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Integrated stochastic optimization approaches for tactical scheduling of trains and railway infrastructure maintenance

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ABSTRACT

This work addresses a tactical railway traffic scheduling problem focused on the optimization of train sequencing and routing decisions and timing decisions related to short-term maintenance works in a railway network subject to disturbed process times. This is modeled as a mixed-integer linear programming formulation in which the traffic flow and track maintenance variables, constraints and objectives are integrated under a stochastic environment. The resulting bi-objective optimization problem is to minimize the deviation from a scheduled plan and to maximize the number of aggregated maintenance works under stochastic disturbances. The two objectives require to schedule competitive train operations versus maintenance works on the same infrastructure elements. Computational experiments are performed on a realistic railway network. We measure the quality of the integrated solutions in terms of their robustness to stochastic perturbations of the train travel times and of the maintenance works. Pareto optimal methods are compared for the bi-objective problem. We also evaluate the impact of introducing routing stability constraints in order to force the trains to keep the same route among the different stochastic disturbed scenarios. The experiments show that forcing the routing stability reduces the routing flexibility and the ability to optimize the two performance indicators when dealing with stochastic disturbances.

Keywords: Railway Traffic Management; Train Scheduling; Infrastructure Maintenance; Disturbance Robustness; Routing Stability; Mixed-Integer Linear Program.

1 Introduction

We daily interface with a lot of different complex systems, like manufacturing, communication or transport systems, with ever increasing complexity. Proper maintenance activities are needed since each component of a complex system is affected by degradation and thus by wear and failures. Maintenance should be performed in every deteriorating system. The goals of Preventive Maintenance (PM) is to ensure any system remains in a specified working condition, basically managing the process of ageing of its components. Preventive maintenance actions are taken before failures happen, typically at pre-specified time intervals, in this perspective different from predictive maintenance (where an individual estimate of the system conditions is available at any time, to drive maintenance actions) and run-to failure or reactive maintenance (only once things are broken, they are fixed). In railway systems, PM is performed to ensure a state that allows trains to run with high safety standard and a high level of service. From 15 to 25 billion euros are being annually spent in Europe for maintenance and renewal of railway infrastructure, and maintenance cost is generally recognized as representing a large part of the railway operation costs [11].

Maintenance works are carried out during a reserved period (non-available for full use by train traffic) on part of a rail network, named *possession*, mostly defined during a rather complex planning process. A general structure of the maintenance planning process is reported in Budai et al. [2] for the Netherlands. The process is organized in the following major decision phases: 1) budget determination; 2) long-term quality prediction; 3) project identification & definition (diagnosis); 4) project prioritization and selection; 5) possession allocation and timetabling of track possession; 6) project combination; 7) short term maintenance and project scheduling; 8) work evaluation and feedback loop. A similar structure is proposed by Lidén in [30], as identified in the Swedish practice. The structure is made of the following steps: 1) major possessions (large infrastructure maintenance activities) are coordinated with the international freight trains according to established prearranged paths; 2) preparation and publishing of the network statement; 3) yearly timetable planning considering the major possession activities; 4) timetable revision planning; 5) planning of minor possession activities; 6) operational planning and control. Overall, the maintenance planning process embraces strategic, tactical and operational phases, where different levels of detail and knowledge of the processes are available.

This paper deals with the integration of train scheduling and maintenance activities through optimization techniques at a tactical stage, where enough detail is available about infrastructure maintenance actions and the train services to put into operation.

The interaction between maintenance and train schedules is obviously critical, especially on high traffic density lines, at both strategic, tactical and operational phases. In fact, those two problems aim to conflicting purposes and the nature of their relationship is competitive, since they simultaneously subtract capacity from the network, one for keeping the network in good state, the other one to make revenues. Railway managers have to deal with a growing demand for transport of goods and people, while ensuring safety, punctuality and reliability of the freight and passenger services. Moreover, a better maintenance planning allows for higher utilization of railway infrastructure and improved services and customer quality [53].

As discussed in a recent survey [30], the management of train traffic and maintenance activities on the railway infrastructure are two key problems for the railway managers. However, these are often treated separately, although the two management issues are strongly interconnected. In what follows we examine the specific optimization problems related to train traffic and infrastructure maintenance management at timetabling, operational and tactical levels.

Typically, the management of railway traffic is based on solving a Train Timetabling Problem (TTP) [6], that is to compute a train schedule specifying the physical route of each train in the network, its arrival, dwell and departure times at each station, and its travel time between consecutive stations, such that some key performance indicators, such as the minimization of the total travel time of the trains in the network in order to cover all services, are optimized. TTP is widely treated in the classical and recent literature (see, e.g. the recent surveys of Cacchiani et al. [4, 5, 6]), and despite different perspectives, all share a common a-priori setup. Timetables are determined more than a year in advance of operations for a set of trains, providing optimality of an objective function of interest for the railway system, with a macroscopic detail of the operations, a very limited (when not absent) inclusion of maintenance processes, and a very limited (when not absent) modelling of possible delays.

In fact, including maintenance determines a partially non-periodic timetabling problem (for cyclic timetables, the cycle time of a timetable, usually an hour, is of much different length than the period of maintenance, typically weeks or months). Moreover, the exact length of maintenance slots and their arrangement are typically unknown at timetabling phase.

Considering delays leads to forms of robust timetabling, where the goal is to determine schedules that try, in case of unexpected events arising in the railway network, to keep good solutions as much as possible, avoiding the propagation of train delays. Robust timetabling typically considers minor disturbances and rescheduling actions, such as fine-tuning the timing of trains (typically no more than a handful of minutes variations), which allow better absorption of unknown delays (see, e.g., [25]).

On the other hand, unpredicted disturbances and disruptions are inevitable during operations. Typically, a disturbance is a small perturbation of the process times, while a disruption is a large disturbance (e.g. a track blockage, a serious accident or bad weather conditions) that requires strong adjustments of traffic, like re-timing, re-sequencing, re-routing services, or even leads to the cancelation of some services (see, e.g., [1, 18, 29, 41]). Once a perturbation occurs (and it can be identified and quantified), the timetable must be adjusted in order to recover feasibility of railway operations, by adjusting the existing timetable. The time horizon to this is typically in the range of a hour. This is the goal of the *a-posteriori* view followed by the Train Rescheduling Problem (TRP) [15, 17, 23, 35], which can be seen as the operational phase of TTP [4].

A different perspective arises at tactical stage, which is an intermediate stage between the two reported so far, with a time horizon of few days. We could refer to this problem as the Tactical Traffic and Possession Scheduling Problem (TTPSP) where decisions about re-timing, re-sequencing and re-routing of trains have to be taken, before the operational day, with some knowledge of maintenance actions and traffic perturbations.

At this level, the impact between perturbations and maintenance is bidirectional, as disturbances have also impact on the infrastructure maintenance management, and maintenance actions themselves can be viewed as a form of unavoidable perturbation to timetabled operations. To this end, we argue that planning of traffic and minor possessions requires a significant coordination effort with train planning plans. Despite that, there are currently no tools to assist the railway managers in the coordination task, and minor possession planning is often carried out regardless of detailed knowledge about the impact on railway traffic flow management.

The current limitations of the railway traffic management practice motivate the current paper, that addresses the problem of integrating train traffic planning and minor possession planning at a tactical level, with inclusion of robustness and stochastic dynamics. This leads to the following practical requirements and problem setting:

1. the maintenance activities are scheduled on the same infrastructure resources and during the same time period required by the trains to perform the services;

2. a (given) preliminary planning of the maintenance actions needs to be adjusted in order to improve the integration of traffic and maintenance processes;

3. estimates of traffic flow perturbations are considered, that can be included in a stochastic optimization framework;

4. the performance indicators related to maintenance activities and to the train services are in competition, and need to be optimized in a multi-objective setting;

5. the quality of the integrated solutions is evaluated in terms of planned and realized performance, for each evaluated scenario.

As regards the first point, we refer to a mathematical modeling based on a network cumulative flow variable based formulation and to a reformulation based on the big-M method. These formulations were proposed by Meng at al. [39] for solving an N-track simultaneous train rerouting and rescheduling problem (i.e. TRP). Meng et al. [37] extended the big-M formulation in order to deal with robust dispatching plans and stable routing decisions. However, previous versions of these formulations neglect the optimization of maintenance aspects and their interaction with traffic flows. This work addresses this aspect by proposing a revised version of the big-M formulation.

As for points 2 and 3, we investigate the integrated solutions to random perturbations of the train travel times and of the maintenance works. These uncertainties are modeled as multiple scenarios weighed with an appropriate probability of occurrence, by which an average performance can be computed. We evaluate stability constraints (as in Meng et al. [37]) which force the traffic to deliver the same service (i.e. to maintain train routing decisions), under still-unknown (at a tactical stage) realizations of the stochastic scenarios. This has the potential to keep train traffic regular, represents an interface to traffic demand, and reduces the complexity of operational rescheduling.

As for point 4, we propose two scalarization methods for the computation of Pareto-optimal solutions based on a standard weighted-sum approach, and ε-constraint methods (iteratively fixing a value of an objective while optimizing the other one, see e.g., [14, 54]).

As for the last point, we refer to computational experiments based on the train scheduling and routing instances introduced for the first time in the INFORMS RAS Competition 2012 [24], and now extended in order to deal with maintenance activities.

This work is organized as follows. Section 2 provides a literature review and research motivations. Section 3 explains the problem characteristics with respect to well-know problems. Section 4 presents the mathematical optimization framework. Section 5 gives the computational results regarding Pareto-optimal, robust and stable solutions. Section

6 discusses the paper contents and suggests directions for further work. Appendix sections report the list of input parameters (Appendix A), present numerical examples of Pareto-optimal solutions (Appendix B) and robust versus stable solutions (Appendix C), illustrate some optimized timetables (Appendix D).

2 Review of the related literature

The literature review proposed in this section provides a brief contextualization of the present work with respect to the recent state-of-the-art on railway operations management, and refers to extensive surveys for a more detailed review of some research streams. We first review papers related to the separate optimization of either traffic flows (i.e. TTP and TRP) or maintenance works. We then focus on the papers related the integration of railway operations management problems. We also cite some papers on the application of multi-objective optimization to railway problems. The section concludes with a discussion of what needs to be addressed in the problem studied in this paper.

TTP and TRP are well treated topics in current and past literatures. Periodic and non-periodic TTPs have been introduced, respectively, by Serafini and Ukovich [52] and Szpigel [55]. The extensive survey papers of Fang et al. [15], Hansen and Pachl [23] and Lusby et al. [35] (Cacchiani et al. [4, 5, 6]) review more recent works on the TRP (TTP). Among the recent TRP literature, we mention two main streams of research: a tactical version of the TRP (see, e.g., [27, 37, 38, 39, 51]) and an operational version of the TRP (see, e.g., [12, 13, 46, 49, 50]). In the tactical level, the TRP includes robustness and stability considerations (see, e.g. [7, 21, 37, 46]) in order to prevent that traffic disturbances make the plans infeasible in practice. In the operational level, the TRP deals with the creation of feasible plans in presence of traffic disturbances. In both levels, several contributions investigate the potential benefits of train routing flexibility (see, e.g., [9, 40, 43]). Our work can be viewed as a tactical TRP approach with flexible routing.

Maintenance planning is also a well-studied problem in the literature. Lee and Cha [28], Lin et al. [33], Gustavsson et al. [22], Manzini et al. [36] Wang et al. [57] and Pargar et al. [42] provide recent and comprehensive analysis of the literature related, respectively, to preventive maintenance policies, preventive maintenance for deteriorating complex repairable systems, preventive maintenance of system components, scheduling preventive maintenance in a production environment with complex machines, classifications of maintenance strategies, grouping for preventive maintenance scheduling. In the railway context, Lidén [30] recognized that few works have been published about how to schedule train traffic and railway maintenance jointly. Previous papers mostly focus on optimizing the maintenance aspect while trains are dispatched in a later stage. However, there is an increasing need to optimize the coordination of train traffic and maintenance activities in order to improve the quality of services and to reduce the railway operating costs. Coordination methods have to be evaluated in an integrated framework that considers the objectives of the different optimization problems.

A few papers have been found regarding the coordination of train scheduling and infrastructure maintenance in the literature. Budai et al. [2] highlight the importance of developing decision support tools, for infrastructure maintenance planners, that are able to suggest optimal schedules of maintenance works. To this aim, they propose a mathematical programming formulation for clustering of maintenance activities on the same link in a network in order to reduce the disturbance of railway traffic. However, the railway traffic is not directly modeled in their formulation. Peng et al. [44] propose a time-space network model and include some constraints about how maintenance works impacts on railway operations while solving a preventive maintenance scheduling problem. Peng et al. [45] extend their previous approach in order to solve a large-scale rail inspection routing and scheduling problem. Forsgren et al. [16] develop a MIP model that reschedules trains such that the impact on traffic flows is as much limited as possible. An original timetable is given as well as a fixed set of track possessions. Vansteenwegen et al. [56] propose an algorithm with the ability to reschedule the train timing and routing decisions for a given schedule of maintenance activities. Albrecht et al. [1] address the problem of developing good quality timetables in which both train movements and scheduled track maintenance activities are simultaneously considered. The approach is shown to be applicable as an operational tool to generate feasible train schedules when disruption occurs. Lidén and Joborn [31] minimize maintenance costs and traffic limitations when dimensioning maintenance windows. However, there is no planned timetable to be-revised, i.e. the timetable is unknown.

Recently, Luan et al. [34] present a first attempt to insert rail maintenance constraints in the train scheduling formulation of [37]. However, the objective function proposed in [34] only addresses train scheduling decisions, disregarding the optimization of maintenance schedules. There are other works on integrated problems but they study different levels of integration. For example, Corman et al. [8] and Dollevoet et al. [13] integrate the schedules related to trains and passengers and optimize both the train schedules and the transfer connections; Giacco et al. [19] and Lai et al. [26] investigate how to improve the integration between rolling stock circulation and maintenance planning.

From the above discussion of the literature, there are a few approaches dealing with the integration of train scheduling and maintenance planning, since the two problems have different modeling characteristics in terms of constraints and objective functions (we assume that *planning* and *scheduling* are referring to the same problem, and use them interchangeably in what follows, for the sole purpose of ensuring clarity). Furthermore, their modeling requires to deal with uncertainties related to both types of operations. The problem of planning trains is subject to rescheduling actions in case of disturbances due to delays of some trains plus differences between planned and realized maintenance works. As a result, the robustness and stability of the traffic flows, i.e. the impact of stochastic phenomena to the actual decisions and their performance, need to be studied in an integrated framework.

Another issue to be addressed is related to the usual assumptions made for planning the maintenance activities. While in most of the reviewed literature the combination of maintenance activities is pre-defined at each site, an advanced approach should be able to compute an optimal timing of the maintenance activities. Furthermore, the simultaneous problem of planning trains and scheduling maintenance activities on common resources, at any time scope, requires to deal with competing objectives.

Some recent literature focuses on multi-objective optimization for the management of public transport operations (see, e.g., weighted-sum method for a collaborative optimization of train stop planning and train scheduling [60], weighted-sum and ε-constraint methods for optimal railway capacity allocation [3], bi-objective conflict detection and resolution approaches for scheduling trains and transfer connections [10], ε-constraint and distance-based methods for a multi-objective and multi-track train scheduling problem

[20], constraint generation procedures to optimize multiple objectives in terminal control area air traffic management [48, 47], compromise approaches to minimize passenger travel time and energy consumption [58, 59, 62] or to minimize passenger delays and train operating costs [61] in busy metro lines). However, there is no previous work that investigates the bi-objective integrated optimization problem addressed in this paper.

