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ON THE SECOND STAGE OF THE CAMBRIDGE CAPITAL CONTROVERSY

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Abstract. The second stage of the Cambridge capital controversy concerns the neo-Walrasian theory of value and distribution. Since production is not understood in this theory as employing factors of production but rather commodities, that is goods and services with date and place of delivery, some scholars maintained that it is not affected by the problems that emerged, during the first stage of the controversy, as regards the conception of capital as a factor of production and the rate of interest as the price for its use. The reply of the 'neo-Ricardians' was based on two arguments. The first regarded the relevance of the new notions of equilibrium adopted in the neo-Walrasian approach, with particular reference to temporary and Arrow–Debreu equilibria, and the second the possibility that the phenomena of re-switching and reverse capital deepening, by affecting the working of the saving-investment market, could cause equilibrium multiplicity and instability also in a neo-Walrasian framework.

Keywords. Arrow–Debreu equilibrium; Cambridge capital controversy; Neo-Walrasian theory; Saving-investment market

1. Introduction

The production process can be viewed either as circular, with commodities produced by means of commodities together with labour and natural resources, or as a 'one-way avenue' from the three factors of production – labour, land and capital – to final commodities (cf. Sraffa, 1960, p. 93). The adoption of one view or the other has important implications for the analysis of income distribution. In particular, the explanation of income distribution grounded on the class conflict is naturally related to the first and its attendant concept of social surplus,[1] whereas the idea that the shares of wages and capital income in the GDP depend on the productivity of factors is clearly linked to the second.

The validity of the one-way view of the production process was challenged during the early stage of the Cambridge capital controversy that spans the period from the publication of Robinson's article of 1953 to the symposium in the *Quarterly Journal of Economics* of 1966. The point at issue in this stage concerned the explanation of the rate of interest as the price for the use of a special factor of production called 'capital'. As is known, the conclusion was that this interpretation of the interest rate is unacceptable because the total value of a set of heterogeneous capital goods cannot be regarded as the quantity of a factor of production (see Section 2 for further details).

It is widely recognized (cf. Garegnani, 2010, 2012; Harcourt, 2015; Lazzarini, 2015) that the Cambridge capital controversy also had a second stage, which started with the Garegnani-Bliss debate in the *Review*

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of Economic Studies in 1970 and has not yet arrived at a definitive conclusion, even though some results have been obtained.

This stage concerns the neo-Walrasian theory of value and distribution, in which, as is known, capital is not seen as a factor of production. In actual fact, this approach sees production processes not as employing factors of production, but rather commodities, understood as goods and services with specified dates and places of delivery.[2] The neo-Walrasians therefore claimed that their theory of supply and demand was immune to the results obtained during the first stage by the 'neo-Ricardians', the economists who follow Sraffa's approach.

The neo-Ricardian reaction to this claim developed along two lines. The first regarded the significance of the notions of equilibrium adopted in the neo-Walrasian approach, with particular reference to temporary and Arrow–Debreu equilibria. The second instead concerned the possibility that the phenomena of re-switching and reverse capital deepening,[3] by affecting the working of the saving-investment market, might bring about equilibrium multiplicity and instability in a neo-Walrasian framework as well.

The present paper addresses the literature on these points. After a brief summary of the first stage of the controversy (Section 2), it undertakes a reconstruction of the central features of the Garegnani-Bliss debate (Section 3) and the neo-Walrasian standpoint (Section 4). As regards the neo-Ricardian response, Section 5 discusses the critique of the new notions of equilibrium. The argument about equilibrium multiplicity and instability caused by the saving-investment market is instead divided into two parts, one regarding Arrow–Debreu models (Section 6) and the other stationary models (Section 7). Appendices A and B focus on technical aspects.

2. The First Stage of the Controversy: A Summing Up

The debates about the notion of capital are as old as its marginalist conception as a factor of production. Cambridge (U.K.) economists began to be involved in these controversies since the 1930s, with the contributions by Sraffa (1932) and Kaldor (1937). What we call the Cambridge capital theory controversy actually started during the 1950s, however, with the publication of Robinson's and Champernowne's articles of 1953 and then the Corfu conference of 1958.[4]

The various surveys of the contributions to this controversy already available include a detailed reconstruction of the debate from the Cambridge (U.K.) standpoint by Harcourt (1969, 1972) and Cohen and Harcourt (2003), and the analyses from a different perspective developed by Stiglitz (1974) and Dixit (1977).[5] This paper will therefore not attempt a complete overview of the first stage of the debate and instead focus solely on some specific points of relevance.

The central core of the controversy emerged with the publication of Sraffa's *Production of Commodities by Means of Commodities* (1960), followed two years later by Samuelson's article on the 'surrogate' production function (Samuelson, 1962). Taking a 'persistent position' of the economy characterized by stationary relative prices and a uniform rate of profit among sectors as his point of reference,[6] Sraffa showed that the link between relative prices and distribution variables, the wage rate and the profit rate in particular, can be complex and unpredictable. For example, in the event of a change of distribution in a certain direction, such as a continuum rise in the rate of profit, the relative price of two commodities may rise and fall in alternating stretches (cf. Sraffa, 1960, p. 38, figure 3).

This result, as Sraffa pointed out (1960, p. 38), actually deprived of any significance the claim that different methods of production of the same commodity can be understood as involving higher (or lower) 'capital intensity', as though this were a property of technical nature. Precisely because the prices of capital goods depend on the wage rate and the profit rate, the ordering of production methods on the basis of their employment of capital per unit of labour could, in general, vary as income distribution changes. Therefore, as in the case of two production methods, the process that initially has the lowest capital intensity can come to have the highest capital intensity for different levels of the distribution variables.

One of the pillars of the neoclassical/marginalist theory of distribution was thus challenged, namely the idea that capital should be regarded as a factor of production, that is as an input, on a par with labour so as to justify a symmetrical explanation of the rate of interest (the general rate of profit) and of the wage rate, both understood as factor prices.[7]

Just two years after the publication of Sraffa's book,[8] Samuelson (1962) tried to put forward a reply. The argument was presented as a sort of 'parable' grounded on the discussion of a particular model in which the same consumption good can be obtained by means of many alternative techniques, each characterized by the employment of capital goods of a specific kind. Any change in the methods of production therefore entails a change in the kind of capital goods employed. In this case, since the quantities of heterogeneous goods cannot be compared, one method cannot be said to employ more 'physical capital' per unit of labour than another. Samuelson attempted, however, to prove the existence of a special substance, a sort of 'jelly' able to take the shape of every possible kind of capital good and from which capital goods derive their productive power. If such a jelly existed, the employment of capital goods would be merely apparent: every time there is a change in the technique in use that implies a greater (net) product per unit of labour, this must be due to an increase in the amount of jelly embodied in the capital goods employed per unit of labour.

Since the optimal technique for firms depends on the rate of interest, Samuelson's attempt would have worked if he had been able to prove that a lower rate of interest was necessarily associated with the use of techniques that, for the same amount of labour, employ a greater amount of jelly and give a greater final output. Levhari's contribution (1965) appeared to work precisely to this end, arguing that if a technique is adopted for a certain interest rate and then abandoned when the rate falls, it is impossible to switch back to that technique at a lower interest rate. He thus asserted the impossibility of the re-switching of techniques and hence also of a non-monotonic relation between the rate of interest and the amount of final output per unit of labour.

As unequivocally established in the 1966 symposium[9] in the *Quarterly Journal of Economics*, however, Levhari's theorem was incorrect. Pasinetti and Garegnani, two economists of the Anglo-Italian school that took Sraffa as its point of reference, as well as Morishima and Sheshinski, were able to prove by means of suitable counter-examples that re-switching is indeed possible despite Levhari's arguments to the contrary (cf. Bruno et al., 1966; Garegnani, 1966; Morishima, 1966; Pasinetti, 1966).

