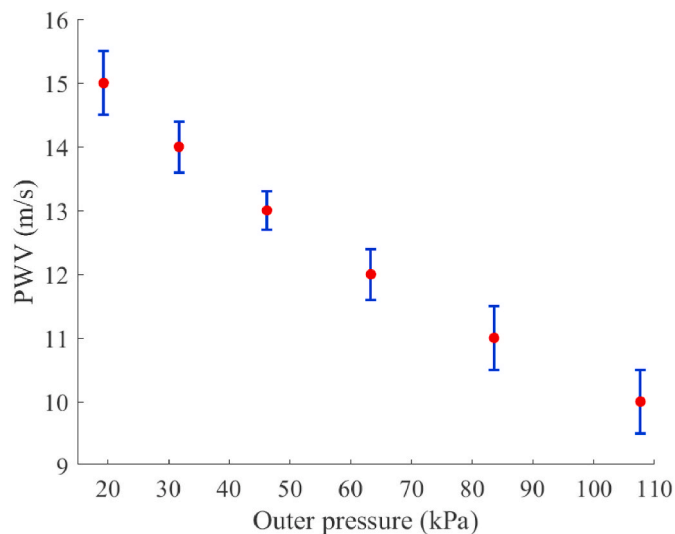
**Table 2**  
Parameters distributions in Monte Carlo Simulations.

Parameter	Measurement unit	Distribution	$\mu \pm$ SD
$r_i$	mm	Uniform	12.5 $\pm$ 0.4
$r_o$	mm	Uniform	14.0 $\pm$ 0.4
$p_i$	kPa	Gaussian	386.8 $\pm$ 0.5
$p_{o,j}$	kPa	Gaussian	107.7 $\pm$ 0.5
			83.6 $\pm$ 0.5
			63.3 $\pm$ 0.5
			46.2 $\pm$ 0.5
$P$	kPa	Uniform	5.3 $\pm$ 0.7
$\rho$	kg·m <sup>-3</sup>	Uniform	1000 $\pm$ 1
$E$	MPa	Gaussian	1.5 $\pm$ 0.1

**Table 3**  
PWV and  $\Delta u_{max}$  results for each  $p_o$  value.

$j$	$p_o$ (kPa)	$\Delta u_{max}$ (mm)	PWV ( $m \cdot s^{-1}$ )	$\epsilon_{PWV\%}$
1	$107.7 \pm 0.5$	$1.00 \pm 0.36$	$10.0 \pm 0.5$	5.0%
2	$83.6 \pm 0.5$	$0.89 \pm 0.36$	$11.0 \pm 0.5$	4.5%
3	$63.3 \pm 0.5$	$0.79 \pm 0.36$	$12.0 \pm 0.4$	3.3%
4	$46.2 \pm 0.5$	$0.71 \pm 0.36$	$13.0 \pm 0.3$	2.3%
5	$31.7 \pm 0.5$	$0.64 \pm 0.36$	$14.0 \pm 0.4$	2.9%
6	$19.3 \pm 0.5$	$0.59 \pm 0.36$	$15.0 \pm 0.5$	3.3%

The values have been reported as mean  $\pm$  SD.  $\epsilon_{PWV\%}$  is the PWV percentage error.



**Fig. 1.** PWV and outer pressure relationship with error bars.

increasing PWV values therefore successfully reproducing the tube stiffening process, simulated through the outer pressure decrease. From the outcomes retrieved in MCSs it is possible to assess the measurement range (e.g., until at least 1 mm) of the displacement transducers which should be able to detect the pulse wave passage along the elastic tube through the maximum radial displacement variation  $\Delta u_{max}$ . Furthermore,  $\Delta u_{max}$  standard deviations allow to establish the accuracy of the abovementioned transducers, e.g., at least tenth of mm.

In addition, from these preliminary results it is also possible to estimate the minimum bandwidth that allows the transducer to detect the pressure wave passage. In particular, by supposing the two transducers diameter of about 10 mm each, as well as a fixed distance  $\Delta s$  of 5 cm between the transducers central axis and, by considering the maximum PWV provided in this study ( $15 m \cdot s^{-1}$ ), the minimum bandwidth can be estimated from (8) to be 300 Hz. Finally, from the considerations above, it can be assessed that (a) the transducers, satisfying those specifications and currently available on the market, are displacement transducers such as inductive, capacitive and optical and (b) the measurement chain should involve an analog to digital converter (ADC) able to sample the input signal at a sampling frequency of at least 1 kHz.

#### 4. Conclusions

In the present study, numerical simulations implemented through an *ad hoc* algorithm developed in MATLAB, for the design of an ideal arterial simulator able to reproduce pulse wave velocity variations have been proposed. As a first step a mathematical model has been derived from consolidated and commonly accepted ones, by considering both geometrical and mechanical quantities involved. In particular, such model resulted from the combination of the Bramwell-Hill formula and the static radial displacement mathematical expression with the periodic

pressure variation simulating the heart pumping action. The second step has been the implementation of a series of Monte Carlo Simulations to retrieve useful data for the metrological characterization (e.g., measurement range, accuracy and bandwidth) of some elements involved in the measurement chain (e.g., pressure manometers, displacement transducers). The outcomes seem to confirm that with decreasing outer pressure, by keeping constant the elastic tube inner pressure, PWV increases and the maximum radial displacement variation decreases, therefore properly simulating the tube stiffening (i.e., progressive pathological condition). Despite the promising results, the algorithm proposed may be improved by (a) studying PWV variations according to changes of a higher number of mechanical quantities involved (e.g., Young Modulus, tube inner pressure), (b) investigating the system output with different geometrical characteristics in sight of a system miniaturization and (c) including further metrological characteristics (e.g., sensitivity, resolution). A further future development is the realization of an *ad hoc* experimental set-up on the basis of the numerical simulations data. Such set-up could be used for the validation of the proposed model by the comparison between the PWV values impressed on the basis of the simulator physical characteristics and the ones experimentally retrieved from the ratio of the distance between the two displacement transducers and the transit time.

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