3 Problem description

The problem studied in this paper can be defined as follows. Given a railway network, a set of planned arrival and departure times of the trains at stations, a set of maintenance activities at some network locations, a time horizon of traffic prediction, the problem is to find an optimal integrated schedule that simultaneously determines the traffic flow and maintenance related aspects. In more detail, the traffic flow aspects are related to scheduling and routing trains, while the maintenance aspects are related to timing these aspects. Clearly, the two aspects are interrelated, since train operations and maintenance activities ask for the same infrastructure resources during the same time horizon.

3.1 Traffic and maintenance

We assume that the rail network is given at a microscopic level of representation, i.e. at the level of block sections. The timetable information on scheduled arrival and departure times of the trains comes from earlier planning phases. Similarly, maintenance activities and their sequencing are given, at the tactical planning level studied in this paper. Instead, the definition of a detailed traffic flow, a detailed maintenance plan and their integration are the variables of the studied problem. From a temporal point of view, this tactical problem has to be solved a few days or weeks before the operational day.

We now introduce the technical terms used in this paper. A *node* is a specific physical point of the network. For example, it can be used to specify the start or end of a block section, a point of convergence or divergence of two or more tracks, a stopping location at a station platform (where a train is allowed to embark/disembark passengers, or to perform loading/unloading of goods). Two nodes are connected by a *cell*, which is therefore by definition limited between a node i and a node j. A cell is often referred as a block section in the literature, thus cell and block section are synonymous. A cell allows the passage of a train at a time, which means implicitly that the capacity of a block section is one.

Based on the above definitions, we refer to a network as a set of sequences of nodes and cells. A *route* in the network is a subset of subsequent cells from a so-called *origin node* to a so-called *destination node* assigned to a train. Each train has one origin and one destination node, but multiple alternative routes from origin to destination. All the routes of each train have the same origin and destination node. A *planned departure time* (earliest start time) and *planned arrival time* (due date time) are respectively defined for the origin and destination nodes. We refer to a *planned dwell time* as the time a train spends on a station cell without moving. A *planned travel time* is the time required by a train to travel on a cell. For each train, the timetable defines a minimum dwell time at each station cell and a minimum travel time at each cell. However, in presence of disturbances, planned departure and arrival times of the trains can be delayed, while travel and dwell times of trains can be larger than the planned values.

The maintenance activities need to be carried out on *maintenance areas*, that are specific cells of the network. Each cell of a maintenance area requires the same *maintenance volume*, i.e. the sum of the processing times of all the maintenance works on that cell. Cells belonging to different maintenance areas may require different maintenance volumes.

In each maintenance area, we assume that maintenance works have a given duration and cannot be carried out simultaneously due to a limited availability of resources. They have to be performed in a pre-defined processing order. However, an optimal processing order between train operations and maintenance works is unknown and needs to be computed for each area. Beyond classic TRP constraints , we thus need to detect and solve conflicting requests of the same cells by different train operations and maintenance works. Specific *capacity* constraints are required to model occupation and release times of each cell. We introduce these constraints via the computation of *safety time intervals*, or safety headways, between pairs of consecutive trains, or pairs of consecutive maintenance works, or mixed pairs of train operations and maintenance works. All train operations and maintenance works have to be carried out within a given *time horizon*.

As regards the performance indicators to be optimized, we consider train traffic flow aspects as well as maintenance related aspects. Since the nominal timetable requires adjustments in an uncertain environment, the goal of our tactical approach to the TRP is to minimize the positive and negative *deviations* from the nominal timetable. Regarding the maintenance aspect, we aim at pairing as many as possible maintenance works in each area. Two works are *paired* if they are processed one immediately after the other. Pairing maintenance works can be considered as a process of establishing possession of the appropriate length. This kind of objective function has been recognized to generate substantial benefits ([2], [31], [32], [42]).

Figure 1 presents an illustrative example regarding the interaction between train scheduling and routing decisions and the allocation of a maintenance work on the maintenance area. The example considers two trains traversing the network in opposite directions. Train 1 travels from right to left and can use two routes: route 1 includes the cell requiring maintenance works, while route 2 is partially overlapping with the route assigned to train 2. The latter train travels from left to right via the assigned route.

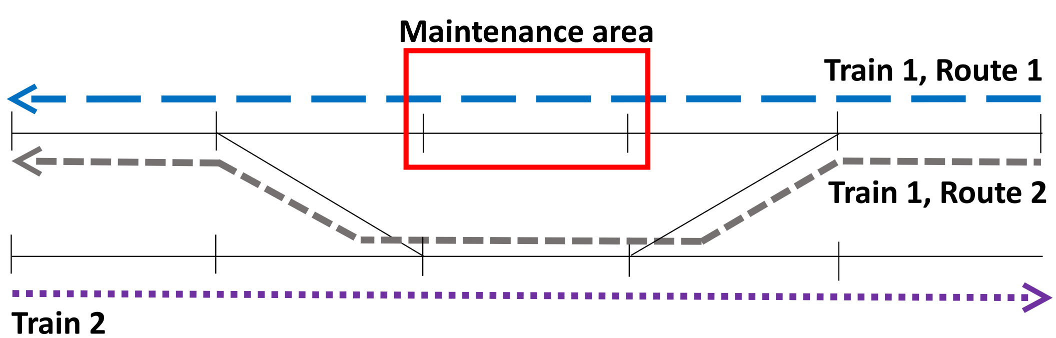

Figure 1: Example of interaction between traffic and maintenance scheduling

Four solutions are possible for the example of Figure 1. Solution A (B): Train 1 uses route 1 and the maintenance work is scheduled before (after) train 1 occupies the cell. Solution C (D): Train 1 is rerouted via route 2 and is scheduled before (after) Train 2. The selection of a solution clearly depends on the importance given to the different objectives and to the level of routing flexibility adopted regarding the management of traffic flows.

3.2 Stochastic and uncertain dynamics

At the tactical level followed in this study, we have some limited knowledge of traffic and maintenance operations, and we should include the fact that both processes may perform different than the plan. This is achieved by considering small perturbations of process times in a stochastic modeling environment. The variability of process times is applied to the travel times of trains and to the duration of maintenance works. We assume that the former times can vary in a considerably smaller window of values compared to the latter times, since typically there is more uncertainty related to the railway infrastructure maintenance. The stochastic environment is modeled by defining a set of *scenarios*: a nominal scenario, i.e. the process times are exactly as in the nominal timetable and there is no uncertainty regarding their values, and some disturbed scenarios, where the process times can be equal, larger or shorter than planned in the nominal timetable, depending on a given variance and a specified probability distribution that we assume Gaussian.

We assess the quality of the solutions computed in this way by considering the weighted average over all scenarios, in a similar way to Corman et al. [7]. The proposed indicators are related to the objective function of the train scheduling aspect of the problem, and to the objective function of the infrastructure maintenance aspect. We optimize the train-related and maintenance-related indicators for each stochastic scenario and compute the average variability of the objective functions (which can be seen as measure of robustness of the performance indicators).

For the disturbed scenarios, we can also look at the variability of the train routing variables (which can be seen as a measure of stability of the solution). This basically determines a stochastic problem where the re-routing decisions kept fixed are a first stage of a stochastic programming, and where the other variables define the second stage. We focus on re-routing as this is one of the strongest actions that can be taken in an operational perspective, and it is not very often performed during rail operations; and moreover because a route decision would allow much more substantial room in the network when maintenance works have to be performed on adjacent tracks, compared to other typical operational actions such as re-sequencing and re-timing, which would then be the second-stage decisions (for instance, following approaches based on D'Ariano et al. [12]). In particular, we use routing stability constraints (as in Meng et al. [37]), forcing trains to keep the same route in all disturbed scenarios.

4 Problem formulation

This section describes the mathematical programming formulation that is proposed in this paper for the integrated problem. We next summarize the main assumptions that characterize the formulation:

- The network is divided into a set of block sections and stations. In each station, we consider up to two alternative tracks, a main and a secondary track for each train.

- Each block section or station track (i.e. a cell) can host at most one train or one maintenance work at a time.

- In each maintenance area, the maintenance works follow a pre-defined order of processing.
- All trains are of the same type and we do not consider prioritization of train sequencing and routing decisions.
- The occupation time, release time, and safety time intervals are modeled at a macroscopic level, i.e. the granularity of train traffic flow representation is in minutes.
- The duration of maintenance works is also modeled at a macroscopic level.
- The travel times of each train are computed for a given speed profile. This can be specific for the train. However, we do not consider adjustments of the travel times due to re-sequencing decisions, causing unscheduled waiting times (which would be performed in an operational perspective). In order words, we do not consider possible modifications of train speed profiles when computing scheduling solutions.
- Trains can pass through a station or have a scheduled stop. However, we measure the deviation from the nominal timetable only at the exit of each train from the network. Furthermore, trains have a planned arrival time at their destination and a planned departure time from their origin.
- We consider a minimum and maximum dwell time for each train on each cell.
- All operations must start after a given start time $T_s = 0$ and must be completed within a given end time $T_e = T$.

The input data can be grouped in the following sets: V is the set of nodes and E is the set of cells of the infrastructure, F is the set of trains, B is the set of maintenance areas, $P_{(i,j)}$ is the set of maintenance works on cell $(i,j) \in E$, and S is the set of random scenarios. A detailed list of the input parameters of the formulation and related subscripts is reported in Tables 14 and 15 of Appendix A.

All the modeled train traffic flows and infrastructure maintenance activities must be completed within a given time horizon T. The possible routes for a train $f \in F$ are given as a set E_f of cells and a set V_f of nodes that the train is allowed to use. Each train f has a traveling direction δ_f in the network. For instance, if train f drives through (i,j) and train f' drives through (j,i), they will travel on the same cell, but in opposite directions. This situation requires to introduce δ_f in order to model some capacity constraints correctly. The origin and destination nodes of f are respectively O_f and D_f. Planned times to get in and out the network for train f are EST_f and PCT_f. For each train, a planned (minimum) travel time $FT_f(i,j,s)$ is given to traverse (travel on) cell (i,j) under scenario $s \in S$. Minimum and maximum dwell times are also defined for train f on cell (i,j) under scenario s. $w_f^{min}(i,j,s)$ and $w_f^{max}(i,j,s)$ are the minimum and maximum dwell time values allowed for a train on cell (i,j). In practise, the travel and dwell times can be subject to random variations depending on the scenario s and its occurrence probability Pr_s.

We have to consider the following safety time intervals: $h_f(i,j)$ is required between the occupation end of train f from cell (i,j) and the release of cell (i,j); $g_f(i,j)$ is required between the preparation of cell (i,j) and the occupation start of train f on cell (i,j); $mh_p(i,j)$ is required before/after the processing of the maintenance work $p \in P_{(i,j)}$.

Therefore, $g_f(i,j)$ and $h_f(i,j)$ represent time periods required for safety reasons. In more detail, a cell (i,j) is released $h_f(i,j)$ time units after the occupation end of train f, while a cell (i,j) needs to be ready for processing $g_f(i,j)$ time units before the occupation start of train f. Regarding the maintenance works, $mh_p(i,j)$ represents the time units required for the pre-processing [post-processing] of a cell (i,j) in case the work p is executed after [before] a train operation. Furthermore, we assume that the time period required for the pre-processing [post-processing] of a cell is null in case the work p is executed after [before] another work.

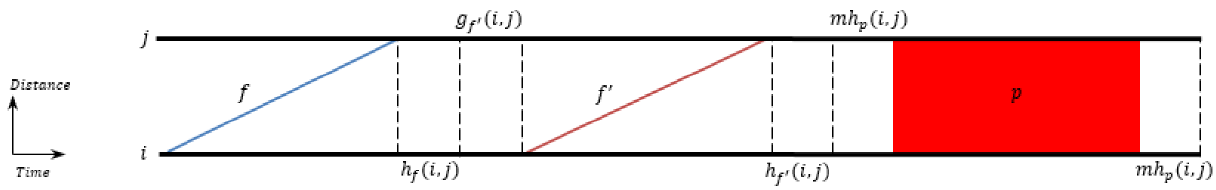

Figure 2: Safety time intervals between two trains f and f' and a maintenance work p

Figure 2 shows an illustrative example of the safety time intervals for two consecutive trains f and f' traveling on a cell (i,j) in the same traveling direction. After train f', a maintenance work p is performed. All the required safety time intervals are shown both for the two consecutive trains and between f' and p.

Each maintenance area $b \in B$ is made by E_b cells and requires a maintenance volume H_b to be carried out within a given time horizon T for the completion of all the activities. The latter value represents the maintenance works to be carried out within T on each cell in E_b. Each maintenance work p on a cell (i,j) has a pre-defined duration $MT_p(i,j,s)$ for scenario s.

The formulation of the integrated problem presents the following decision variables:

Traffic aspect : Timing, sequencing and routing variables for modeling the trains in the network. Specifically, the timing variables of a train f are related to its entrance, exit, planned travel and dwell times on each cell.

Maintenance aspect : Timing and pairing variables for modeling the maintenance works. Specifically, the timing variables of a maintenance work are related to its start and end of processing on each cell, while the pairing variables are used to measure whether two consecutive maintenance works are processed one immediately after the other one.

Integration aspect : Sequencing variables between maintenance works and trains for modeling their integration. These variables are required in order to schedule both the traffic and maintenance aspects on common cells.

The decision variables are specified in Table 1.

4.1 Objective functions

The integrated problem requires to optimize the view point of competing entities: the dispatchers of the train traffic flows and the managers of the maintenance activities are asking for the same infrastructure resources in overlapping time periods.

We model the objective function of the dispatchers as the minimization of the total deviation from the nominal timetable. The total deviation is defined for all trains as the absolute difference between the actual arrival time and the planned arrival time at their destination node. This objective function can be formulated as follows:

$$\min f_1 = \min \sum_{s \in S} [Pr_s \sum_{f \in F} \sum_{i:(i,D_f) \in E_f} \mid d_f(i, D_f, s) - PCT_f \mid] \tag{1}$$

We surrogate the infrastructure managers' objective as the maximization of the number of paired works, since the maintenance works should be performed in as little time as possible in order to save personnel and tool costs. The number of paired works is defined for all maintenance areas as the number of consecutive maintenance works that are processed one immediately after the other. This objective function can be formulated as follows:

$$\max f_2 = \max \sum_{s \in S} [Pr_s \sum_{b \in B} \sum_{(i,j) \in E_b} \sum_{p=2,\ldots,|P_{(i,j)}|} y_p(i, j, s)] \tag{2}$$

For both the objective functions, we consider all scenarios according to their probability of occurrence.