The possibility of re-switching made Samuelson's parable untenable. The fact that the same production technique of the final good can be adopted (i.e. optimal) for two different levels of the interest rate but not some levels between them proved the groundlessness of the view of capital as a factor of production to be employed together with labour in variable proportions, and hence also of the rate of interest as the price to pay for its use. As Samuelson himself admitted in the final paper of the symposium:

> the simple tale told by Jevons, Böhm-Bawerk, Wicksell, and other neoclassical writers—alleging that, as the interest rate falls in consequence of abstention from present consumption in favor of future, technology must become in some sense more 'roundabout,' more 'mechanized,' and 'more productive'—cannot be universally valid. (Samuelson, 1966, p. 568.)

Moreover,

> There often turns out to be no unambiguous way of characterizing different processes as more 'capital-intensive,' more 'mechanized,' more 'roundabout,' except in the *ex post* tautological sense of being adopted at a lower interest rate (p. 582.)

This can be seen as the end of the idea of capital as a factor of production, at least outside the models in which the existence of just one commodity is assumed. There is no 'ultimate substance' or 'jelly' from which capital goods derive their productivity. The only thing that can take the shape of every possible set

of capital goods is the amount of purchasing power that covers their value. This cannot be regarded as the quantity of a factor of production, however, for the reasons given by Sraffa in his book.

3. The Garegnani-Bliss Debate in the *Review of Economic Studies*

Once this had been established, any neoclassical economists wishing to go on regarding the interest rate as a price set by capital supply and demand would have been faced with the problem of possible 'reverse capital deepening', that is lower rather than higher demand for capital (for a given employment of labour) when the rate of interest decreases. Reverse capital deepening could therefore be seen as a possible (further) cause of equilibrium instability.

Samuelson touched on this possibility briefly at the end of the 1966 symposium, suggesting that instability and multiplicity problems could arise in Modigliani's life-cycle model or in a 'Solow-Harrod growth model' (Samuelson, 1966, p. 578). This critical argument was then employed by Garegnani in an article published in 1970 in the *Review of Economic Studies* (Garegnani, 1970a).

Garegnani's discussion of re-switching and reverse capital deepening as possible causes of equilibrium instability is presented in a somewhat peculiar way. Starting from an equilibrium position, he assumes an increase in capital supply due to positive net saving. This means that there is now an excess supply of capital at the initial level of the interest rate. The excess supply of capital causes the interest rate to fall. The decrease of the rate of interest is supposed to cause an increase in the demand for capital, but the opposite takes place in the event of reverse capital deepening: the demand for capital decreases and the excess supply can become even greater. He concludes as follows:

> we are forced to the conclusion that a change, however small, in the 'supply' or 'demand' conditions of labour or capital (saving) may result in drastic changes of r and w. That analysis would even force us to admit that r may fall to zero or rise to its maximum, and hence w rise to its maximum or to fall to zero, without bringing to equality the quantities supplied and demanded of the two factors. (Garegnani, 1970, p. 426)

As stated above, Garegnani intended to highlight the possibility of a problem of equilibrium instability. Even though he clarified his position and reformulated this point in other papers (see in particular Garegnani, 1983, 1990), it is unclear, however, in the passage quoted above, whether the problem concerned comparative statics, equilibrium stability or even the very existence of equilibrium.

It was in the last sense that Bliss (1970) understood it in his comment on Garegnani's article published in the same journal, which was, as a result, mainly grounded on the existence proof of equilibrium for Arrow–Debreu models. As he wrote at the beginning,

> Professor Garegnani in his paper makes a claim which, to economists familiar with the modern theory of general equilibrium, will seem rather surprising. He supposes an economy with many capital goods in stationary long-run equilibrium at rate of interest r^*. He then asks himself whether, following a change in demand leading to "a tendency to positive saving", there exists a new equilibrium of supply and demand consistent with the new demand functions. He concludes that no such equilibrium need exist (p. [4]25-28). Now an equilibrium of supply and demand certainly might not exist, but we know from the work of G. Debreu [1959] that the conditions required for existence are rather weak, and that existence can be guaranteed whenever demand functions are continuous and the technology a convex one (e.g. a linear technology). Both these conditions obtain in Garegnani's model. How then does he arrive at his conclusion? (Bliss, 1970, p. 437)

According to Bliss, Garegnani obtained his result because he focused on 'stationary long-run equilibria' and, by so doing, failed to perceive the possible existence of equilibria in which relative prices and the rate of interest change over time. The important thing now, however, is not so much to go into the details

of the Garegnani-Bliss debate as to stress that it can be regarded as the ideal dividing line between the first and second stages of the Cambridge capital controversy. Both of the primary features characterizing the second stage of the capital controversy were in fact introduced in that debate: (i) re-switching and reverse capital deepening were indicated as possible causes of equilibrium instability and (ii) attention shifted to the neo-Walrasian models, where capital is not understood as a factor of production and the rate of interest as the price for its use.

Moreover, in this second stage, only one Cambridge was involved: the one in England. The neoclassical side of the controversy was no longer represented by Samuelson and the other MIT economists, who were replaced by Bliss, Hahn,[10] and, some years later, Mandler.

4. Neo-Walrasians versus Neo-Ricardians

As seen in Section 2, the results obtained in the first stage of the controversy concerned the view of capital as a factor of production and the rate of interest as the price for its use. They therefore left the neo-Walrasian theory of value essentially untouched, as these problematic conceptions are no part of it.

The neo-Walrasians therefore proclaimed with great emphasis that their theory of commodity prices is not affected by the Sraffian – or neo-Ricardian, as they prefer to say – critiques of the neoclassical/marginalist theory of capital. In particular, Bliss (1975) and Hahn (1975,[11] 1982b) maintained that in neo-Walrasian general equilibrium models, starting from given initial endowments of commodities and assuming complete current and forward markets opened simultaneously at the initial date, it is possible to arrive at the determination of the intertemporal price system with no need to aggregate capital goods either in value terms or by means of any special 'jelly'. Moreover, they claimed that the critical arguments used by the neo-Ricardians were aimed at naïve versions of the theory, those appropriate for first-year students, which employ aggregation for the sake of simplicity, but have no impact on more sophisticated versions such as the neo-Walrasian. In Hahn's words:

> The neo-Ricardians, by means of the neoclassical theory of the choice of technique, have established that capital aggregation is theoretically unsound. Fine. Let us give them an alpha for this. The result has no bearing on the mainstream of neoclassical theory simply because it does not use aggregates. It has a bearing on the vulgar theories of textbooks. But textbooks are not the frontier of knowledge. (Hahn, 1975, p. 363)

On the one hand, the neo-Walrasians therefore admitted, as Samuelson already had in his summing-up of 1966, that the conception of capital as a factor of production is 'theoretically unsound'. It can have a place only in 'the vulgar theories of textbooks'.[12] They were also aware that this particular factor of production was invented with the aim of explaining the rate of interest as the price for its use. As Bliss writes:

> It is true, of course, that capital, meaning here the physical goods which are used in capitalistic production, is typically held by its owners as a store of wealth and a source of income. But it does not follow that the economic theorist should regard it exclusively in this light. Wherein lies the temptation to do so? It is to be found in a closely associated view; namely, that the rate of interest is the price of capital. The value which accrues from a sale is the product of price and quantity sold. Hence if the rate of interest is the price of capital, the quantity of capital must be the wealth on which an interest yield is calculated. It will be shown shortly why this view is incorrect, but to cut a long story short, the conclusion may be announced at once. The rate of interest is not the price of capital. (Bliss, 1975, pp. 6–7)

On the other, however, they wanted to defend the supply-and-demand explanation of value and distribution. In particular, Hahn (1982b) argued, by means of a two-period and two-good example,

that the results (existence and Pareto-efficiency) obtained for the Arrow–Debreu equilibrium were 'not based on any aggregation hypothesis' (p. 371) and therefore not at risk. He thus drew the following conclusion:

> unless one wishes to claim that aggregation is essential if a theory is to be called neoclassical, so that Arrow-Debreu for instance are not neoclassical, none of this [i.e. the neo-Ricardian critique] has any bearing on the main issue of this lecture. Sraffa performed a service in showing how neoclassical arguments can be used to show neoclassical aggregation parables to be in logical difficulties. But that cannot help with a critique of marginal theory. (Hahn, 1982b, p. 373)

The Sraffian economists' response to Bliss's and Hahn's claims developed along two distinct lines. The first, which will be considered in the next section, concerned the significance of the new neo-Walrasian notions of equilibrium, with particular reference to Arrow–Debreu equilibrium. The second, addressed in Sections 6 and 7, sought instead to show the need, also in neo-Walrasian models, for a specific market for saving and investment that is actually analogous, at least as regards the problems that can arise there, to the market for the factor 'capital' in the traditional version of the neoclassical/marginalist theory.