Symbol	Description
$x_f(i, j, s)$	Binary variable for train routing. $x_f(i, j, s) = 1$, if cell (i, j) is chosen for train f under scenario s. Otherwise, $x_f(i, j, s) = 0$.
$o(f, f', i, j, s)$	Binary variable for train sequencing. $o(f, f', i, j, s) = 1$ if train f travels from i to j and is scheduled before train f' (traveling in any direction) on cell (i, j) under scenario s. Otherwise, $o(f, f', i, j, s) = 0$.
$o(f, f', j, i, s)$	Binary variable for train sequencing. $o(f, f', j, i, s) = 1$ if train f travels from j to i and is scheduled before train f' (traveling in any direction) on cell (j, i) under scenario s. Otherwise, $o(f, f', j, i, s) = 0$.
$a_f(i, j, s)$	Integer variable for the entrance time of f on cell (i, j) under scenario s.
$d_f(i, j, s)$	Integer variable for the exit time of f from cell (i, j) under scenario s.
$t_f(i, j, s)$	Integer variable for the sum of planned travel and dwell times of f on cell (i, j) under scenario s.
$l_p(f, i, j, s)$	Binary variable for sequencing maintenance works and trains. $l_p(f, i, j, s) = 1$, if train f is scheduled after execution of work p on cell (i, j) or (j, i) under scenario s. Otherwise, $l_p(f, i, j, s) = 0$.
$y_p(i, j, s)$	Binary variable for pairing maintenance works. $y_p(i, j, s) = 1$, if maintenance works $p-1$ and p are processed one immediately after the other one on cell (i, j) under scenario s. Otherwise, $y_p(i, j, s) = 0$.
$a_p(i, j, s)$	Integer variable for the start time of maintenance work p on cell (i, j) under scenario s.
$d_p(i, j, s)$	Integer variable for the end time of maintenance work p on cell (i, j) under scenario s.

Table 1: List of decision variables

4.2 Constraints

This section describes the constraints of the integrated problem. For clarity reasons, they are grouped into nine sets, concerning specific constraints regarding the studied problem. We next explain them set by set. The constraints are defined for all scenarios.

Set I: Flow balance constraints.

- Flow balance constraints at the origin nodes:

$$\sum_{j:(O_f,j)\in E_f} x_f(O_f, j, s) = 1 \qquad \forall f \in F, \forall s \in S \tag{3}$$

- Flow balance constraints at intermediate nodes:

$$\sum_{i:(i,j)\in E_f} x_f(i, j, s) = \sum_{k:(j,k)\in E_f} x_f(j, k, s) \qquad \forall f \in F, \forall j \in V_f \backslash \{O_f, D_f\}, \forall s \in S \tag{4}$$

- Flow balance constraints at the destination nodes:

$$\sum_{i:(i,D_f)\in E_f} x_f(i, D_f, s) = 1 \qquad \forall f \in F, \forall s \in S \tag{5}$$

In set I, constraints (3) and (5) ensure that each train f occupies exactly one cell that starts from the origin node of f and one cell that ends in the destination node of f. Constraints (4) ensure that if a train f occupies a cell that ends in an intermediate node j, then train f must also occupy a cell that starts from node j.

Set II: Time-space network constraints.

- Entrance time constraints at the origin nodes:

$$a_f(O_f, j, s) \geq EST_f \ x_f(O_f, j, s) \qquad \forall f \in F, \forall j : (O_f, j) \in E_f, \forall s \in S \tag{6}$$

- Cell-to-cell transition constraints:

$$\sum_{i:(i,j)\in E_f} d_f(i, j, s) = \sum_{k:(j,k)\in E_f} a_f(j, k, s) \qquad \forall f \in F, \forall j \in V_f \backslash \{O_f, D_f\}, \forall s \in S \tag{7}$$

- Mapping constraints between time-space network and physical network:

$$x_f(i, j, s) - 1 \leq a_f(i, j, s) \leq x_f(i, j, s)M \qquad \forall f \in F, (i, j) \in E_f, \forall s \in S \tag{8}$$

$$x_f(i, j, s) - 1 \leq d_f(i, j, s) \leq x_f(i, j, s)M \qquad \forall f \in F, (i, j) \in E_f, \forall s \in S \tag{9}$$

In set II, constraints (6) specify that the routing of each train must start from its origin node after its earliest start time. Constraints (7) ensure that the exit time of each train from a cell (i, j) must coincide with the entrance time of the train in the following cell (j, k). Clearly, the end node of cell (i, j) must be equal to the star node of cell (j, k), i.e. the intermediate node j. Constraints (8) and (9) allow that the arrival and departure times of each train f are larger than zero only on the cells traversed by the selected route.

Set III: Travel and dwell time constraints.

- Entrance and exit time constraints:

$$t_f(i,j,s) = d_f(i,j,s) - a_f(i,j,s) \qquad \forall f \in F, (i,j) \in E_f, \forall s \in S \tag{10}$$

- Minimum and maximum dwell time constraints:

$$t_f(i,j,s) \geq [w_f^{min}(i,j,s) + FT_f(i,j,s)]\ x_f(i,j,s) \qquad \forall f \in F, \forall (i,j) \in E_f, \forall s \in S \tag{11}$$

$$t_f(i,j,s) \leq [w_f^{max}(i,j,s) + FT_f(i,j,s)]\ x_f(i,j,s) \qquad \forall f \in F, \forall (i,j) \in E_f, \forall s \in S \tag{12}$$

In set III, constraints (10) specify that the sum of travel and dwell times of each train f on a cell (i,j) must be equal to the difference between the exit time and the entrance time of f on (i,j). Constraints (11) and (12) specify that the travel and dwell times of each train f on a cell (i,j) must be equal or larger (smaller) than its planned travel time plus its minimum (maximum) planned dwell time. Specifically, the difference between the minimum and maximum planned dwell times is larger than zero, only if there is a scheduled stop in the timetable for train f on cell (i,j).

Set IV: Mapping constraints between train sequencing and routing.

$$x_f(i,j,s) + x_{f'}(i,j,s) - 1 \leq o(f,f^{'},i,j,s) + o(f^{'},f,i,j,s) \leq 3 - x_f(i,j,s) - x_{f'}(i,j,s)$$
$$\forall f,f^{'} \in F : f \neq f^{'}, \delta_f = \delta_{f'};\ \forall (i,j) \in E_f \cap E_{f'};\ \forall s \in S \tag{13}$$

$$x_f(i,j,s) + x_{f'}(j,i,s) - 1 \leq o(f,f^{'},i,j,s) + o(f^{'},f,i,j,s) \leq 3 - x_f(i,j,s) - x_{f'}(j,i,s)$$
$$\forall f,f^{'} \in F : f \neq f^{'}, \delta_f \neq \delta_{f'};\ \forall i,j \in V : (i,j) \in E_f \wedge (j,i) \in E_{f'}; \forall s \in S \tag{14}$$

$$o(f,f^{'},i,j,s) \leq x_f(i,j,s)$$
$$\forall f,f^{'} \in F : f \neq f^{'}, \delta_f = \delta_{f'}; \forall (i,j) \in E_f \cap E_{f'}; \forall s \in S \tag{15}$$

$$o(f,f^{'},i,j,s) \leq x_{f'}(i,j,s)$$
$$\forall f,f^{'} \in F : f \neq f^{'}, \delta_f = \delta_{f'}; \forall (i,j) \in E_f \cap E_{f'}; \forall s \in S \tag{16}$$

$$o(f,f^{'},i,j,s) \leq x_f(i,j,s)$$
$$\forall f,f^{'} \in F : f \neq f^{'}, \delta_f \neq \delta_{f'}; \forall i,j \in V : (i,j) \in E_f \wedge (j,i) \in E_{f'}; \forall s \in S \tag{17}$$

$$o(f,f^{'},i,j,s) \leq x_f(j,i,s)$$
$$\forall f,f^{'} \in F : f \neq f^{'}, \delta_f \neq \delta_{f'}; \forall i,j \in V : (i,j) \in E_f \wedge (j,i) \in E_{f'}; \forall s \in S \tag{18}$$

For set IV, constraints (13) and (14) specify how an ordering decision between two trains f and $f^{'}$ on the same cell (i,j) or (j,i) is modeled, respectively when f and $f^{'}$ travel in the same direction ($\delta_f = \delta_{f'}$) or opposite directions ($\delta_f \neq \delta_{f'}$). Specifically, if f preceeds $f^{'}$, the binary variables related to train sequencing decisions are as follows: $o(f,f^{'},i,j,s) = 1$ and $o(f^{'},f,i,j,s) = 0$. Alternatively, $o(f,f^{'},i,j,s) = 0$ and

$o(f', f, i, j, s) = 1$. Therefore, both in constraints (13) and (14), the sum of the ordering variables must be equal to 1 when an ordering decision is taken on cell (i, j) between trains f and f'. However, this type of constraints must be *activated* only when both trains f and f' use the same cell, i.e. both the route of train f and the route of train f' contain cell (i, j). Table 2 shows how constraints (13) work in detail. The first column shows the case in which a train sequencing decision must be activated between trains f and f', i.e. when both routing variables of f and f' are equal to 1. The other two columns represent the case in which at least one of the two trains does not use cell (i, j). Constraints (14) work similarly.

Constraint Bounding	$x_f(i,j,s) = 1 \wedge x_{f'}(i,j,s) = 1$	$x_f(i,j,s) = 0 \vee x_{f'}(i,j,s) = 0$	$x_f(i,j,s) = 0 \wedge x_{f'}(i,j,s) = 0$
Left bound	1	0	-1
Right bound	1	2	3

Table 2: Values for the left and right bounds of constraints (13)

Constraints (15), (16), (17) and (18) complete the mapping between the train sequencing variables and the train routing variables, i.e. the cell occupation variables. This is achieved by specifying, in all cases, that the value of an ordering variable between trains f and f' can never be larger than the value of the routing variables of trains f and f'. In other words, train f can be scheduled first or after than train f' on a cell (i, j) only if both trains f and f' use that cell.

Set V: Capacity constraints between trains.

$$a_{f'}(i,j,s) + [3 - x_f(i,j,s) - x_{f'}(i,j,s) - o(f,f',i,j,s)]M \geq d_f(i,j,s) + g_{f'}(i,j) + h_f(i,j)$$
$$\forall f, f' \in F : f \neq f', \delta_f = \delta_{f'}; \forall (i,j) \in E_f \cap E_{f'}; \forall s \in S \tag{19}$$

$$a_f(i,j,s) + [3 - x_f(i,j,s) - x_{f'}(i,j,s) - o(f',f,i,j,s)]M \geq d_{f'}(i,j,s) + g_f(i,j) + h_{f'}(i,j)$$
$$\forall f, f' \in F : f \neq f', \delta_f = \delta_{f'}; \forall (i,j) \in E_f \cap E_{f'}; \forall s \in S \tag{20}$$

$$a_{f'}(j,i,s) + [3 - x_f(i,j,s) - x_{f'}(j,i,s) - o(f,f',i,j,s)]M \geq d_f(i,j,s) + g_{f'}(j,i) + h_f(i,j)$$
$$\forall f, f' \in F : f \neq f', \delta_f \neq \delta_{f'}; \forall i,j \in V : (i,j) \in E_f \wedge (j,i) \in E_{f'}; \forall s \in S \tag{21}$$

$$a_f(i,j,s) + [3 - x_f(i,j,s) - x_{f'}(j,i,s) - o(f,f',i,j,s)]M \geq d_{f'}(j,i,s) + g_f(i,j) + h_{f'}(j,i)$$
$$\forall f, f' \in F : f \neq f', \delta_f \neq \delta_{f'}; \forall i,j \in V : (i,j) \in E_f \wedge (j,i) \in E_{f'}; \forall s \in S \tag{22}$$

Sets V represent a set of typical safety constraints for the management of traffic flows. The aim is to impose a minimum safety time interval between two consecutive trains f and f' on a cell (i, j), as shown in Figure 2. Specifically, constraints (19) [constraints (20)] model the case in which f preceeds f' [f' preceeds f] and the two trains travel on the cell (i, j) in the same direction ($\delta_f = \delta_{f'}$). This type of constraints is activated when the variables $x_f(i,j,s)$, $x_{f'}(i,j,s)$ and $o(f,f',i,j,s)$ are equal to one, i.e. the big-M value disappears from the constraint. The activation means that the entrance time $a_{f'}(i,j,s)$ of f' on cell (i,j) must be equal or greater than the exit time $d_f(i,j,s)$ of f from cell (i,j) plus the safety time interval $h_f(i,j)$ required by f to release the cell and the safety time interval $g_{f'}(i,j)$ required by f' to be ready to start its occupation of the cell. Similarly, constraints (21) and (22) model the minimum safety time intervals required between two trains traveling on cell (i,j) in opposite directions ($\delta_f \neq \delta_{f'}$).

Set VI introduces the capacity constraints required to model the minimum safety time interval between a train f and a maintenance work p on the same cell (i,j). We assume that a fictitious direction is assigned to each work p on each cell (i,j), and all works are processed according the same direction. We distinguish the case in which f is performed in the same direction of p, as modeled by constraints (23) and (24), and the case in which f and p are performed in opposite directions, as modeled by constraints (25) and (26).

Constraints (23) [constraints (24)] model the case in which p is scheduled before f [f is scheduled before p] on cell (i,j) and the following minimum safety time interval is required: the entrance time $a_f(i,j,s)$ of f [the start time $a_p(i,j,s)$ of p] on cell (i,j) must be equal or greater than the end time $d_p(i,j,s)$ of p [the exit time $d_f(i,j,s)$ of f] on cell (i,j) plus the safety time interval $mh_p(i,j)$ required after [before] the processing of the maintenance work p and the safety time interval $g_f(i,j)$ [$h_f(i,j)$] required by f to be ready to start its occupation of the cell [to release the cell].

Set VI: Capacity constraints between trains and maintenance works.

$$\begin{aligned} & a_f(i,j,s) + [2 - x_f(i,j,s) - l_p(f,i,j,s)]M \geq d_p(i,j,s) + mh_p(i,j) + g_f(i,j) \\ & \forall f \in F, \forall b \in B, \forall (i,j) \in E_f \cap E_b, \forall p \in P_{(i,j)}, \forall s \in S \end{aligned} \tag{23}$$

$$\begin{aligned} & a_p(i,j,s) + l_p(f,i,j,s)M \geq d_f(i,j,s) + [mh_p(i,j) + h_f(i,j)]x_f(i,j,s) \\ & \forall f \in F, \forall b \in B, \forall (i,j) \in E_f \cap E_b, \forall p \in P_{(i,j)}, \forall s \in S \end{aligned} \tag{24}$$

$$\begin{aligned} & a_f(j,i,s) + [2 - x_f(j,i,s) - l_p(f,i,j,s)]M \geq d_p(i,j,s) + mh_p(i,j) + g_f(j,i) \\ & \forall f \in F, \forall b \in B, \forall (i,j) : (j,i) \in E_f \wedge (i,j) \in E_b, \forall p \in P_{(i,j)}, \forall s \in S \end{aligned} \tag{25}$$

$$\begin{aligned} & a_p(i,j,s) + l_p(f,i,j,s)M \geq d_f(j,i,s) + [mh_p(i,j) + h_f(j,i)]x_f(j,i,s) \\ & \forall f \in F, \forall b \in B, \forall (i,j) : (j,i) \in E_f \wedge (i,j) \in E_b, \forall p \in P_{(i,j)}, \forall s \in S \end{aligned} \tag{26}$$

Variables		*Constraints* (23)	*Constraints* (24)
$l_p(f,i,j,s) = 1$	$x_f(i,j,s) = 1$	*activated*	*not activated*
	$x_f(i,j,s) = 0$	*not activated*	*not activated*
$l_p(f,i,j,s) = 0$	$x_f(i,j,s) = 1$	*not activated*	*activated*
	$x_f(i,j,s) = 0$	*not activated*	*not activated*

Table 3: The activation cases regarding constraints (23) and (24)

Table 3 shows the cases in which constraints (23) and (24) are activated. A constraint (23) [constraint (24)] is activated when p preceeds f [f preceeds p] on cell (i,j), i.e. $l_p(f,i,j,s) = 1$ [$l_p(f,i,j,s) = 0$], and a route is selected for f, i.e. $x_f(i,j,s) = 1$.

We note that if $l_p(f,i,j,s) = 0$ and $x_f(i,j,s) = 0$ then $a_p(i,j,s) \geq 0$ (i.e. the corresponding constraint (24) is not activated), since the corresponding constraint (9) enforces $d_f(i,j,s) \leq 0$.

Constraints (25) and (26) work in a similar way to constraints (23) and (24), except for considering the case in which f and p are performed in opposite directions.

Set VII models some constraints related to the processing of the maintenance works and their pairing in each maintenance area. Constraints (27) ensure that the start time of a work on a cell is after the end time of the previous work (if any) on the same cell. Constraints (28) ensure that the duration of a maintenance work is equal to the difference between the start time and the end time of the work. Constraints (29) and (30) model the possible pairing of two maintenance works on a cell. We recall that two works are paired if and only if the end time of the previous work $p-1$ on a cell (i,j) is equal to the start time of the subsequent work p on the same cell under scenario s. The pairing is verified by setting $y_p(i,j,s) = 1$. In this case, the corresponding constraint (29) is activated, while the corresponding constraint (30) is not activated. The case $y_p(i,j,s) = 0$ activates the corresponding constraint (30) and does not activate the corresponding constraint (29).

Set VII: Maintenance works constraints.

- Consecutive maintenance works:

$$\begin{aligned} & a_p(i,j,s) \geq d_{p-1}(i,j,s) \\ & \forall b \in B, \forall (i,j) \in E_b, \forall p \in P_{(i,j)} \backslash \{1\}, \forall s \in S \end{aligned} \tag{27}$$

- Duration of the maintenance works:

$$\begin{aligned} & MT_p(i,j,s) = d_p(i,j,s) - a_p(i,j,s) \\ & \forall b \in B, \forall (i,j) \in E_b, \forall p \in P_{(i,j)}, \forall s \in S \end{aligned} \tag{28}$$

- Pairing of the maintenance works:

$$\begin{aligned} & M[1 - y_p(i,j,s)] \geq a_p(i,j,s) - d_{p-1}(i,j,s) \\ & \forall b \in B, \forall (i,j) \in E_b, \forall p \in P_{(i,j)} \backslash \{1\}, \forall s \in S \end{aligned} \tag{29}$$

$$\begin{aligned} & -My_p(i,j,s) \leq a_p(i,j,s) - d_{p-1}(i,j,s) - 1 \\ & \forall b \in B, \forall (i,j) \in E_b, \forall p \in P_{(i,j)} \backslash \{1\}, \forall s \in S \end{aligned} \tag{30}$$

Set VIII represents the deadline constraints related to the time horizon T. Constraints (31) model the maximum (allowed) end time for each maintenance work, while constraints (32) model the maximum (allowed) time for each train to exit from the network.

Set VIII: Deadline constraints.

- Maintenance work completion:

$$d_p(i,j,s) \leq T \qquad \forall b \in B, \forall (i,j) \in E_b, \forall p \in P_{(i,j)}, \forall s \in S \tag{31}$$

- Train exit time from the network:

$$d_f(i,D_f,s) \leq T \qquad \forall f \in F, \forall s \in S \tag{32}$$

Set IX models the routing stability constraints. These constraints enforce that the route selected for each train in the nominal scenario $s = 1$ is applied to all the other $|S| - 1$ scenarios. In this way, each train keeps the same route among the $|S|$ scenarios.

Set IX: Routing stability constraints.

$$x_f(i,j,s) = x_f(i,j,s-1) \qquad \forall f \in F, \forall (i,j) \in E_f, \forall s \in S \backslash \{1\} \tag{33}$$

4.3 Weighted-sum formulation

This section presents a first method to combine the two objective functions of the integrated problem. We propose a formulation based on a revised version of the weighted-sum method, in which both the objective functions are directly optimized proportionally to the assigned weights. This is achieved by means of two input parameters α_1 and α_2 defined by the decision maker. The following three conditions hold: $\alpha_1 \geq 0$, $\alpha_2 \geq 0$, $\alpha_1 + \alpha_2 = 1$.

Figure 3 reports the main steps of the weighted-sum method proposed in this work. We recall that f_1 and f_2 are the objective functions of the dispatchers and of the infrastructure managers. We let f_1^* (f_2^*) be the value of the optimal solution of the integrated problem with $\min f_1$ ($\min f_2$). Also, we let $f_1^{'}$ ($f_2^{'}$) be the value of the performance indicator f_1 (f_2) for the optimal solution of the integrated problem with the weighted-sum method, i.e. with the objective function $\min Z$.

Weighted-sum method
Input: α_1, α_2
Begin
Step 1: $\min f_1$ subject to constraints (3)$-$(33), and set $\beta_1 = f_1^*$
Step 2: $\max f_2$ subject to constraints (3)$-$(33), and set $\beta_2 = f_2^*$
Step 3: set $\gamma_1 = \alpha_1/\beta_1$, $\gamma_2 = \alpha_2/\beta_2$, $Z = \gamma_1 f_1 - \gamma_2 f_2$
Step 4: $\min Z$ subject to constraints (3)$-$(33)
Step 5: Return the pair ($f_1^{'}$, $f_2^{'}$)
end

Figure 3: Sketch of the weighted-sum method

The proposed weighted-sum method is based on five steps: the first two steps are required for the computation of f_1^* and f_2^*; the third step normalizes the values α_1 and α_2 and sets the function Z to be minimized; the fourth and fifth steps compute and return the values $f_1^{'}$ and $f_2^{'}$. The decision maker can vary the values α_1 and α_2 and repeat steps 3$-$5 in order to search for new compromise solutions, giving greater relevance to either the traffic or the maintenance aspect.

We observe that the weighted-sum method identifies a non-dominated solution by giving weights to the objective functions. However, this method requires the investigation of numerous values of the parameters α_1 and α_2 for the identification of the Pareto frontier, since it is difficult to set α_1 and α_2 values in order to obtain a Pareto-optimal solution in a desired region of the objective space.

4.4 ε-constraint formulation

This section describes an alternative method for the computation of non-dominated solutions. We propose an iterative procedure based on the ε-constraint method. At each iteration, we solve a single-objective formulation for the integrated problem, in which one performance indicator is directly optimized in the objective function, while the other performance indicator is indirectly optimized via the insertion of an additional bound constraint in the single-objective formulation.

Figure 4 explains the main steps of the interactive procedure. At each iteration i, the values $f_1^*(i)$ and $f_2^*(i)$ are computed as for the weighted-sum method. In addition, we let $f_2^{''}(i)$ be the value of the performance indicator f_2 regarding the optimal solution of the integrated problem with $\min f_1$ at iteration i.

ε-constraint method

Begin

Step 1: set $i = 1$

Step 2: $\min f_1$ subject to constraints (3)$-$(33), and set $\beta_1 = f_1^*(i)$, $\varphi_2 = f_2^{''}(i)$

Step 3: $\max f_2$ subject to constraints (3)$-$(33), and set $\beta_2 = f_2^*(i)$

Step 4: insert the pair (β_1, φ_2) in the set of solution values Φ

Step 5: **while** $(\varphi_2 < \beta_2)$ **do**

Begin

set i ++

$\min f_1$ subject to constraints (3)$-$(33) plus constraint: $f_2 > \varphi_2$

set $\beta_1 = f_1^*(i)$, $\varphi_2 = f_2^{''}(i)$

insert the pair (β_1, φ_2) in the set of solution values Φ

end

Step 6: return the non-dominated pairs from the set Φ

end

Figure 4: Sketch of the ε-constraint method

The proposed ε-constraint method is based on six steps: step one is the initialization of a counter of the number of iterations; steps two and three are identical to the first two steps of the weighted-sum method; step four inserts the solution computed in step 1 (in terms of the values of the two performance indicators) in a set Φ; step five is the iterative phase of the procedure that solves a new formulation with the addition of a bound constraint, stores this solution in Φ, and updates the iteration counter; step six filters the solutions in Φ and returns the non-dominated solutions.

5 Computational experiments

This section presents computational results on a realistic test case based on the benchmark instances proposed for the INFORMS RAS 2012 Competition [24]. The instances are extended in order to deal with the constraints and objectives of the integrated problem. The experiments are performed on a computer with processor Intel Xeon (3.4 GHz) and 32 GB Ram. We used Windows 7 and IBM-ILOG-CPLEX MILP solver 12.7.

Section 5.1 describes the test case and the data used for the generation of the instances. Section 5.2 gives a set of preliminary results in order to assess the computational performance of the solver for various types of instances. Section 5.3 presents a first round of experiments with a deterministic setting of the integrated problem. The bi-objective problem is studied by comparing the results obtained for the weighted-sum and the ε-constraint methods in terms of Pareto-optimal solutions. Section 5.4 shows the results obtained for a second round of experiments with a stochastic problem setting. We investigate the robustness of the solutions computed with constraints regarding stability of control actions, i.e. constraints (33), providing a quantitative assessment on the impact of uncertainty towards the performance of the models (robustness and stability).

5.1 Test case description

The computational experiments are proposed for a realistic test case based on an adaptation of the railway data introduced for the INFORMS RAS 2012 Competition [24]. We next describe the network, maintenance and traffic flow data.

Network data: Figure 5 shows the railway network that consists of 84 nodes and 81 cells, i.e. $|V| = 84$ and $|E| = 81$. The network is partially single-track (*main 0*) and partially double-track (*main 1* plus *main 2*). There are four sidings (one siding) for train re-routing on the single-track (double-track).

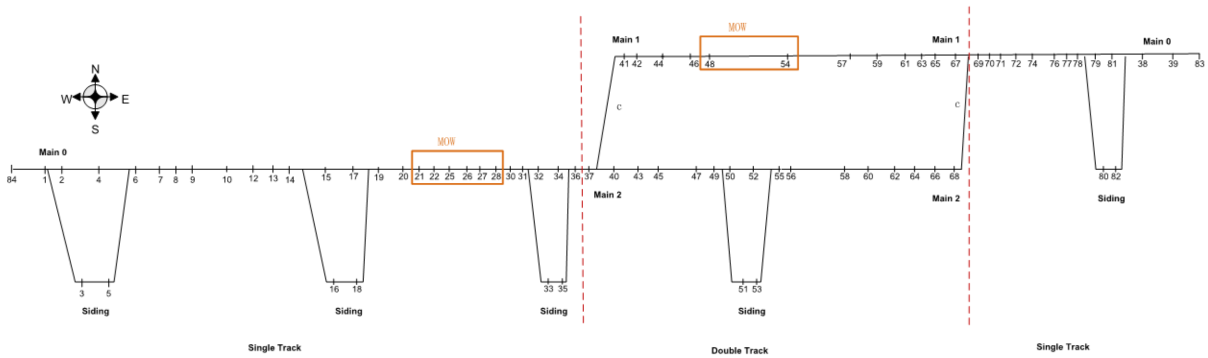

Figure 5: The studied railway network

Maintenance data: Figure 5 also reports the maintenance areas (*Maintenance of Way, MOW*) of the network. We have two maintenance areas, i.e. $|B| = 2$, and six maintenance works to be performed on some cells of each area, i.e. $|P_{(i,j)}| = 6$. Each maintenance work lasts 25 minutes, i.e. $MT_p(i, j, 1) = 25$. It follows that the each area requires 150 minutes of maintenance works, i.e. $H_b = 150$.

Traffic flow data: Each train traversing the network of Figure 5 has the following routing alternatives. The odd-numbered trains travel from left to right, with the option to change their routing in the first, third and/or fourth siding. The even-numbered trains travel from right to left, with the option of change their routing in the second and/or fifth siding. The travel time of each train in each cell varies between 0 and 6 minutes. The dwell times are set equal to 0. The safety time intervals are set as follows: $g_f(i, j) = 0$, $h_f(i, j) = 3$ minutes, $mh_p(i, j) = 2$ minutes.

5.2 Computation time versus instance size

In this section, the solver is assessed when varying the number of trains, i.e. between 2 and 20 trains, to be scheduled in the network. We assume that the traffic is not disturbed. Table 4 presents the following quantitative information on the tested instances: train routing ($x_f(i,j,s)$), train sequencing ($o(f,f',i,j,s)$), sequencing of maintenance works and trains ($l_p(f,i,j,s)$), pairing of maintenance works ($y_p(i,j,s)$).

Type of Variable	$x_f(i,j,s)$	$o(f,f',i,j,s)$	$l_p(f,i,j,s)$	$y_p(i,j,s)$
2-train case	122	488	72	36
4-train case	244	1952	144	36
6-train case	366	4392	216	36
8-train case	488	7808	288	36
10-train case	610	12200	360	36
12-train case	732	17568	432	36
14-train case	854	23912	504	36
16-train case	976	31232	576	36
18-train case	1098	39528	648	36
20-train case	1220	48800	720	36

Table 4: Key binary variables for the investigated instances

The computational assessment is reported on Figure 6 and is based on two indicators: one related to the time required to find the best known solution (grey curves), and another one related to the time to proof optimality (black curves). Both indicators are reported in seconds, and we give up to one hour of computation time to the solver. Specifically, Figure 6 shows six plots: case a (c) [e] one routing is fixed for each train and the model optimizes f_1 (f_2) [f_1 and f_2]; case b (d) [f] each train has a flexible routing (i.e. routing alternatives) and the model optimizes f_1 (f_2) [f_1 and f_2]. For e and f cases, the two performance indicators have equal weight and the resulting bi-objective problem is solved with the weighted-sum method.

From the results of Figure 6, we conclude that the solver can easily solve the c and d cases, since the maintenance-related variables are much less compared to the train-related variables, and it is thus simple searching for an optimal solution for the former variables. When optimizing the train-related variables, the problem is still easy to solve for the a case, while it becomes difficult for the b case when dealing with a large number of trains. This is due to the train-related variables that exponentially increase when alternative routings are available for each train. The most complicated cases are e and f, since the bi-objective problem is harder to solve than the single-objective problems. In Sections 5.3 and 5.4, further experiments will be performed on the instances with 10 trains that (in the undisturbed case) are solved to optimality in a quite modest computation time.