5. The New Notions of Equilibrium

Theories of value have traditionally focused attention on a price system understood as the centre around which the prices that regulate actual trade gravitate. This idea represented, and indeed still does, the link between reality, that is the prices determined in any trade session of a mercantile exchange, and one of the most abstract parts of economic theory, namely the theory of value.

The gravitation of actual prices around a theoretical central level was initially conceived by Adam Smith and then acknowledged by economists with no exceptions, including the founders of the neoclassical/marginalist approach. The 'long-run equilibrium' or the 'normal equilibrium' were in fact understood as the theoretical positions towards which the actual magnitudes tend. In Walras's words:

> It never happens in the real world that the selling price of any given product is absolutely equal to the cost of the productive services that enter into that product, or that the effective demand and supply of services or products are absolutely equal. Yet equilibrium is the normal state, in the sense that it is the state towards which things spontaneously tend under a régime of free competition in exchange and in production. (Walras, 1954, p. 224)

At a later time, however, with the rise of the neo-Walrasian approach, new notions of equilibrium were introduced whose link with reality is, at the best, dubious.

5.1 *Temporary and Arrow–Debreu equilibria*

According to the neo-Walrasian approach, commodities delivered at different dates typically have different relative prices. In the temporary equilibrium model, markets are open at the beginning of each period and agents take their decisions – consumption and production plans – on the basis of their expectations about the prices that will regulate trading when the markets reopen in future periods. In the Arrow–Debreu model, markets for every present and future delivery are instead open at the beginning of the first period only and never reopen, so that all the problems associated with the of formation of price expectations are avoided by assumption.[13]

Since these equilibrium models are not designed to determine a system of prices understood as the centre around which actual prices tend to gravitate, but rather different commodity prices for every possible date of delivery, the above-mentioned link between theory and reality breaks down.

Within the traditional conception, theoretical prices are generally different from the actual ones observed at different dates, and this is perfectly compatible with the tendency of the latter to orbit around the former. If instead, as in the neo-Walrasian approach, equilibrium is formed by prices of goods and services delivered period by period, then there is no possibility of the adjustment (or tendency) of actual prices towards theoretical ones. In the case of the Arrow–Debreu model in particular, either the actual prices of the commodities delivered at a certain date, say period t, correspond through some coincidence to those initially determined by equilibrium for delivery at that date, or the next date, $t + 1$, will arrive and the actual and theoretical prices for delivery in t will definitively remain different. In other words, since adjustment or gravitation processes take time, they do not appear to be possible with reference to prices of commodities with specific delivery dates.

This problem was initially mentioned very briefly by Malinvaud, who wrote that, in the Arrow–Debreu framework, '[t]here might be some difficulties with stability since time must then enter both the definition of equilibrium, and the process of convergence toward equilibrium' (Malinvaud 1961, p. 152).

The point was then developed by Garegnani during a conference held in 1974 at the State University of New York in Buffalo, the proceedings of which were published in 1976. In particular, Garegnani (1976, pp. 36–39) argued that the lack of persistency of the conditions determining the equilibrium level of commodity relative prices – so that different relative prices are determined for different dates of delivery – would prevent that level from being conceived as a centre of gravitation for actual prices. In the neo-Walrasian models, the forces driving the changes of equilibrium prices for commodities with different delivery dates are at work simultaneously with those that should push actual prices towards the equilibrium position. As a result, before the adjustment to the equilibrium level of prices for delivery in a certain period is completed, the next period arrives (equilibrium) relative prices change, and the adjustment remains incomplete.

Garegnani therefore concluded that the abandonment of the traditional method due to the rise of the neo-Walrasian approach took place at the expense of the relevance of the theory of value, which became a kind of intellectual toy devoid of any importance for the analysis of reality. This is indeed sometimes admitted even by neo-Walrasian authors, who say that the Arrow–Debreu equilibrium is simply a benchmark against which the real economy can be measured in order to highlight problems of market incompleteness or inefficiency, not a position towards which it tends in the long run (see, for example, Geanakoplos, 1987, p. 117).

Garegnani's point was then taken up and developed by Petri, in a number of papers (1978, 1991, 2004, 2017), and by other scholars.[14] The problem stressed by Petri (2004, pp. 39–53) concerns, in particular, the 'impermanence' of the data on which the equilibrium is determined. Within the theoretical tâtonnement process, invented by Walras and adopted by most of the neo-Walrasian scholars (cf. Hahn, 1982a), the search for the equilibrium price vector is driven by an auctioneer that calls a set of prices and allows trades only if agents' decisions are mutually consistent at those prices, that is if they are market-clearing prices. Trades at disequilibrium prices cannot be excluded, however, if the adjustment process is to have something to do with reality.[15] Disequilibrium prices and disequilibrium decisions can therefore have an unpredictable effect on some of the data on which the equilibrium depends, especially the endowments of commodities and their distribution across agents. As a result, the equilibrium prices for future delivery change during the process of adjustment.

5.2 *The 'Hicksian Divide'*

Looking at the history of modern general equilibrium theory, Garegnani claimed that there was an 'Hicksian divide' in the evolution of the neoclassical/marginalist approach. More precisely, he maintained that the difficulties encountered by the traditional marginalist theory with reference to capital as a factor of production were indeed at the origin of the neo-Walrasian drift[16] and a central part in determining this turning point was played by Hicks (cf. Garegnani, 1976, 1990 and especially 2010 and 2012).

As is known, Hicks's initial attempt, in his *Theory of Wages* (1932), to develop an explanation of income distribution based on the marginal productivity of factors had met with strong objections due to the particular nature of the factor 'capital'. In a well-known review of the book, Shove (1933, pp. 470−71) claimed that Hicks neither gave an appropriate definition of capital nor said how quantities of that factor are to be measured, and that it would therefore be impossible to follow his reasoning until those points had been cleared up. These criticisms prompted Hicks to rethink his approach radically and embark on a new path in *Value and Capital* ([1939] 1946).

On the one hand, this involved taking up some features of the theory put forward by Walras, one of the founders of the marginalist approach. We refer in particular to the given endowment of commodities taken by Walras as the starting point for his supply-and-demand analysis instead of the given quantities of the factors of production (labour, land, and capital) on which the initial versions of the theory were mostly based.

On the other, however, since this arbitrary endowment of commodities is generally incompatible with the persistency (stationarity) of the system of relative prices,[17] Hicks was forced to abandon the traditional idea of equilibrium as a centre of gravitation and adopt a new conception in which relative prices are allowed to change at every possible date of delivery.[18] He thus introduced the notions of temporary equilibrium, in particular, but also of 'equilibrium over time', which corresponds to the modern Arrow–Debreu equilibrium.[19]

6. Saving-Investment Equilibrium: Part One

As mentioned at the end of Section 4, the second argument introduced by neo-Ricardian scholars in response to Bliss and Hahn was based on the possibility that the phenomena of re-switching and reverse capital deepening would bring about equilibrium multiplicity or instability in neo-Walrasian models as well, even though they do not treat capital as a factor of production. Here we shall refer primarily to the contributions Garegnani presented in a number of seminars, conferences and papers (see Garegnani, 2000, 2003, 2005a, 2005b, 2011), but we must also mention similar or connected points developed by Schefold[20] (2000, 2003, 2005a, 2005b) and Parrinello (2005, 2008, 2011). The basic idea of this argument was that neo-Walrasian equilibria would require equality between saving and investment for each period. Investment being understood as the value of the capital goods demanded by firms, the problems affecting the demand for capital in value terms could therefore arise again through the investment function or curve.