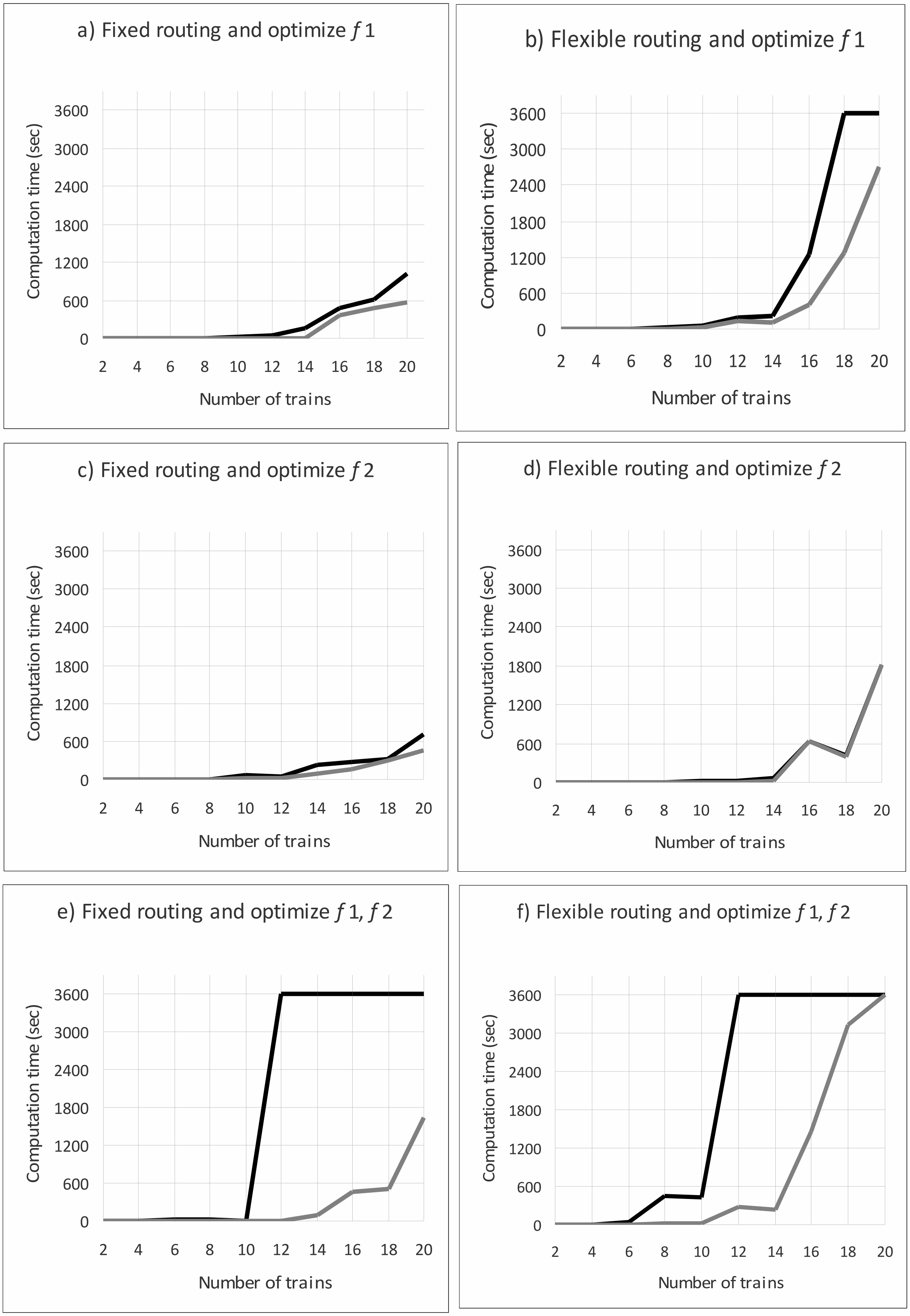

Figure 6: Time to find the best solution (in grey) and to proof optimality (in black)

5.3 Search for Pareto-optimal solutions

This sections studies the bi-objective problem for the instance (Table 4) with $|F| = 10$ trains and $T = 520$ minutes of time horizon, both in case of fixed train routing (Tables 5 and 6) and flexible train routing (Tables 7 and 8). We compare the results obtained by the weighted-sum (Tables 5 and 7) and ε-constraint (Tables 6 and 8) methods.

For the case with fixed train routing, Table 5 shows the computation time (in seconds) required by the weighted-sum method when varying α_1 and the value of f_1 and f_2 indicators. In total, we tested several dozen settings of α_1 but we only obtained five non-dominated solutions for $\alpha_1 = 0.95$, 0.8, 0.5, 0.37, 0.2 (as reported in bold in Table 5).

Table 6 presents the results obtained by the ε-constraint method in case of fixed train routing. For each iteration, we report similar information as in Table 5. In 19 iterations, this method identified nine non-dominated solutions (as reported in bold in Table 6). Figures 11 and 12 of Appendix D show the non-dominated solutions obtained at iteration 10 (with the best value of f_1) and iteration 19 (with the best value of f_2) of Table 6.

For the case with flexible train routing, we tested several settings of α_1 for the weighted-sum method. However, the solver reached most of the times the given time limit of computation (i.e. one hour). Table 7 only reports three cases: the two extreme values (α_1 equal to 0 and 1) and an intermediate value ($\alpha_1 = 0.5$, reported in bold) corresponding to a non-dominated solution.

Table 8 presents the results obtained by the ε-constraint method in case of flexible train routing. For this method, the time limit of computation is never reached. In 22 iterations, it identified six non-dominated solutions (as reported in bold in Table 8). Figures 13 and 14 of Appendix D show the non-dominated solutions obtained at iteration 17 (with the best value of f_1) and iteration 22 (with the best value of f_2) of Table 8.

Overall, we have the following observations: the ε-constraint method outperforms the weighted-sum method in terms of number of non-dominated solutions. Furthermore, the instances generated by the ε-constraint method are easier to solve, since this method treats the bi-objective problem as an iterative single-objective optimization problem.

α_1	Comp. Time (sec)	f_1	f_2
1	30.69	350	8
0.95	234.24	**350**	**21**
0.8	204.22	**351**	**22**
0.5	364.94	**355**	**23**
0.37	773.22	**392**	**25**
0.2	222.34	**548**	**30**
0	13.84	1212	30

Table 5: Weighted-sum method for the case with 10 trains and fixed routing

Iteration	Comp. Time (sec)	f_1	f_2
1	30.69	350	8
2	36.64	350	12
3	34.52	350	14
4	35.70	350	15
5	50.78	350	16
6	53.92	350	17
7	46.33	350	18
8	42.98	350	19
9	46.58	350	20
10	60.73	**350**	**21**
11	1602.86	**351**	**22**
12	117.92	**355**	**23**
13	341.27	**374**	**24**
14	339.08	**392**	**25**
15	2100.51	520	26
16	636.97	**520**	**27**
17	272.19	**524**	**28**
18	113.66	**532**	**29**
19	42.52	**548**	**30**

Table 6: ε-constraint method for the case with 10 trains and fixed routing

α	Comp. Time (sec)	f_1	f_2
1	76.89	168	5
0.5	436.00	**168**	**25**
0	27.80	1178	30

Table 7: Weighted-sum method for the case with 10 trains and flexible routing

Iteration	Comp. Time (sec)	f_1	f_2
1	76.89	168	5
2	47.81	168	10
3	90.58	168	11
4	72.16	168	12
5	203.80	168	13
6	257.69	168	14
7	65.48	168	15
8	68.70	168	16
9	67.88	168	17
10	112.61	168	18
11	97.53	168	19
12	102.81	168	20
13	364.47	168	21
14	135.59	168	22
15	575.86	168	23
16	1420.92	168	24
17	631.14	**168**	**25**
18	2701.30	**352**	**26**
19	2014.53	**376**	**27**
20	1805.11	**394**	**28**
21	2648.55	**418**	**29**
22	315.81	**449**	**30**

Table 8: ε-constraint method for the case with 10 trains and flexible routing

5.4 Impact of uncertainty towards stability and robustness

This section presents computational results on the stochastic modeling environment of Section 3.2.

We recall that a set of scenarios is considered, around a nominal case where maintenance and train traffic follow the plan, combined with some disturbed case studies. In the latter case studies, the maintenance and train traffic follow the nominal duration, but some (known) trains have a starting (known) delay. Those case studies are as follows:

Disturbed case study I. The two trains i and j with the smallest planned (earliest) entrance (start) time are delayed by 100 minutes, i.e. $EST'_i = EST_i + 100$ and $EST'_j = EST_j + 100$.

Disturbed case study II. The two trains i and j with the smallest planned (earliest) entrance (start) time are delayed by 100 minutes, i.e. $EST'_i = EST_i + 100$ and $EST'_j = EST_j + 100$. The two trains h and k with the largest planned (earliest) entrance (start) time are anticipated by 100 minutes, i.e. $EST'_h = EST_h - 100$ and $EST'_k = EST_k - 100$.

The scenario set is instead constructed as follows. It considers a Nominal (labelled No) scenario and scenarios with longer or shorter length of process times (labelled Pessimistic (Pe) or Optimistic (Op), respectively), for both maintenance and train traffic. The latter scenarios have the following characteristics:

Traffic flows. The travel time of trains is disturbed on the basis of a Gaussian distribution. For each train, a negative (positive) variation of the travel time in the No scenario is randomly chosen in a time window: [-10%, 0%] ([0%, +10%]).

Maintenance works. The duration of works is disturbed on the basis of a Gaussian distribution. A negative (positive) variation of their duration in the No scenario is randomly chosen in a time window: [-30%, 0%] ([0%, +30%]).

In case study I the disturbed works are related to the maintenance area (MOW) involving nodes 21–28, while in case study II the disturbed works are related to both maintenance areas. We consider for all cases $|F| = 10$ trains, $T = 520$ minutes of time horizon and routing flexibility.

Table 9 summarizes the characteristics of train traffic flows and maintenance works regarding the two disturbed case studies (I and II) investigated in this section.

Case Study	Delayed Trains	Disturbed Travel Times	Disturbed Works
I	First 2	All Trains (No, Pe, Opt)	Nodes 21–28
II	First 2 and Last 2	All Trains (No, Pe, Opt)	Nodes 21–28 and 48–54

Table 9: Key characteristics of the disturbed case studies

Tables 10 and 11 (12 and 13) present the best known solutions computed by the solver within 3 hours for case study I (II). In order to consider both objective functions with equal weight, we use the weighted-sum method of Figure 3 with $\alpha_1 = \alpha_2 = 0.5$.

In Tables 10 and 12 (Tables 11 and 13), we consider the following probabilities for the three scenarios: $Pr_{No} = 0.33$ (0.7), $Pr_{Pe} = 0.33$ (0.15), and $Pr_{Op} = 0.33$ (0.15).

Stoch. Scenarios	Nominal (No)		Pessimistic (Pe)				Optimistic (Op)			
Perf. Indicators	f_1	f_2	f_1	f_2	*Stab.1*	*Stab.2*	f_1	f_2	*Stab.1*	*Stab.2*
Constr.(33) *On*	380	24	506	24	100	100	305	25	100	100
Constr.(33) *Off*	366	25	486	24	97	70	298	25	99	90

Table 10: Case Study I: Results for $Pr_{No} = 0.33$, $Pr_{Pe} = 0.33$, $Pr_{Op} = 0.33$

Stoch. Scenarios	Nominal (No)		Pessimistic (Pe)				Optimistic (Op)			
Perf. Indicators	f_1	f_2	f_1	f_2	*Stab.1*	*Stab.2*	f_1	f_2	*Stab.1*	*Stab.2*
Constr.(33) *On*	366	24	586	24	100	100	301	24	100	100
Constr.(33) *Off*	366	25	490	23	96	60	304	25	99	90

Table 11: Case Study I: Results for $Pr_{No} = 0.7$, $Pr_{Pe} = 0.15$, $Pr_{Op} = 0.15$

The measured variability of f_1 and f_2 values between the nominal and evaluated scenarios is considered a measure of robustness for what concerns the objective function. Moreover, we also investigate the variability of the decision variables , by inserting or not the stability constraints for routing, i.e. constraints (33) *On* or constraints (33) *Off*, in *Pe* and *Op* scenarios.

The routing stability *Stab.1* (*Stab.2*) is measured, in percentage, as the number of times that each train is assigned to the same resources of its nominal route (respectively, to the whole of its nominal route) in the evaluated scenarios.

We, moreover, consider different relative probabilities for the scenarios, which vary from 75% nominal to a case in which all scenarios are equiprobable (i.e. 33% for nominal, 33% for a pessimistic case, 33% for an optimistic case).

From the results in Tables 10 and 11, we have the following observations:

- Constraints (33) *On/Off*: Setting train routing flexibility between the different scenarios (i.e., removing constraints (33) from the formulation) is often beneficial in terms of f_1 and f_2 values. The advantage is more evident in Table 10, where f_1 values always improve and f_2 values never deteriorate. However, using train routing flexibility generates instability, that we measure in terms of deterioration of *Stab.1* and *Stab.2* values. The instability is the largest for the *Pe* scenario of Table 11.

- *No/Pe/Op* scenarios: Comparing the performance improvement when constraints (33) are off versus on, the *No* scenario improves in terms of both f_1 and f_2 values (Table 10), while the *Pe* and *Op* scenarios improve either f_1 or f_2 value.

- Probability of occurrence: The equiprobability setting (Table 10) is always equal or better (in terms of both f_1 and f_2 values) than the other setting (Table 11) when constraints (33) are off, while this is not the case when constraints (33) are on.

Stoch. Scenarios	Nominal (No)		Pessimistic (Pe)				Optimistic (Op)			
Perf. Indicators	f_1	f_2	f_1	f_2	*Stab.1*	*Stab.2*	f_1	f_2	*Stab.1*	*Stab.2*
Constr.(33) *On*	419	24	677	23	100	100	265	25	100	100
Constr.(33) *Off*	366	24	623	23	97	80	259	25	98	80

Table 12: Case Study II: Results for $Pr_{No} = 0.33$, $Pr_{Pe} = 0.33$, $Pr_{Op} = 0.33$

Stoch. Scenarios	Nominal (No)		Pessimistic (Pe)				Optimistic (Op)			
Perf. Indicators	f_1	f_2	f_1	f_2	*Stab.1*	*Stab.2*	f_1	f_2	*Stab.1*	*Stab.2*
Constr.(33) *On*	384	24	689	23	100	100	294	24	100	100
Constr.(33) *Off*	366	24	681	23	96	70	287	25	98	80

Table 13: Case Study II: Results for $Pr_{No} = 0.7$, $Pr_{Pe} = 0.15$, $Pr_{Op} = 0.15$

From the results in Tables 12 and 13, we have the following observations:

- Constraints (33) *On*/*Off*: Train routing flexibility is always beneficial in terms of f_1 values. Sometimes it also improves the f_2 value. As for the previous case study, using train routing flexibility deteriorates the *Stab.1* and *Stab.2* values.

- $No/Pe/Op$ scenarios: Comparing the performance improvement when constraints (33) are off versus on, the three scenarios always improve the f_1 value. The Op scenario of Table 13 also improves the f_2 value.

- Probability of occurrence: When constraints (33) are off, equiprobability (Table 12) presents a larger improvement of f_1 compared to the other setting. However, Pr_{No} = 0.7 (Table 13) helps to better minimize f_1 for No when constraints (33) are on. Regarding f_2, equiprobability is always equal or better than the other setting.

6 Conclusions and further research

This paper investigates mathematical models and methods for managing a bi-objective problem of practical interest for railway managers who have to deal with the integration of train traffic flow and maintenance work decisions. To address this problem, we present a mathematical model in which the railway infrastructure is modeled at a microscopic level of detail. State-of-the-art train scheduling approaches are used as a basis for the model proposed in this paper. The novel modeling features are related to constraints and objective functions for the integration of train and maintenance schedules.

Computational experiments on the INFORMS RAS Competition 2012 [24] instances show that the proposed methodology can be used to identify Pareto-optimal solutions, to investigate the potential benefits of using train routing flexibility in order to optimize train-related and maintenance-related objectives, to quantify the impact of routing stability in case of various stochastic disturbance scenarios, and to find a trade-off between indicators of solution stability and robustness to the disturbance scenarios.

Future work should address a number of research directions. Even more detailed mathematical formulations of both train traffic flow and maintenance process could be investigated. Problem-dedicated algorithms and solution methods could be developed in order to handle even more detailed mathematical formulations and to further improve the management of practical instances with more trains and complex railway infrastructures, which might include also temporary speed restrictions or need for global rerouting or cancellation of train services due to disruptions.