6.1 *Garegnani on the Saving-Investment Market and Arrow–Debreu Equilibria*

Garegnani's analysis refers primarily to the model introduced by Hahn in his article of 1982. It is an Arrow–Debreu model with two commodities and two dates of delivery.[21]

It is assumed that there is no produced input and the initial endowments of commodities delivered in the first period (period 0) are partly consumed directly by households in that period and partly employed by firms, along with the endowment of labour, in order to produce consumption goods delivered in the second period (period 1).

For each commodity delivered in 0, Garegnani defines the amount 'saved' as the difference between its endowment and the quantity directly consumed. Saving is thus understood by Garegnani as the value of the quantities of commodities 'saved'. Investment is instead defined as the value of the commodities that firms wish to employ as capital goods in period 0.

It is therefore evident that the market-clearing conditions for commodities delivered in 0 entail equality between saving and investment as understood by Garegnani. A new system of equilibrium conditions is therefore built in which the two market-clearing conditions for the commodities delivered in 0 are removed and replaced by the condition of equality between saving and investment, defined as stated

above, and a special condition about the ratio in which the two commodities delivered in 0 are demanded, which must, in equilibrium, be equal to the ratio of their initial endowments. Garegnani then demonstrates the equivalence – as far as the solutions are concerned – of this new system of equilibrium conditions with the original one.

According to Garegnani, this new way of writing the system of equilibrium conditions reveals the need for an equality between saving and investment in Arrow–Debreu models. Even if capital is no longer understood as a factor of production, a condition of equilibrium between the supply of and demand for capital, intended as the value of commodities saved and invested, would still remain in these models. Moreover, Garegnani claims that reverse capital deepening could affect the direction of change of the amount of investment as the (own) rate of interest (of the numéraire commodity)[22] varies and could, by so doing, cause problems of equilibrium multiplicity and instability.[23]

6.2 *Mandler's Response*

The neo-Walrasian reaction to this thesis was mainly driven by Michael Mandler, who attended Garegnani's and Schefold's presentations during the workshop 'General Equilibrium: Problems, Prospects, Alternatives', organized by Fabio Petri in Siena in the summer of 1999.[24] A few years later, in 2002, within the framework of a symposium in *Metroeconomica* on the book *Critical Essays on Piero Sraffa's Legacy in Economics* (Kurz, ed., 2000), he put forward an initial assessment of Garegnani's and Schefold's[25] contributions. In particular, he concluded that:

> Schefold and Garegnani clarify that a convincing Sraffian critique of general equilibrium theory must focus on the stability and plausibility of equilibrium rather than existence. They pursue this project in different ways, Schefold by examining how equilibrium prices move through time when an economy's endowments change, Garegnani by studying the out-of-equilibrium behaviour at a point in time. I have argued that neither strategy succeeds, at least not when the consumer demand side of the model is sufficiently well behaved. (Mandler 2002, p. 220)

In 2005, *Metroeconomica* organized a second symposium expressly devoted to this controversy with articles by Garegnani, Schefold, Mandler and Parrinello.[26] On this occasion, referring to the Arrow–Debreu model introduced by Hahn and then taken up by Garegnani, Mandler (2005) proved that if household demand functions satisfy, on aggregate, the weak axiom of revealed preference (WARP),[27] then equilibrium multiplicity and instability are (generically)[28] ruled out independently of any possible hypothesis about the technical coefficients of production (see Appendix A for some analytical details).

In other words, Mandler's analysis showed that problems of multiplicity and instability can only arise in the Hahn–Garegnani model from phenomena concerning households' choices, especially the combinations of income effects across households that prevent the WARP from being satisfied at the level of market demand functions.[29] Firms' decisions, including those about production plans and the employment of inputs, therefore essentially play no part in causing multiple and unstable equilibria.

6.3 *Are Saving and Investment Possible in the Arrow–Debreu Model?*

Contrary to what Garegnani had attempted to claim, reverse capital deepening cannot therefore be regarded, in the Arrow–Debreu model, as a possible further cause of equilibrium multiplicity and instability that operates by means of the saving-investment market. This conclusion enables us, however, to highlight an important shortcoming of the Arrow–Debreu equilibrium model. As pointed out by Schefold (2007) in a critical assessment of Garegnani's contribution, there is in fact no room for real saving and investment in the Arrow–Debreu framework.[30]

As already outlined in Section 5, the Arrow–Debreu is a particular kind of neo-Walrasian model in which it is assumed that spot and forward markets are: (i) complete, that is it is possible to exchange all commodities for every possible date of delivery; (ii) of finite number, that is there is a finite number of possible delivery dates and (iii) open simultaneously in a single instant, the initial instant of the first period. The last feature has important implications for the point examined here.

On the one hand, given that firms can sell the output they will produce in the same moment as they buy the inputs they will employ, costs do not have to be paid in advance by means of capital[31] but can be financed directly out of revenues. In actual fact, even before Schefold's discussion of Garegnani's analysis, the impossibility of considering the investment of capital within an Arrow–Debreu economy was argued – perhaps for the first time – by Currie and Steedman (1990, chapter 7), who claimed that meaningful analysis of the investment of capital and its return can be developed only in models with spot markets open at every date. In these models, agents need to trade assets in order to move their purchasing power over time. This need is instead totally absent in the Arrow–Debreu framework, as the idea of transferring wealth over time has no real meaning there (Currie and Steedman, 1990, p. 147).

On the other, since all the markets are open for a single instant, the entire purchasing capacity of households must be used at that moment to buy commodities that will then be delivered on different dates. Saving in order to transfer purchasing power to some future date would thus actually be impossible in the Arrow–Debreu model, as no transaction can take place after the initial instant. In this framework, income and expenditure can exist in the one moment in which markets are open.

It therefore appears evident that neither firms' investment nor households' saving can be conceived of in this model if they are understood in the usual (consistent) way. Garegnani's very peculiar definition of saving and investment blurred the vision of this fact.[32]

6.4 *Is Re-Switching Possible in the Arrow–Debreu Model?*

As seen in Section 2, reference was made in the first stage of the capital theory controversy primarily to positions characterized by stationary relative prices, that is a situation in which the same relative price system applies both to inputs and to outputs. In this framework, as shown by Sraffa (1960), the link between distribution variables and relative prices is very close. For a given set of methods of production in use, once the rate of interest (or the wage rate) is known, the price vector is determined accordingly. As a result, if several sets of production methods – that is alternative techniques – are possible, the cost-minimizing one can be determined for each level of the rate of interest. This makes it possible to study the switching (and re-switching) of techniques for different interest rate levels.

By contrast, in the Arrow–Debreu model, commodities delivered at different dates have different relative prices.[33] As a result, there are more commodity prices to determine than outputs. Costs therefore have a weaker influence on prices and the link between the latter and the distribution variables is not as close as within a stationary framework. In particular, it has been demonstrated (Fratini, 2015), on the assumption of non-stationary relative prices, that if a level of the wage rate is taken as given, then neither the price vector nor the set of methods in use is univocally determined. Many different price vectors, supporting the use of different techniques, can be associated with the same wage rate.

In actual fact, the concept of a switch point between two techniques loses its meaning outside the case of stationary prices. The possibility of re-switching therefore appears to be restricted to cases in which the stationarity of relative prices is assumed or implied by features of the model.

7. Saving-Investment Equilibrium: Part Two

Despite its failure to achieve its purpose, Garegnani's attempt was very fruitful. First, as stressed in Section 6.3, criticism of Garegnani's conclusions showed that the Arrow–Debreu model is not the right

framework to address decisions on investment and saving, as both of these are closely related to the possibility of market trades at different dates. Capital, understood as the amount of value that allows firms to pay for inputs before their outputs are sold, cannot exist if all the transactions, for present and future delivery, take place in the same moment. Second, it paved the way for a better statement of Garegnani's own point. Once it was clear that re-switching and reverse capital deepening cannot be found in the Arrow–Debreu framework, it also became clear that they could instead arise in neo-Walrasian models based on different assumptions.

Even though the relative prices of the commodities delivered in t can in general differ from those of the commodities delivered in $t + 1$ in the neo-Walrasian theory, models can be conceived of in which, by hypothesis, relative prices remain stationary, one in particular being the model considered by Malinvaud (1953, section IV) and Bliss (1975, chapter 4).[34] Moreover, if chaotic dynamics are excluded, the positions of stationary equilibrium should be precisely those towards which the paths of (sequential) neo-Walrasian equilibrium tend over a sufficiently long period of time. As a result, if the analysis addresses these stationary equilibria, re-switching not only proves to be possible but its role in determining the multiplicity and instability of solutions can also be identified, precisely what was initially and unsuccessfully attempted with reference to the Arrow–Debreu equilibrium.

In these models, the stationarity of relative prices period by period requires the absence of accumulation of capital goods (per unit of labour).[35] As a result, the equality between gross saving and the value of the capital goods to replace must be included within the system of conditions for a stationary equilibrium (see Appendix B). This condition has characteristics very similar to the condition of equilibrium between the supply of and demand for capital. In particular, it is expected that the interest rate level will adjust in such a way as to make net accumulation nil, that is to bring about equality between gross saving and the value of capital goods employed by firms.[36] Hicks described this adjustment as follows:

> A fall in the rate of interest would encourage the adoption of longer processes, requiring the use (at any moment) of larger quantities of intermediate products. But since we are in a stationary state, there can be no tendency for the stock of capital to increase or diminish; constancy of the stock thus gives us one relation between its size and the rate of interest. Also, if entrepreneurs do not desire to increase or diminish their stock, their net borrowing must be nil. If the demand and supply for loans are to be in equilibrium, net saving must therefore also be nil. The rate of interest must therefore be fixed at a level which offers no incentive for net saving or dis-saving. (Hicks [1939], 1946, pp. 118–119)

At the beginning of the above passage, Hicks claims that a fall in the interest rate would encourage the adoption of more capital-intensive methods of production. This does not always occur, however, as we know that reverse capital deepening is possible. Moreover, re-switching is also possible due to the stationarity of relative prices. The effects of the occurrence of reverse capital deepening and re-switching in the neo-Walrasian stationary models are discussed in Fratini (2007) and (2013).

With reference to an overlapping-generation model with linear methods of production and many different kinds of capital goods, the analysis can be focused on stationary equilibria and in particular on the levels of the interest rate[37] for which the amount of gross saving (per worker) corresponds to the value of the bundle of (circulating) capital goods employed (per worker). For this model, reverse capital deepening can be seen as a further cause of equilibrium multiplicity in addition to income effects.

More precisely, since wages are the income out of which savings are made, when the rate of interest approaches to its maximum level, the wage rate and the amount of saving tend towards zero. This means that the saving curve is either monotonically decreasing or bell-shaped. In both cases, there can be multiple equilibria even in the event of a well-behaved (monotonically decreasing) investment curve. If the saving curve is bell-shaped, however, multiple equilibria in the rising stretch must necessarily be associated

with the occurrence of reverse capital deepening, which can therefore be regarded as a specific source of equilibrium multiplicity.

Moreover, on some assumptions of regularity concerning the saving function,[38] re-switching can be regarded as a cause of equilibrium local instability. In other words, with the equilibrium level as the starting point, a small drop in the rate of interest entails – if re-switching occurs – an excess of gross saving with respect to the value of the capital goods employed.

Garegnani's argument that even though they do not regard capital as a factor of production, neo-Walrasian models are not immune to problems similar to those that emerged in the first stage of the debate therefore ultimately proved correct, albeit in the framework of stationary rather than Arrow–Debreu models.

8. Conclusions

As implicitly admitted by Samuelson (1966, p. 582) in writing that there is no unambiguous way to say that one method of production is more capital-intensive than another, capital is not a factor of production and the rate of interest is therefore not the price for its use. This is what was established during the first stage of the Cambridge capital controversy. As seen in Section 4, neo-Walrasian and neo-Ricardian (Sraffian) economists agree on this point.[39]

The disagreement between the two schools concerns whether this conclusion also has any impact on the neo-Walrasian theory of value and distribution. In particular, Bliss and Hahn maintained that the neo-Walrasian theory is not affected by the neo-Ricardian criticism because it regards production as a process employing commodities, that is goods and services with date and place of delivery, rather than factors of production.

The response to this claim was primarily developed by Garegnani and other neo-Ricardian scholars along two lines. The first is based on the fact that neo-Walrasian equilibrium prices cannot be understood as the central level around which the prices that regulate actual trades tend to orbit. The point therefore concerns the relevance of the neo-Walrasian theory for the interpretation of real economic phenomena. The second instead seeks to show that reverse capital deepening and re-switching can also occur in the neo-Walrasian models, due to the saving-investment market, and affect equilibrium multiplicity and instability.

As regards the second, the analysis initially took the Arrow–Debreu model as its particular point of reference. As argued by Currie and Steedman (1990) and by Schefold (2007), however, meaningful notions of saving and investment cannot be found within this framework because of the assumption of complete markets open for a single instant. Mandler (2005) was thus able to prove that problems of equilibrium multiplicity and instability cannot occur in this model if consumers' decisions are well-behaved, that is if they satisfy the WARP in aggregate. More recent studies have shown, however, that problems of multiplicity and instability linked to re-switching can arise in a different class of neo-Walrasian models, namely stationary models characterized by zero net accumulation in each period.

Finally, even though the research is still in progress, there is a conclusion that can surely be drawn from analysis of the first and second stages of the Cambridge capital controversy. Empirical studies claim that in the most industrialized countries, the share of capital incomes in the total national income is in the interval of 25−30% (see Piketty 2014, p. 222, figure 6.5). Jevons, Böhm-Bawerk and the other founders of the neoclassical/marginalist approach would say that these capital incomes are what firms pay to households for the use of a factor of production and that their amount therefore depends on the marginal productivity of 'capital'. Thanks to the first stage of the capital controversy, it is known that this view is 'theoretically unsound', as Hahn put it. It is therefore reasonable to wonder whether the modern mainstream economic theory can provide us with a better explanation of the nature and the determinants

of capital incomes. Serious doubts are emerging about this possibility, however, in the second stage of the controversy.