References

[1] Albrecht, A.R., Panton, D.M., Lee, D.H. (2013) Rescheduling rail networks with maintenance disruptions using problem space search. *Computers and Operations Research* **40**(3) 703–712.

[2] Budai, G., Huisman, D., Dekker, R. (2006) Scheduling preventive railway maintenance activities. *Journal of the Operational Research Society* **57**(9) 1035–1044.

[3] Burdett, R.L. (2015) Multi-objective models and techniques for analysing the absolute capacity of railway networks. *European Journal of Operational Research* **245**(2) 489–505.

[4] Cacchiani, V., Huisman, D., Kidd, M., Kroon, L.G., Toth, P., Veelenturf, L.P., Wagenaar, J.C. (2014) An overview of recovery models and algorithms for real-time railway rescheduling. *Transportation Research Part B* **63**(1) 15–37.

[5] Cacchiani, V., Galli, L., Toth, P. (2015) A tutorial on non-periodic train timetabling and platforming problems. *EURO Journal on Transp. and Logistics* **4**(3) 285–320.

[6] Cacchiani, V., Toth, P. (2012) Nominal and robust train timetabling problems. *European Journal of Operational Research* **219**(3) 727–737.

[7] Corman, F., D'Ariano, A., Hansen, I.A. (2014) Evaluating disturbance robustness of railway schedules. *Journal of Intelligent Transportation Systems: Technology, Planning, and Operations* **18**(1) 106–120.

[8] Corman F., D'Ariano, A., Marra, A.D., Pacciarelli, D., Samà, M. (2017) Integrating Train Scheduling and Delay Management in Real-time Railway Traffic Control. *Transportation Research Part E* **105**(1) 213–239.

[9] Corman, F., D'Ariano, A., Pacciarelli, D., Pranzo, M. (2010) A tabu search algorithm for rerouting trains during rail operations. *Transportation Research Part B* **44**(1) 175–192.

[10] Corman, F., D'Ariano, A., Pacciarelli, D., Pranzo, M. (2012) Bi-objective conflict detection and resolution in railway traffic management. *Transportation Research Part C* **20**(1) 79–94.

[11] Community of European Railway and Infrastructure Companies (CER) (2012) Annual Report 2012-2013. Increased efficiency through the reduction of rail track maintenance costs. Brussels, Belgium. *http://www.cer.be/sites/default/files/publication/FINAL_CER_AR2013_DEF.pdf*

[12] D'Ariano, A., Pacciarelli, D., Pranzo, M. (2007) A branch and bound algorithm for scheduling trains in a railway network. *European Journal of Operational Research* **183**(2) 643–657.

[13] Dollevoet, T., Corman, F., D'Ariano, A., Huisman D. (2014) An iterative optimization framework for delay management and train scheduling. *Journal of Flexible Services and Manufacturing* **26**(4) 490–515.

[14] Ehrgott, M. (2005) Multicriteria optimization, Springer, Berlin.

[15] Fang, W., Yang, S., Yao, X. (2015) A survey on problem models and solution approaches to rescheduling in railway networks. *IEEE Transactions on Intelligent Transportation Systems* **16**(6) 2997–3016.

[16] Forsgren, M., Aronsson, M., Gestrelius, S. (2013) Maintaining tracks and traffic flow at the same time. *Journal of Rail Transport Planning and Management* **3**(3) 111–123.

[17] Fu, L., Dessouky, M. (2017) Models and algorithms for dynamic headway control. *Computers & Industrial Engineering* **103** 271–281.

[18] Gao, Y., Kroon, L.G., Schmidt, M., Yang, L. (2016) Rescheduling a metro line in an overcrowded situation after disruptions. *Transportation Research Part B* **93** 425–449.

[19] Giacco, G.L., D'Ariano, A., Pacciarelli, D. (2014) Rolling stock rostering optimization under maintenance constraints. *Journal of Intelligent Transport Systems: Technology, Planning, and Operations* **18**(1) 95–105.

[20] Ghoseiri, K., Szidarovszky, F., Asgharpour, M.J. (2004) A multi-objective train scheduling model and solution. *Transportation Research Part B* **38**(10) 927–952.

[21] Goverde, R.M.P. (2007) Railway timetable stability analysis using max-plus system theory. *Transportation Research Part B* **41**(2) 179–201.

[22] Gustavsson, E., Patriksson, M., Strömberg, A.-B., Wojciechowski, A., Önnheim, M. (2014) Preventive maintenance scheduling of multi-component systems with interval costs. *Computers and Industrial Engineering* **76** 390–400.

[23] Hansen, I.A., Pachl, J. (Eds.) (2014). Railway Timetabling & Operations. Eurailpress, Hamburg, Germany.

[24] INFORMS RAS (2012) Problem description and released data set for *Problem Solving Competition 2012*. http://www.informs.org/Community/RAS/Problem-Solving-Competition/2012-RAS-Problem-Solving-Competition

[25] Kroon, L.G., Maróti, G., Helmrich, M.R., Vromans, M., Dekker, R. (2008) Stochastic improvement of cyclic railway timetables. *Transp. Research Part B* **42**(6) 553–570.

[26] Lai, Y.C., Fan, D.C., and Huang, K.L. (2015) Optimizing Rolling Stock Assignment and Maintenance Plan for Passenger Railway Operations. *Computers and Industrial Engineering* **85** 284–295.

[27] Larsen, R., Pranzo, M., D'Ariano, A., Corman, F., Pacciarelli, D. (2014) Susceptibility of Optimal Train Schedules to Stochastic Disturbances of Process Times. *Flexible Services and Manufacturing Journal* **26**(4) 466–489.

[28] Lee, H., Cha, J.H. (2016) New stochastic models for preventive maintenance and maintenance optimization. *European Journal of Operational Research* **255**(1) 80–90.

[29] Li, S., Dessouky, M., Yang, L., Gao, Z. (2017) Joint optimal train regulation and passenger flow control strategy for high-frequency metro lines. *Transportation Research Part B* **99**(1) 113–137.

[30] Lidén, T. (2015) Railway infrastructure maintenance - a survey of planning problems and conducted research. *Transportation Research Procedia* **10** 574–583.

[31] Lidén, T., Joborn, M. (2016) Dimensioning windows for railway infrastructure maintenance: Cost efficiency versus traffic impact. *Journal of Rail Transport Planning and Management* **6**(1) 32–47.

[32] Lidén, T., Joborn, M. (2017) An optimization model for integrated planning of railway traffic and network maintenance. *Transportation Research Part C* **74** 327–347.

[33] Lin, Z.-L., Huang, Y.-S., Fang, C.-C. (2015) Non-periodic preventive maintenance with reliability thresholds for complex repairable systems. *Reliability Engineering and System Safety* **136** 145–156.

[34] Luan, X., Miao, J., Meng, L., Corman, F., Lodewijks, G. (2017) Integrated optimization on train scheduling and preventive maintenance time slots planning. *Transportation Research Part C* **80** 329–359.

[35] Lusby, R.M., Larsen, J., Ehrgott, M., Ryan, D. (2011) Railway track allocation: models and methods. *OR Spectrum* **33**(4) 843–883.

[36] Manzini, R., Accorsi, R., Cennerazzo, T., Ferrari, E., Maranesi, F. (2015) The scheduling of maintenance. A resource-constraints mixed integer linear programming model. *Computers & Industrial Engineering* **87** 561–568.

[37] Meng, L., Luan, X., Zhou, X. (2016) A train dispatching model under a stochastic environment: stable train routing constraints and reformulation. *Networks and Spatial Economics* **16**(3) 791–820.

[38] Meng, L., Zhou, X. (2011) Robust single-track train dispatching model under a dynamic and stochastic environment: a scenario-based rolling horizon solution approach. *Transportation Research Part B* **45**(7) 1080–1102.

[39] Meng, L., Zhou, X. (2014) Simultaneous train rerouting and rescheduling on an N-track network: A model reformulation with network-based cumulative flow variables. *Transportation Research Part B* **67** 208–234.

[40] Mu, S., Dessouky, M. (2011) Scheduling freight trains traveling on complex networks. *Transportation Research Part B* **45**(7) 1103–1123.

[41] Narayanaswami, S., Rangaraj, N. (2013) Modelling disruptions and resolving conflicts optimally in a railway schedule. *Computers & Industrial Engineering* **64**(1) 469–481.

[42] Pargar, F., Kauppila, O., Kujala, J. (2017) Integrated scheduling of preventive maintenance and renewal projects for multi-unit systems with grouping and balancing *Computers & Industrial Engineering* **110** 43–58.

[43] Pellegrini, P., Marlière, G., Rodriguez, J. (2014) Optimal train routing and scheduling for managing traffic perturbations in complex junctions. *Transportation Research Part B* **59**(1) 58–80.

[44] Peng, F., Kang, S., Li, X., Ouyang, Y., Somani, K., Acharya, D. (2010) A heuristic approach to the railroad track maintenance scheduling problem. *Computer-Aided Civil and Infrastructure Engineering* **26**(2) 129–145.

[45] Peng, F., Ouyang, Y., Somani, K. (2013) Optimal routing and scheduling of periodic inspections in large-scale railroad networks. *Journal of Rail Transport Planning & Management* **3**(4) 163–171.

[46] Quaglietta, E., Corman, F., Goverde, R.M.P. (2013) Stability analysis of railway dispatching plans in a stochastic and dynamic environment. *Journal of Rail Transport Planning & Management* **3**(4) 137–149.

[47] Samà, M., D'Ariano, A., Corman, F., Pacciarelli, D. (2017) A variable neighborhood search for fast train scheduling and routing during disturbed railway traffic situations. *Computers and Operations Research* **78**(1) 480–499.

[48] Samà, M., D'Ariano, A., D'Ariano, P., Pacciarelli, D. (2014) Optimal aircraft scheduling and routing at a terminal control area during disturbances. *Transportation Research Part C* **47**(1) 61–85.

[49] Samà, M., D'Ariano, A., D'Ariano, P., Pacciarelli, D. (2017) Scheduling models for optimal aircraft traffic control at busy airports: Tardiness, priorities, equity and violations considerations. *Omega* **67**(1) 81–98.

[50] Samà, M., Pellegrini, P., D'Ariano, A., Rodriguez, J., Pacciarelli, D. (2016) Ant colony optimization for the real-time train routing selection problem. *Transportation Research Part B* **85**(1) 89–108.

[51] Samà, M., Pellegrini, P., D'Ariano, A., Rodriguez, J., Pacciarelli, D. (2017) On the tactical and operational train routing selection problem. *Transportation Research Part C* **76**(1) 1–15.

[52] Serafini, P., Ukovich, W. (1989) A mathematical model for periodic scheduling problems. *SIAM Journal on Discrete Mathematics* **2**(4) 550–581.

[53] Shift2Rail Joint Undertaking (2015) Strategic Master Plan, Brussels, Belgium.

[54] Steuer, R.E. (1989) Multiple criteria optimization: Theory, computation, and application, Krieger, Malabar.

[55] Szpigel, B. (1973) Optimal Train Scheduling on a Single Track Railway. M. Ross (Ed.), Operational Research, Amsterdam, The Netherlands, pp. 343–352.

[56] Vansteenwegen, P., Dewilde, T., Burggraeve, S., Cattrysse, D. (2016) An iterative approach for reducing the impact of infrastructure maintenance on the performance of railway systems. *European Journal of Operational Research* **252**(1) 39–53.

[57] Wang, L., Chu, J., Wu, J. (2007) Selection of optimum maintenance strategies based on a fuzzy analytic hierarchy process. *International Journal of Production Economics* **107**(1) 151–163.

[58] Wang, Y., Tang, T., Ning, B., van den Boom, T.J.J., De Schutter, B. (2015) Passenger-demands-oriented train scheduling for an urban rail transit network. *Transportation Research Part C* **60** 1–23.

[59] Xu, X., Li, K., and Li, X. (2016) A multi-objective subway timetable optimization approach with minimum passenger time and energy consumption. *Journal of Advanced Transportation* **50** 69–95.

[60] Yang, L., Qi, J., Li, S., Gao, Y. (2016) Collaborative optimization for train scheduling and train stop planning on high-speed railways. *Omega* **64** 57–76.

[61] Yin, J., Tang, T., Yang, L., Gao, Z., Ran, B. (2016) Energy-Efficient Metro Train Rescheduling with Uncertain Time-Variant Passenger Demands: An Approximated Dynamic Programming Approach. *Transportation Research Part B* **91** 178–210.

[62] Yin, J., Yang, L., Tang, T., Gao, Z., Ran, B. (2017) Dynamic passenger demand oriented metro train scheduling with energy-efficiency and waiting time minimization: mixed-integer linear programming approaches. *Transportation Research Part B* **97** 182–213.

Appendix A. List of notations

Symbol	Description
i, j, k	Node index, $i, j, k \in V$. V is the set of nodes.
e	Cell index, $e = (i, j) \in E$. E is the set of cells.
f	Train index, $f \in F$. F is the set of trains. V_f (E_f) are the nodes (cells) used by train f.
b	Maintenance area index, $b \in B$. B is the set of maintenance areas.
p	Maintenance work index, $p \in P_{(i,j)}$. $P_{(i,j)}$ is the set of works on (i, j).
s	Scenario index, $s \in S$. S is the set of random scenarios.

Table 14: List of subscripts for input parameters

Symbol	Description
T	Time horizon for the completion of all the activities.
δ_f	Direction of traveling of train f.
O_f	Origin node of train f.
D_f	Destination node of train f.
EST_f	Planned (earliest) entrance (start) time of train f at its origin node.
PCT_f	Planned (due) arrival (completion) time of f at its destination node.
$h_f(i, j)$	Safety time interval between the occupation end of train f from cell (i, j) and the release of cell (i, j).
$g_f(i, j)$	Safety time interval between the preparation of cell (i, j) and the occupation start of train f on cell (i, j).
$mh_p(i, j)$	Safety time interval required before/after the processing of the maintenance work p on cell (i, j).
Pr_s	Occurrence probability of scenario s.
$w_f^{min}(i, j, s)$	Minimum dwell time of train f on cell (i, j) under scenario s.
$w_f^{max}(i, j, s)$	Maximum dwell time of train f on cell (i, j) under scenario s.
$FT_f(i, j, s)$	Planned travel time of f to traverse cell (i, j) under scenario s.
$MT_p(i, j, s)$	Duration of maintenance work p on cell (i, j) under scenario s.
E_b	Set of cells of maintenance area b.
H_b	Maintenance volume to be carried out on each area b.
M	A number sufficiently larger than T.

Table 15: List of other input parameters

Appendix B. Example: Pareto-optimal solutions

A numerical example illustrates the methods proposed for the computation of Pareto-optimal solutions. We consider a deterministic setting of the formulations with a single (nominal) scenario $|S| = 1$. Also, we assign a particular route to each train.