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Notes

1. According to the standard definition, the social surplus is the difference between the bundle of commodities obtained as social output and the quantities of commodities necessary to enable the repetition of the production process (cf. Garegnani, 1984, pp. 292–293).
2. For the sake of simplicity, the possibility of different 'events' or 'states' at the moment of delivery is not considered in this paper.
3. In order to avoid possible misunderstandings, 're-switching' is to be understood as the case in which the same set of production methods is in use for two different levels of the rate of interest but not for some levels between them, and 'reverse capital deepening' as the case in which a higher level of the rate of interest (relatively to the wage rate) leads to higher investment of capital per unit of labour.
4. The International Economic Association held a conference in Corfu (Greece) on the theory of capital in 1958. A number of very important economists took part, including Sraffa. For a record of the conference, see Hicks (1960) and Lutz and Hague (1961).
5. The most important contributions to this debate are also collected in Bliss, Cohen and Harcourt's three-volume edition *Capital Theory* (2005), which includes two introductions, one by Bliss and the other by Cohen and Harcourt.
6. This 'normal' or 'long-run' position has constituted the framework for recognition of the persistent determinants of commodity relative prices since Adam Smith's time. It was only recently, as we shall see below, that this method was abandoned with the advent of the neo-Walrasian notions of equilibrium. We shall return to this point in Section 5.
7. The conception of the rate of interest as the price that brings the supply of and demand for capital into equilibrium is typical of almost all the traditional versions of the neoclassical/marginalist theory. For example, Marshall explicitly writes that 'interest, being the price paid for the use of capital in any market, tends towards an equilibrium level such that the aggregate demand for capital in that market, at that rate of interest, is equal to the aggregate stock forthcoming there at that rate' (Marshall [1890] 1920, p. 534).
8. Actually, in his essay in honor of Åkerman, published in 1961, Samuelson already mentioned Sraffa's book among the relevant contributions to the theory of capital (cf. Samuelson 1961, p. 422).
9. In actual fact, the subjects of the symposium were already addressed during the first conference of the International Econometric Society, held in Rome in 1965, where Pasinetti presented an initial version of his 1966 article. Cf. Pasinetti (1966, p. 503).
10. Bliss was at Christ's College, Cambridge, when he wrote his comment on Garegnani's article and Hahn, as is known, spent most of his professional life in Cambridge.
11. Hahn's 1975 article was the reply to Harcourt's review (Harcourt, 1975) of his book on the theory of distribution (Hahn, 1972).
12. Surprisingly enough, Hahn appears to put the aggregate textbook model and the one-commodity model on the same footing in his article of 1982 (see Hahn, 1982b, p. 370). This is somewhat dangerous, as the latter is in fact simply based on the very special and unrealistic assumption of the presence of just one kind of capital good. The former instead entails the conception of capital as a factor of production with all the attendant logical difficulties.

13. The reader might think that this assumption is irrelevant because 'if markets were reopened at later dates for the same Arrow–Debreu commodities, then no additional trade would take place anyway' (Geanakoplos 1987, p. 122). It is not so simple, however, because if agents knew that markets would open again, they would behave differently from the outset and all the problems connected with expectations and speculative trades, upon which a vast literature exists, would arise. These problems are instead completely avoided within the Arrow–Debreu framework.
14. Similar arguments can also be found in Garegnani (1990), Currie and Steedman (1990), Kurz and Salvadori (2003) and Bilancini and Petri (2008).
15. The unrealistic features of the tâtonnement process are widely discussed by Fisher (1982, 1987), who also analysed a number of alternative adjustment processes. However, Fisher does not seem to be awere of the problems raised by Garegnani and Petri, which do not depend so much on the type of adjustment process as on the intertemporal nature of the equilibrium price vector.
16. For the origin of the new notions of equilibrium, see also Milgate (1979) and Gehrke (2003).
17. Criticism of the Walrasian theory of equilibrium with capital formation lies beyond the scope of the present paper. Interested readers are therefore referred to Garegnani (1990, pp. 11−23).
18. Hicks presented the abandonment of the traditional method as a great achievement for the general equilibrium theory, namely as the shift from 'Economic Statics' to 'Economic Dynamics'. For a critical assessment of this view, see Trabucchi (2013).
19. According to the reconstruction provided by Petri (1991), after the publication of the second edition of *Value and Capital* (1946), Hicks himself became gradually aware of the limitation of the 'temporary equilibrium method'. In particular, Petri argues that, in *Capital and Growth* (1965), Hicks included the incompatibility with a time-consuming process of adjustment among the 'serious defects' of the temporary equilibrium method.
20. Schefold's argument is quite different from Garegnani's. He starts from a given initial long-run position, an equilibrium in which relative prices remain stationary, and assumes that an increase in the supply of labour takes place through the immigration of workers from abroad. He then introduces a second long-run position in which the higher supply of labour is fully employed but the gross outputs remain unchanged. The intertemporal equilibrium considered by Schefold is the path linking the first long-run position to the second. Schefold's conclusion is that re-switching entails the instability of the intertemporal equilibrium path so conceived because, in the case of re-switching, the use of more labour-intensive methods of production would require an increase in the wage rate, whereas the initial excess supply of labour entails a decrease.
21. In Hahn (1982b), the commodities are called 'wheat' and 'barley' and the dates are '1976' and '1977'. In Garegnani's articles, the commodities are 'A' and 'B' and the dates '0' and '1'. See Appendix A for a complete description of the model.
22. As is known, in the Arrow–Debreu model, the own factor of interest $(1 + r^i_{t,t+1})$ of a commodity i between two dates, t and $t + 1$, is the relative price of this commodity delivered in t in terms of the same commodity delivered in $t + 1$. In other words, $p^i_t / p^i_{t+1} = (1 + r^i_{t,t+1})$. As a result, if there are two commodities and two dates of delivery, as in the Hahn–Garegnani model, then there are two own rates of interest. In Garegnani's argument, the own rate of interest of the numéraire commodity (commodity 'B') is treated as an independent variable in order to study how saving and investment react to its variations.
23. Clearly, if an economy has multiple equilibria, none of them can be globally stable.
24. A collection of the papers presented during the workshop was published in Hahn and Petri (2003).
25. See footnote 20 for a recontruction of Schefold's (2000) argument.
26. Parrinello (2005) argued that even though atemporal and intertemporal equilibria are formally equivalent (cf. Malinvaud 1961), the adjustment process needed to achieve the equilibrium price vector is different in the two cases. In his view, this conclusion could open the way to the possibility of instability due to reverse capital deepening.

27. According to the WARP, if a household can afford the consumption plans $\mathbf{x}'$ and $\mathbf{x}''$ for an initial price vector $\mathbf{p}'$, and prefers to consume $\mathbf{x}'$, then it will decide to consume $\mathbf{x}''$ only for a price vector $\mathbf{p}''$ such that $\mathbf{x}'$ is no longer affordable. For a more technical discussion, see definition A.2 in Appendix A. Cf. also Mas-Colell et al. (1995, pp. 28−36).
28. While the aggregate WARP cannot rule out equilibrium indeterminacy, it is known that Arrow–Debreu economies generally have a finite number of equilibria. See Kehoe (1980) on this point and Fratini (2008) for further considerations.
29. Mandler's result was not completely new. It had already been proved that the aggregate WARP entails the (generic) uniqueness of equilibrium for Arrow–Debreu economies with constant-returns-to-scale production technology. See Kehoe (1980), Mas-Colell et al. (1995, pp. 606−610) and Schefold (1997, pp. 461−470).
30. In criticising Garegnani's contribution, Schefold remarks that appropriate (and relevant) notions of saving and investment are incompatible with the working of the Arrow–Debreu model. He claims that: '[Garegnani's] approach to the theory of saving is at odds with the conception of intertemporal equilibrium. There is no room for saving as unspent income without a definite commitment to acquire future goods – if necessary, contingent on the state of nature, with uncertainty as in Debreu (1959). Hence there is here no need for a macroeconomic coordination of savings and investment; equilibrium can be found in individual markets. Saving in a world with Keynesian uncertainty, by contrast, is a monetary phenomenon. Markets for future goods are absent. Uncertainty may be a sufficient motive to save in a disequilibrium where not even prices are uniform. Hence, the aggregation of capital to make savers indifferent between capital goods is not necessary for the process of saving to take place, as is clear from Keynesian theory' (Schefold 2007, p. 171).
31. If we get rid of the idea that capital is a factor of production, its real role emerges with clarity. As the classical economists and Marx distinctly saw, capital is what allows entrepreneurs to finance the beginning of the production process, to cover all those costs that must be paid in advance of the sale of output and the obtaining of revenues. Revenues, therefore, will allow capitalists to recover the sums paid in advance, but leaving also a profit, that is a surplus of the revenues above the costs.
32. Besides, reading a working-paper he published in 2009, it emerges quite clearly that although Garegnani referred his analysis to an Arrow–Debreu model, he argued as if markets opened in each period. He seems to conceive the Arrow–Debreu model as a sort of sequence of temporary equilibrium models and, therefore, he claims that households are subject to as many budget constraints as periods. Clearly, budget constraints would be meaningless if markets were not open and, accordingly, no trade could occur.

    Actually, as stressed by Schefold, '[Garegani's (2003)] paper is difficult to read because [...] it provides an idiosyncratic synthesis of heterogeneous elements of different theoretical traditions' (Schefold 2007, p. 130).
33. It can be pointed out in passing that once goods with different dates of delivery are understood as different commodities, as they are in the neo-Walrasian models, production is still a one-way process from inputs to outputs even though inputs are commodities and not factors of production. In the neo-Walrasian theory, inputs are in fact commodities different from outputs. This emerges quite clearly from the analogy between atemporal and intertemporal interpretations of the Arrow–Debreu model (Malinvaud, 1961).
34. A similar model is also addressed in Bloise and Reichlin (2009).
35. If input prices are to be stationary, their relative scarcity must remain constant. As a result, the proportions between the quantities of different kinds of capital goods and between the quantity of each capital good and that of labour must remain unaltered. While this is compatible with a situation of steady growth, we shall refer here to a stationary economy for the sake of simplicity. In other words, the rate of growth of the labour force will be assumed to be nil.

36. For the sake of simplicity, it can be assumed that all the capital goods are circulating.
37. As is known, assuming the stationarity of relative prices, the wage rate and the price vector, for each technique, can be expressed as functions of the rate of interest.
38. Saving is typically an amount of value, the difference between income and expenditure in a certain period. However, when an equilibrium is reached, saving is converted, period by period, into the set of capital goods employed. Therefore, if the interest rate slightly increases starting from an equilibrium level, we can assume that the change in the amount of saving is greater than the change in the value of the equilibrium vector of assets (capital goods). Under this assumption, in the model considered in Fratini (2013), instability can only occur in case of re-switching (negative real Wicksell effects).
39. Some neo-Walrasian authors maintain an ambiguous attitude in this regard. On the one hand, they clearly state, as we have seen, that '[t]he rate of interest in not the price of capital' (Bliss 1975, p. 7). On the other, they say that aggregating capital goods in value terms (by means of a given price vector) makes it possible to conceive the 'marginal product' of this aggregate 'capital' and to establish its equality with the rate of interest (see Bliss 1975, pp. 109–110). Is this the same thing as saying that the rate of interest is the price for the use of the aggregate factor 'capital'? If so, then Bliss is contradicting himself. If not, then the equality discussed by Bliss is essentially meaningless.

    Another example of the same inconsistency can be found in Dixit (1977).
40. There is no need to distinguish between row and column vectors, as these can be identified from their position in the 'row-times-column' multiplication.
41. Proof: If $\mathbf{p}'$ and $\mathbf{p}''$ are two equilibrium price vectors, the production plans $\mathbf{y}' = \mathbf{z}(\mathbf{p}')$ and $\mathbf{y}'' = \mathbf{z}(\mathbf{p}'')$ belong to the set $\mathbf{Y}$. As a result, $\mathbf{p}'\cdot\mathbf{y}'' \leq 0$ and $\mathbf{p}''\cdot\mathbf{y}' \leq 0$. This implies $\mathbf{p}'\cdot\mathbf{z}(\mathbf{p}'') \leq 0$ and ≤ 0. If the WARP holds for the excess-demand function $\mathbf{z}(\mathbf{p})$, however, then this is possible only when $\mathbf{z}(\mathbf{p}') = \mathbf{z}(\mathbf{p}'')$ ■
42. Proof: If $\mathbf{p}'$ and $\mathbf{p}''$ are two equilibrium price vectors, then $\mathbf{p}'\cdot\mathbf{y} \leq 0$ and $\mathbf{p}''\cdot\mathbf{y} \leq 0$, which implies $\mathbf{p}'''\cdot\mathbf{y} \leq 0 \ \forall \ \mathbf{y} \in \mathbf{Y}$.

    Moreover, if the WARP holds for the excess-demand function z(p), then, because of proposition A.1, $\mathbf{z}(\mathbf{p}') = \mathbf{z}(\mathbf{p}'') = \mathbf{y}^*$. Therefore $\mathbf{p}'\cdot\mathbf{y}^* = \mathbf{p}''\cdot\mathbf{y}^* = 0$, which implies $\mathbf{p}'''\cdot\mathbf{y}^* = 0$.

    Finally, $\mathbf{z}(\mathbf{p}') = \mathbf{z}(\mathbf{p}'') = \mathbf{y}^*$ and $\mathbf{p}'''\cdot\mathbf{y}^* = 0$ imply $\mathbf{p}'''\cdot\mathbf{z}(\mathbf{p}') = \mathbf{p}'''\cdot\mathbf{z}(\mathbf{p}'') = 0$. Therefore, if $\mathbf{z}(\mathbf{p}''') \neq \mathbf{z}(\mathbf{p}') = \mathbf{z}(\mathbf{p}'') = \mathbf{y}^*$, then the WARP would imply $\mathbf{p}'\cdot\mathbf{z}(\mathbf{p}''') > 0$ and $\mathbf{p}''\cdot\mathbf{z}(\mathbf{p}''') > 0$. This is impossible, however, as it would imply $\mathbf{p}'''\cdot\mathbf{z}(\mathbf{p}''') = \sigma\,\mathbf{p}'\cdot\mathbf{z}(\mathbf{p}''') + (1-\sigma)\mathbf{p}''\cdot\mathbf{z}(\mathbf{p}''') > 0$, which is incompatible with Walras's law. Therefore, $\mathbf{z}(\mathbf{p}''') = \mathbf{z}(\mathbf{p}') = \mathbf{z}(\mathbf{p}'') = \mathbf{y}^*$ ■
43. For the sake of simplicity, all the capital goods considered in our model are circulating.
44. In this model, as there is no saving by elderly people for clearly evident reasons, the total amount of gross saving in each period corresponds to the difference between the income and consumption expenditure of the young.

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Appendix A: The Hahn–Garegnani model

The model considered by Hahn (1982b) and Garegnani (2000, 2003, 2011) is an Arrow–Debreu model in which there are two commodities, say *a* and *b*, and two dates of delivery, 0 and 1. The initial endowments

of the economy include given quantities of commodities a and b delivered in 0 and a given quantity of labour to perform during period 0: $\omega = [\bar{A}_0, \bar{B}_0, \bar{L}, 0, 0]$.[40]

The endowments of commodities a and b available in 0 are partly consumed directly in that period and partly employed, with labour, in the production of commodities a and b delivered in 1. In accordance with the customary notation, $\mathbf{y} = [-A_0, -B_0, -L, A_1, B_1]$ is a production plan. A production plan is technically feasible if and only if it belongs to the production set Y. In particular, it can be assumed there are m linear production methods or activities whose technical coefficients are the columns of a $5\times m$ matrix $\mathbf{M}$. In this case, $Y \equiv \{\mathbf{y} \in R^5 : \mathbf{y} = \mathbf{M}v,\ \mathbf{v} \in R^m_+\}$, where $\mathbf{v}$ is a vector of activity levels.

On the consumption side, given households' preferences and endowments, their market demand for commodities is a function of the price vector $\mathbf{p} = [p_{a0}, p_{b0}, w, p_{a1}, p_{b1}] \in R^5_+$. Namely, $\mathbf{x}(\mathbf{p}) = [x_{a0}(\mathbf{p}), x_{b0}(\mathbf{p}), 0, x_{a1}(\mathbf{p}), x_{b1}(\mathbf{p})]$. The market excess-demand function can also be defined as $\mathbf{z}(\mathbf{p}) \equiv \mathbf{x}(\mathbf{p}) - \boldsymbol{\omega}$.