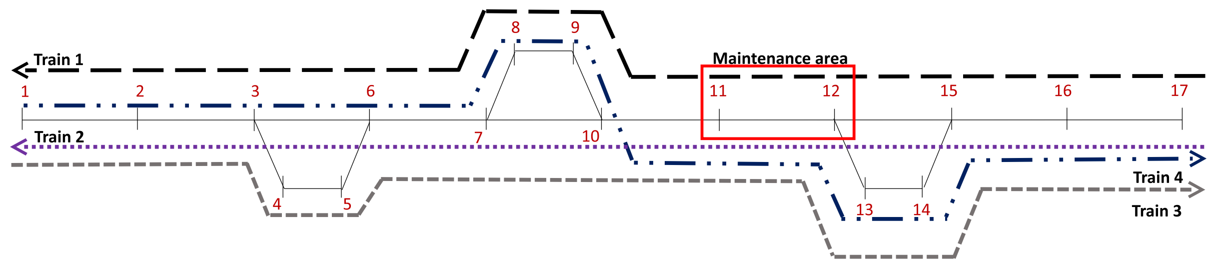

Figure 7: The network, the maintenance area and the trains of the example

Figure 7 shows the railway network that is composed of $|V| = 17$ nodes and $|E| = 19$ cells. The cell (11, 12) is subject to maintenance activities. Four trains ($|F| = 4$) are traversing the network and the route assigned to each train (named 1, 2, 3, or 4) is illustrated in Figure 7. The time horizon for the completion of all the maintenance works in the maintenance area and for the exit of all trains from the network is set as 75 time units ($|T| = 75$).

The example considers a maintenance area ($|B| = 1$) and three maintenance works to be performed on cell (11, 12) ($|P_{(11,12)}| = 3$). Each maintenance work is performed on the same maintenance cell ($b = 1$) and lasts 10 time units ($MT_p(11, 12, 1) = 10$ with $p = 1, 2, 3$). Therefore, the maintenance volume on the maintenance area is 30 ($H_1 = 30$).

Table 16 provides the planned entrance time EST_f at the origin node, the planned arrival time PCT_f at the destination node, and the traveling direction δ_f of each train f.

Train	1	2	3	4
EST_f	2	12	30	35
PCT_f	30	50	60	79
δ_f	1	2	1	2

Table 16: Input parameters for the four trains

Table 17 reports the planned travel time of each train f to traverse each cell of the network under the nominal scenario $s = 1$. For the sake of simplicity, we assume that all trains have the same planned travel time in each cell. The safety time intervals are set as follows: $g_f(i, j) = 0$, $h_f(i, j) = 3$, $mh_p(i, j) = 2$.

Cells of the railway network	$FT_f(i, j, 1)$
(1,2)(3,4)(5,6)(8,9)(11,12)(12,15)(12,13)(16,17)	1
(3,6)(4,5)(7,8)(7,10)(10,11)(13,14)(14,15)	2
(2,3)(6,7)(9,10)(15,16)	3

Table 17: Planned travel time in each cell for the nominal scenario $s = 1$

Table 18 gives the size of key binary variables for: train routing ($x_f(i,j,s)$), train sequencing ($o(f,f',i,j,s)$), sequencing of maintenance works and trains ($l_p(f,i,j,s)$), pairing of maintenance works ($y_p(i,j,s)$).

Type of Variable	$x_f(i,j,s)$	$o(f,f',i,j,s)$	$l_p(f,i,j,s)$	$y_p(i,j,s)$
Number of Variables	50	608	12	3

Table 18: Size of the problem instance in terms of key binary variables

Table 19 reports the optimal solutions for the weighted-sum method for 11 values of the parameter α_1 ($\alpha_2 = 1 - \alpha_1$), while Table 20 the optimal solutions for the ε-constraint method. The solutions are reported in terms of the value of performance indicators f_1 and f_2. The latter method requires three iterations and computes a non-dominated solution at each iteration, while the former method returns two non-dominated solutions only.

As a general remark, the weighted-sum method has the advantage to easily set the importance of each performance indicator in the objective function, while it is difficult to identify the set of non-dominated solutions by varying the value of parameters α_1 and α_2.

α_1	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0
f_1	17	17	17	17	17	29	29	29	29	29	35
f_2	0	0	0	0	0	2	2	2	2	2	2

Table 19: Values of f_1 e f_2 for various settings of α_1 in the weighted-sum method

Iteration	1	2	3
f_1	17	23	29
f_2	0	1	2

Table 20: Values of f_1 e f_2 for the various iterations of the ε-constraint method

Figures 8, 9 and 10 illustrate a time-space diagram for the non-dominated solutions obtained at the first, second and third iterations of the ε-constraint method.

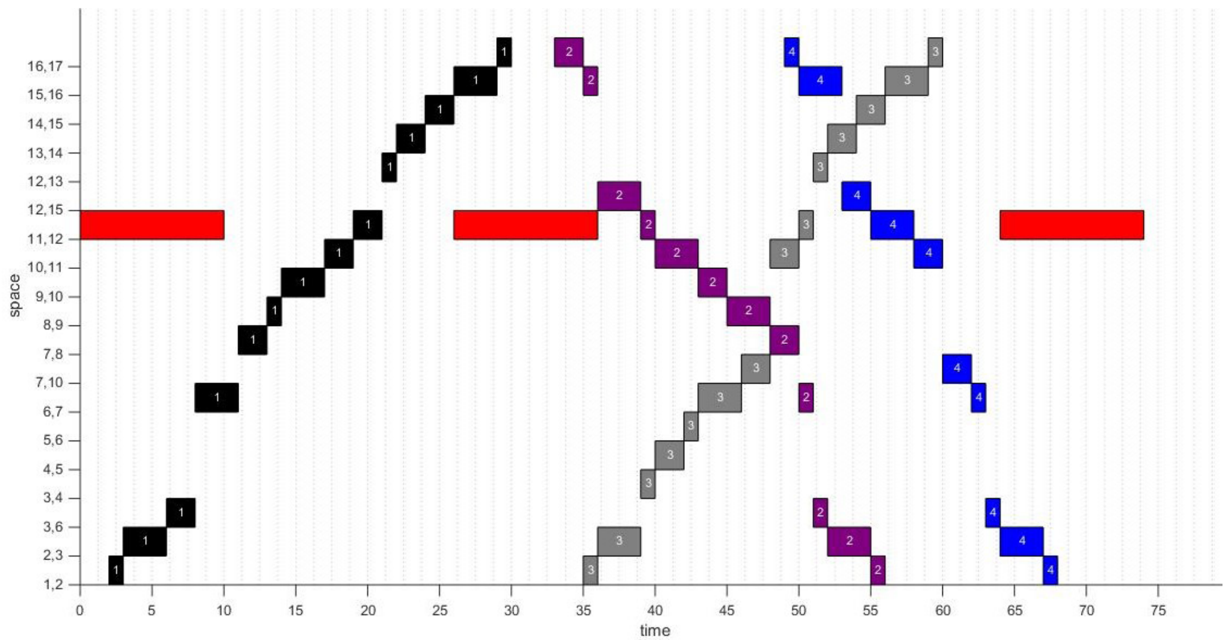

Figure 8: Solution of the first iteration of the ε-constraint method

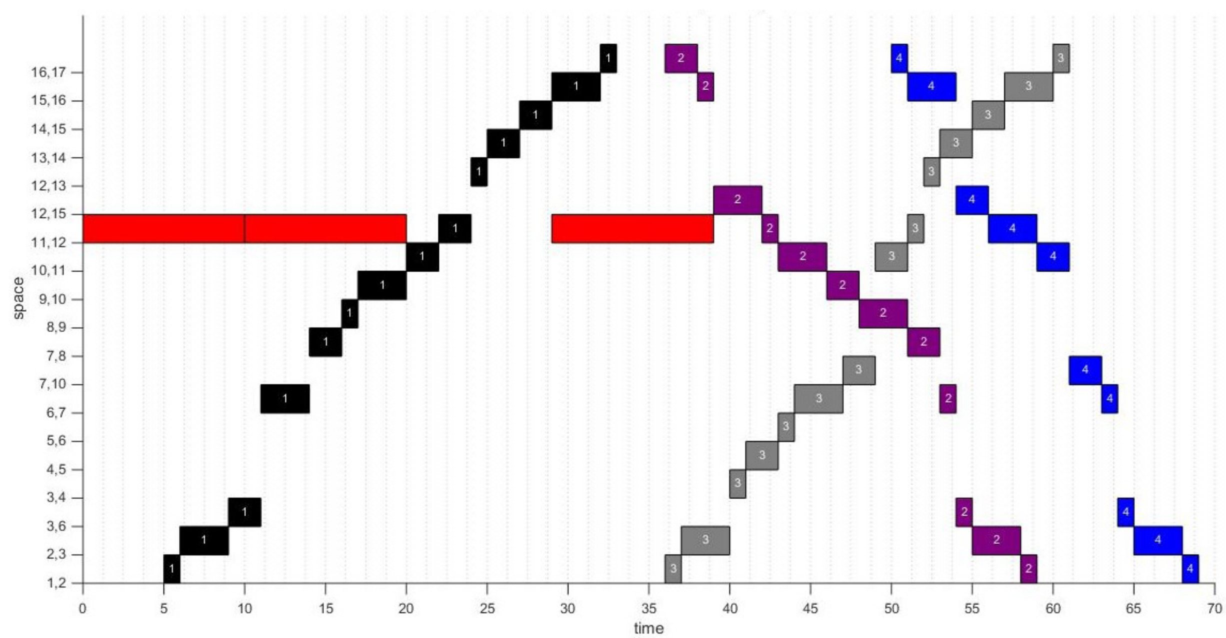

Figure 9: Solution of the second iteration of the ε-constraint method

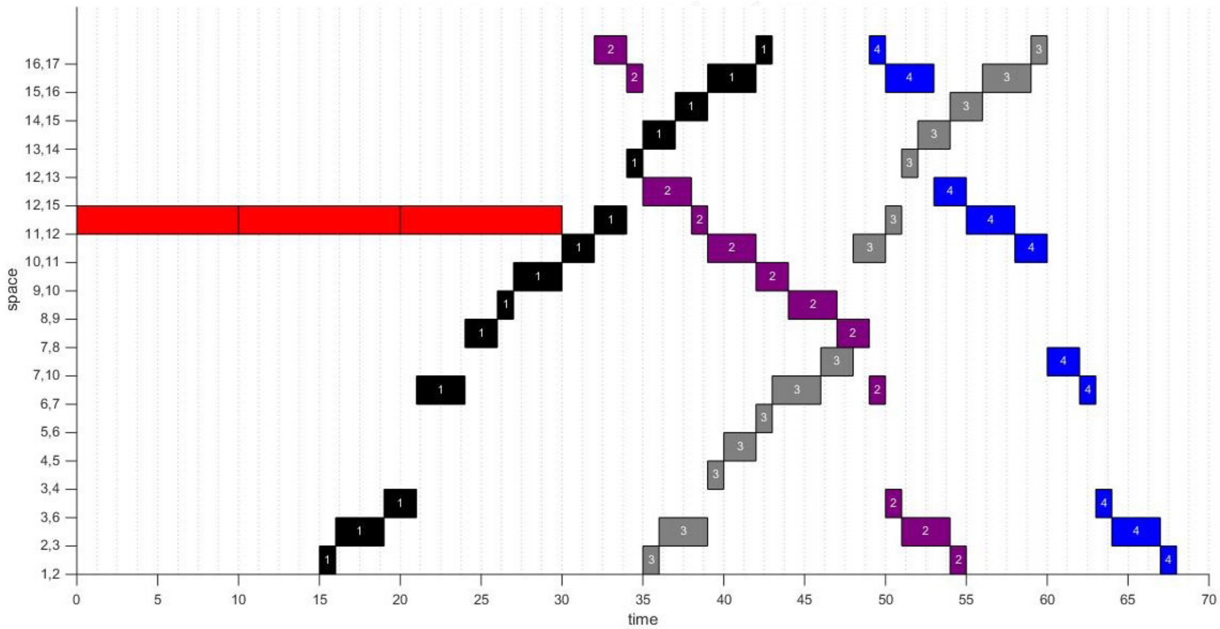

Figure 10: Solution of the third iteration of the ε-constraint method

Appendix C. Example: Robust versus stable solutions

The numerical example of Appendix B is generalized to the stochastic case with multiple scenarios and with alternative routings available for each train. Under this setting, we study the robustness of the nominal solution to random perturbations of the train travel times and of the maintenance works. We also investigate the impact of the routing stability constraints, i.e. constraints (33), for the stochastic scenarios.

The four trains of the example can traverse the network of Figure 7 by using all possible combinations of routing alternatives. Specifically, each train has 8 routing alternatives, since the network of the example includes 3 sidings and all the possible permutations.

Three scenarios ($|S| = 3$) are considered: the nominal (No) scenario of Appendix B plus two other scenarios. The latter scenarios, named pessimistic (Pe) and optimistic (Op), are generated as follows. The pessimistic (optimistic) scenario is obtained by increasing (decreasing) by 10% the travel time of trains 1 and 2 (trains 3 and 4) compared to the nominal scenario. In the pessimistic (optimistic) scenario, the processing time of the maintenance works is also enlarged (reduced) by 30% compared to the nominal scenario.

Tables 21 and 22 present the optimal solutions computed for the formulation generated by the weighted-sum method of Figure 3 with $\alpha_1 = \alpha_2 = 0.5$ (for this setting of α_1 and α_2 we have $f_1^* = 9$ and $f_2^* = 2$). In Table 21 (Table 22), we consider the following probabilities for the three scenarios: $Pr_{No} = 0.33$ (0.7) for the nominal, $Pr_{Pe} = 0.33$ (0.15) for the pessimistic and $Pr_{Op} = 0.33$ (0.15) for the optimistic. The three scenarios have thus equal probability in Table 21, while in Table 22 a higher probability of occurrence is assigned to the nominal scenario compared to the other two scenarios.

In Tables 21 and 22, the robustness of an optimal solution computed for a Pe/Op scenario is measured as the variability of the value of performance indicators f_1 and f_2 between the nominal and the evaluated scenario. The routing stability *Stab.1* (*Stab.2*) is measured, in percentage, as the number of times that each train is assigned to the same resources of its nominal route (to its nominal route) in Pe and Op scenarios. Clearly, the routing stability of a scenario is 100% when constraints (33) are enforced.

Stoch. Scenarios	Nominal (No)		Pessimistic (Pe)				Optimistic (Op)			
Perf. Indicators	f_1	f_2	f_1	f_2	*Stab.1*	*Stab.2*	f_1	f_2	*Stab.1*	*Stab.2*
Constr.(33) *On*	15	1	18	1	100	100	17	1	100	100
Constr.(33) *Off*	9	1	9	1	43	25	10	1	45	0

Table 21: Results for $Pr_{No} = 0.33$, $Pr_{Pe} = 0.33$, $Pr_{Op} = 0.33$

Stoch. Scenarios	Nominal (No)		Pessimistic (Pe)				Optimistic (Op)			
Perf. Indicators	f_1	f_2	f_1	f_2	*Stab.1*	*Stab.2*	f_1	f_2	*Stab.1*	*Stab.2*
Constr.(33) *On*	9	1	31	1	100	100	20	1	100	100
Constr.(33) *Off*	9	1	10	1	22	0	13	1	46	0

Table 22: Results for $Pr_{No} = 0.7$, $Pr_{Pe} = 0.15$, $Pr_{Op} = 0.15$

From the results of Tables 21 and 22, we have the following observations: better results are often achieved for f_1 (while f_2 does not vary in this example) when a scenario has a higher probability of occurrence; the solutions are more robust in case of equal probability; a trade-off exists between the performance robustness and the solution stability.

Appendix D. Optimal timetables

This appendix presents the time-space diagrams regarding four non-dominated solutions for the case study described in Section 5.3.