Definition A.1. *For this economy, a price vector $\boldsymbol{p}^* \in R^5_+$ is an equilibrium if and only if: (i) $\exists\, \boldsymbol{y}^* \in \mathrm{Y}$: $\boldsymbol{z}(\boldsymbol{p}^*) = \boldsymbol{y}^*$; (ii) $\boldsymbol{p}^* \cdot \boldsymbol{y} \leq 0\ \forall\ \boldsymbol{y} \in \mathrm{Y}$.*

Since Arrow and Debreu (1954) proved that at least one equilibrium exists for this model, Hahn's conclusion is that commodity relative prices, own rates of interest – which are again commodity relative prices, as $(1 + r_a) = p_{a0}/p_{a1}$ and $(1 + r_b) = p_{b0}/p_{b1}$ – and wage rate are determined with no reference to capital as a factor of production.

Garegnani's reply, as stated above, is that reverse capital deepening can be a cause of equilibrium multiplicity in this economy because of the implicit presence of a market for saving and investments. He defines the amount of saving as the value of the part of the initial endowments of commodities that is not directly consumed in period 0: $S(\mathbf{p}) \equiv p_{a0}[\bar{A}_0 - x_{a0}(\mathrm{p})] + p_{b0}[\bar{B}_0 - x_{b0}(\mathbf{p})]$. As regards investments, if $A_0(\mathrm{p})$ and $B_0(\mathbf{p})$ are the inputs of commodities a and b employed, with an amount of labour $\bar{L}$, in order to produce the quantities of outputs $x_{a1}(\mathbf{p})$ and $x_{b1}(\mathbf{p})$, in Garegnani's analysis the amount of investment is $I(\mathbf{p}) \equiv p_{a0}A_0(\mathrm{p}) + p_{b0}B_0(\mathbf{p})$.

Garegnani then replaces the market clearing conditions for commodities a and b delivered in 0 with the two following conditions: (i) $S(\mathbf{p}) = I(\mathbf{p})$; (ii) $\bar{A}_0/\bar{B}_0 = [x_{a0}(\mathrm{p}) + A_0(\mathrm{p})]/[x_{b0}(\mathbf{p}) + B_0(\mathbf{p})]$. As solutions of Garegnani's modified system are equivalent to solutions of the original system, he claims that reverse capital deepening, by affecting the shape of the investment function, can bring about equilibrium multiplicity.

As can be easily proved for this model, however, if the weak axiom of revealed preference (WARP) holds for the excess-demand function $\mathbf{z}(\mathbf{p})$, then the equilibrium is (generically) unique.

Definition A.2. *If $\boldsymbol{p}'$ and $\boldsymbol{p}''$ are two price vectors such that $\boldsymbol{p}' \neq \boldsymbol{p}''$, the WARP holds for the excess-demand function $\boldsymbol{z}(\boldsymbol{p})$ if $\boldsymbol{z}(\boldsymbol{p}') \neq \boldsymbol{z}(\boldsymbol{p}'')$ and $\boldsymbol{p}' \cdot \boldsymbol{z}(\boldsymbol{p}'') \leq 0$ imply $\boldsymbol{p}'' \cdot \boldsymbol{z}(\boldsymbol{p}') > 0$.*

Proposition A.1. *Let $\boldsymbol{p}'$ and $\boldsymbol{p}''$ be two equilibrium price vectors of the economy. If the WARP holds for the excess-demand function $\boldsymbol{z}(\boldsymbol{p})$, then $\boldsymbol{z}(\boldsymbol{p}') = \boldsymbol{z}(\boldsymbol{p}'') = \boldsymbol{y}^*$.*[41]

Proposition A.2. *Let $\boldsymbol{p}'$ and $\boldsymbol{p}''$ be two equilibrium price vectors of the economy. If the WARP holds for the excess-demand function $\boldsymbol{z}(\boldsymbol{p})$, then every price vector $\boldsymbol{p}''' = \sigma\boldsymbol{p}' + (1 - \sigma)\boldsymbol{p}''$, with $0 < \sigma < 1$, is an equilibrium price vector.*[42]

The second proposition tells us that if improbable cases of equilibrium indeterminacy are ruled out, then an economy whose excess-demand function $\mathbf{z}(\mathbf{p})$ satisfies the WARP has a unique equilibrium independently of any assumption about technical coefficients of production in the matrix $\mathbf{M}$.

Appendix B: The neo-Walrasian stationary model

In addition to Arrow–Debreu equilibrium, the neo-Walrasian approach includes a number of different models based on different assumptions. We shall focus here on an adapted version of the 'semi-stationary' model studied, among others, by Malinvaud (1953, section IV) and Bliss (1975, chapter 4). It is a recursive production model in which no initial moment exists and every period is identical to both the previous and the following one.

There are N different commodities. A vector of outputs $\mathbf{q} \in R_+^N$ emerges at each date from the production processes started in the previous period. The output vector is divided into two parts: $\mathbf{q} = \mathbf{x} + \mathbf{k}$. The vector $\mathbf{x} \in R_+^N$ is consumed by households during the period. The vector $\mathbf{k} \in R_+^N$ is made up of the commodities employed as inputs[43] together with the available labour force $\bar{L}$. The employment of $\mathbf{k}$ and $\bar{L}$ will give a vector of outputs $\mathbf{q}$ in the subsequent period.

On the consumption side, we shall refer to a standard overlapping generation model. $\bar{L}$ identical individuals are born at each date and live for two periods, namely youth and old age. At birth, each individual has no endowment other than a unit of labour services to perform during youth.

Let $\mathbf{p} \in R_+^N$ be a stationary price vector and w and r respectively the wage rate and the interest rate. Each individual decides on his or her consumption plan so as to maximize his or her intertemporal utility subject to the budget constraint. From consumers' decisions it is possible to obtain the demand function for consumption goods, $\mathbf{x}(\mathbf{p},w,r)$, with $\mathbf{x}$: $R_+^{N+2} \rightarrow R_+^N$, and the gross saving function $s(\mathbf{p},w,r)$.[44]

On the production side, let $\mathbf{A}$ and ℓ be respectively the $N \times N$ matrix of commodity input coefficients and the $N \times 1$ vector of labour input coefficients. Given the technical coefficients ($\mathbf{A}$, ℓ), the labour force of the economy $\bar{L}$ and the functions $\mathbf{x}(\mathbf{p},w,r)$ and $s(\mathbf{p},w,r)$, the equilibrium conditions can be set for our stationary model.

Definition B.1. *For this economy, an output vector $\boldsymbol{q}^* \in R_+^N$, a price vector $\boldsymbol{p}^* \in R_+^N$, a wage rate w^* and an interest rate r^* are a stationary equilibrium if and only if the following obtain:*

i. market clearing conditions

$$\mathbf{x}\left(\boldsymbol{p}^*, w^*, r^*\right) = \boldsymbol{q}^* \cdot (\mathbf{I} - \mathbf{A})$$
$$\boldsymbol{q}^* \cdot \ell = \bar{L}$$

ii. zero-profit conditions

$$\boldsymbol{p}^* - \mathbf{A} \cdot \boldsymbol{p}^* \left(1 + r^*\right) - \ell w^* = 0$$

iii. a zero-net-accumulation condition

$$s\left(\boldsymbol{p}^*, w^*, r^*\right) - \boldsymbol{q}^* \cdot \mathbf{A} \cdot \boldsymbol{p}^* = 0.$$

Once a numéraire is adopted, that is price vectors are normalized, the zero-profit conditions allow us to express the price vector and the wage rate as functions of the rate of interest. Therefore, in turn, the LHS of the zero-net-accumulation condition becomes a function of the rate of interest and we can look for the interest rate levels that make the net accumulation zero.

It has been proved (Fratini, 2007, 2013) that if alternative methods of production for the same commodity are allowed, re-switching is possible and can affect the shape of the net-accumulation function so as to cause equilibrium multiplicity and instability.