- Figure 11: iteration 10 ($f_1 = 350$, $f_2 = 21$) of Table 6 (with fixed routing);
- Figure 12: iteration 19 ($f_1 = 548$, $f_2 = 30$) of Table 6 (with fixed routing);
- Figure 13: iteration 17 ($f_1 = 168$, $f_2 = 25$) of Table 8 (with flexible routing);
- Figure 14: iteration 22 ($f_1 = 449$, $f_2 = 30$) of Table 8 (with flexible routing).

From the solutions reported in the four figures, we have the following observations. Assessing the solutions in terms of the number of pairings of maintenance works, the f_2-gap is more evident when the train routing is fixed, i.e. Figure 11 versus Figure 12. Assessing the solutions in terms of the total deviation of all trains from the nominal timetable, the f_1-gap is more evident when the train routing is flexible, i.e. Figure 13 versus Figure 14. The different trend obtained for the two performance indicators is due to the fact that train routing flexibility offers additional alternatives to reduce train deviations from the nominal timetable (i.e. a better f_1 minimization), even if the number of alternatives reduces when f_2 is set equal to the best possible value. Furthermore, train routing flexibility also helps to compute a more compact train schedule and, therefore, to increase the number of pairings of maintenance works (i.e. a better f_2 maximization). The latter result is evident when comparing Figure 11 with Figure 13.

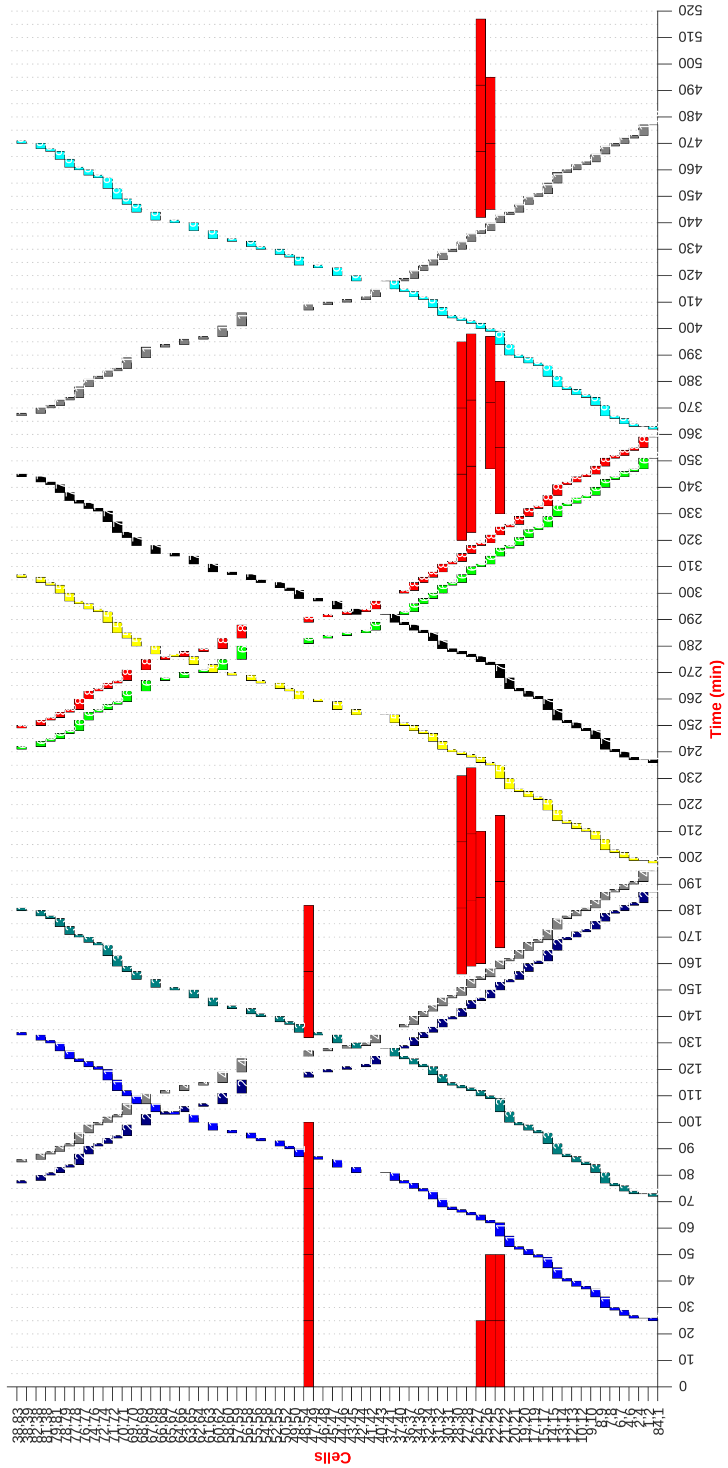

Figure 11: Iteration 10 of Table 6

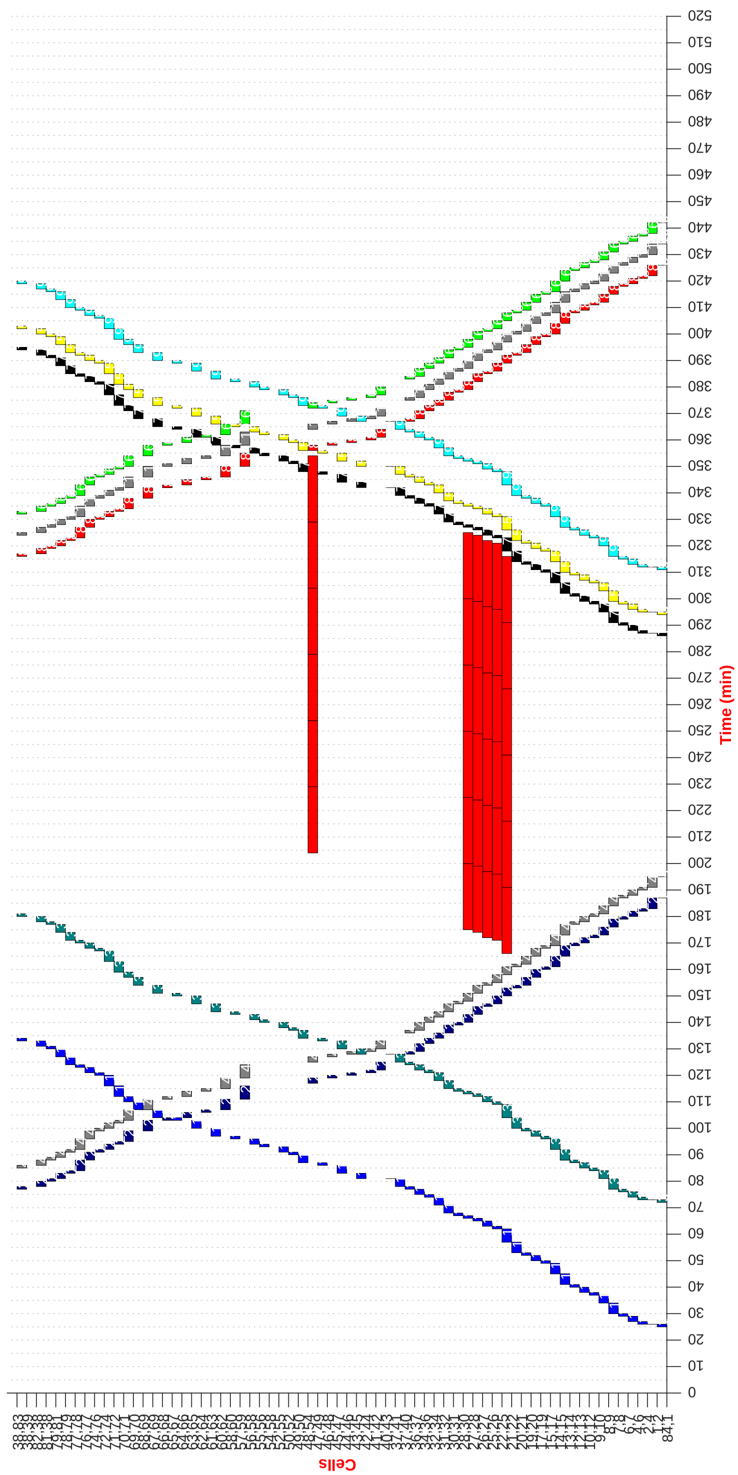

Figure 12: Iteration 19 of Table 6

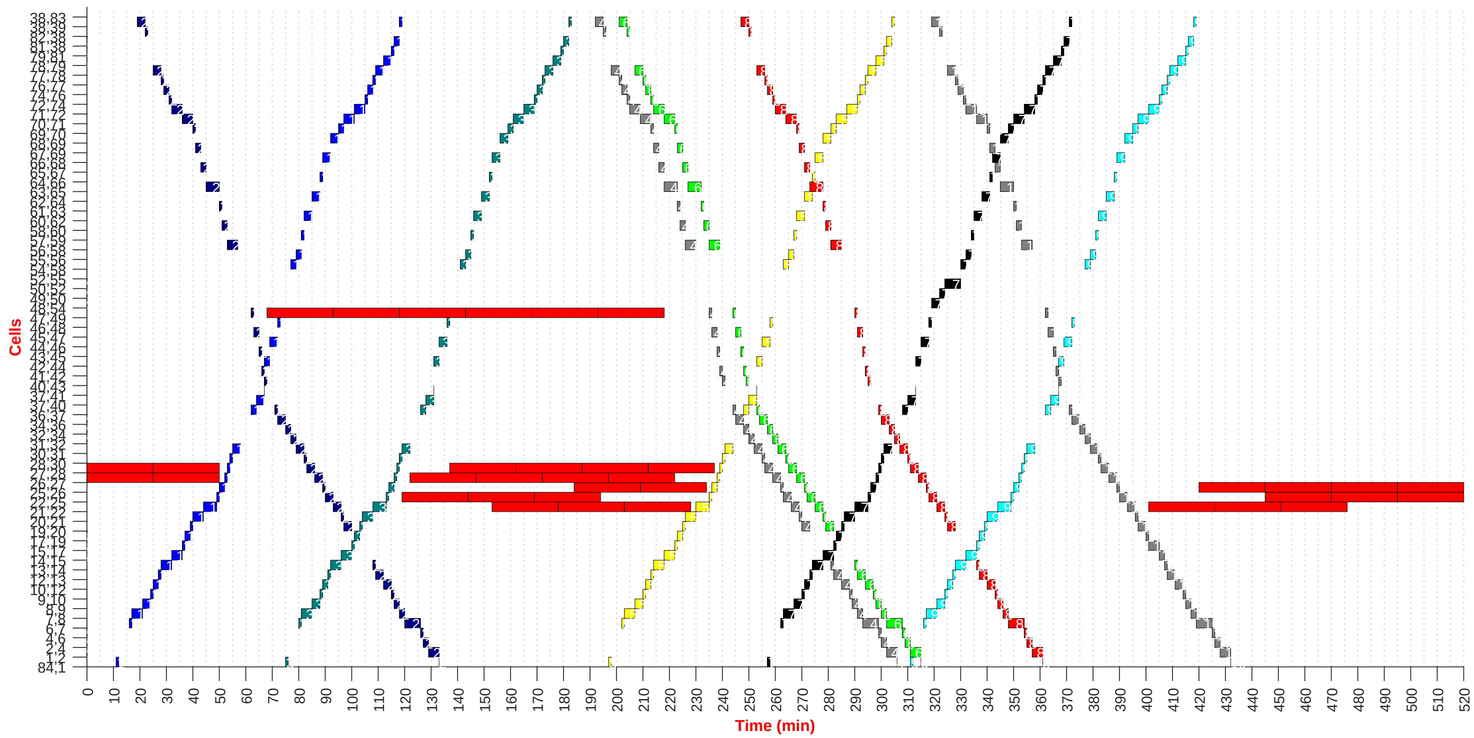

Figure 13: Iteration 17 of Table 8

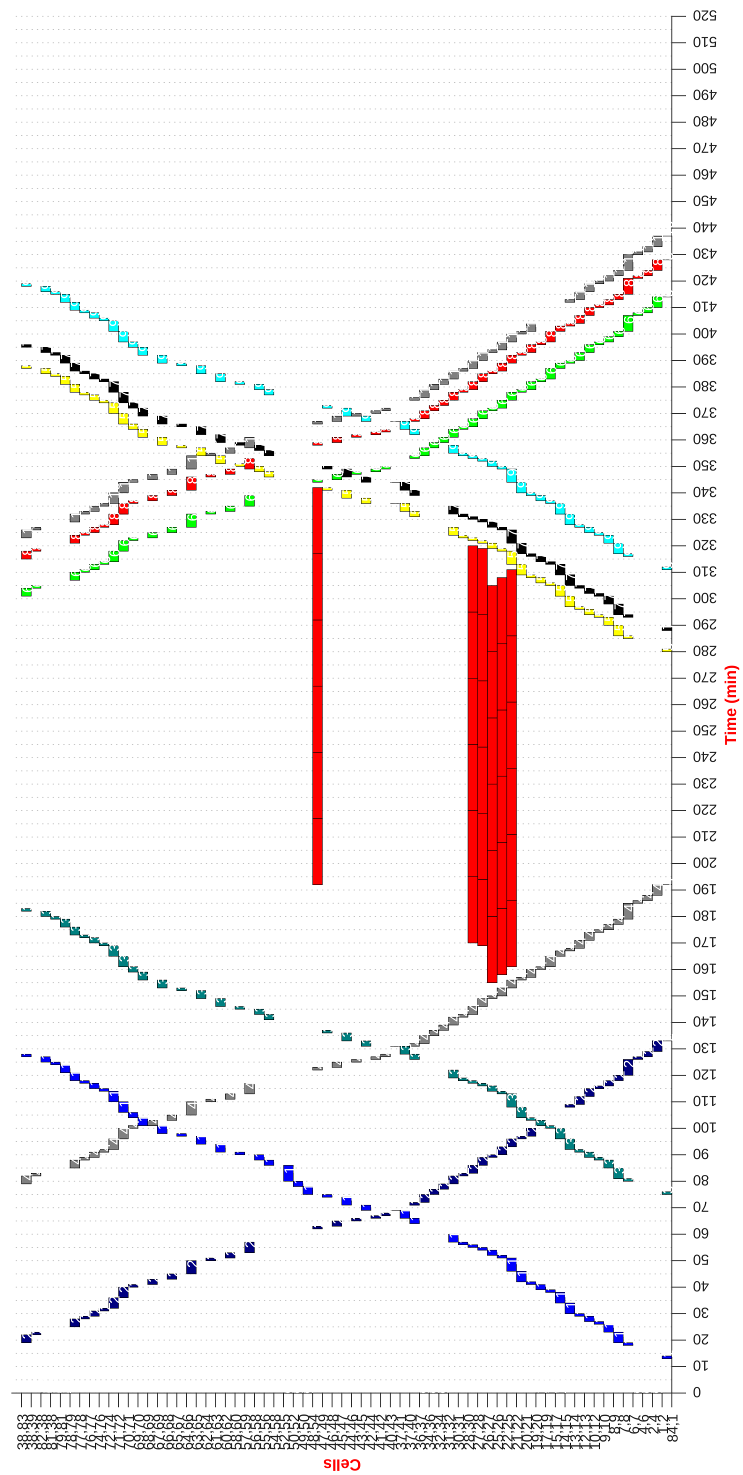

Figure 14: Iteration 22 of Table 8