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Hybrid TM/TE Cylindrical Leaky Waves: Electromagnetic Radiation at Broadside from Radially Periodic Structures

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***Abstract*—The nature of broadside radiation from planar, radially periodic structures excited at the center is investigated in this paper by considering an annular microstrip antenna operating in the microwave range. This kind of antenna, capable of radiating either a scanned conical pattern or a broadside pencil beam, is typically designed by enforcing that its linearized version supports a single TM leaky mode. However, a single TM cylindrical leaky wave is known to be incapable of radiating a directive broadside beam. To solve the conundrum, a generalized pencil-of-function analysis of the excited field is performed. It is found that a non-negligible TE component is present, which exhibits a radial decay different from the TM component but the same complex radial wavenumber. Based on the geometrical symmetries of the structure, an electromagnetic model for the surface currents excited on the annular microstrips is developed and validated by full-wave results for both the currents and the associated radiation patterns. The generality of the adopted approach sheds new light on the nature of the electromagnetic field radiated at broadside by the general class of annular, radially periodic structures, including novel terahertz antennas and plasmonic structures used for achieving extraordinary transmission at optical frequencies.**

***Index Terms*—Broadside radiation, cylindrical leaky wave (CLW), radially periodic structures.**

I. Introduction

ELECTROMAGNETIC radiation from planar structures with azimuthal symmetry and radial periodicity, the so-called 'bull-eye' (BE) structures, has been extensively studied and employed [1], [2]. BE structures are based on the excitation of cylindrical leaky waves (CLWs) [3] that radiate while propagating radially along the surface. Hence, they are two-dimensional leaky-wave antennas (2-D LWAs) [4], [5] belonging to a class intermediate between that of uniform, non-periodic Fabry-Perot cavity antennas [6]–[8] and that of 2-D periodic LWAs [9]–[11]: whereas all these structures radiate through CLWs, BE antennas are uniquely characterized by being periodic in a *single*, radial direction. Due to their intrinsic dispersive features, BE antennas provide for a simple and cost-effective alternative to planar arrays for the generation of highly directional pencil beams, required in microwave, millimeter- and sub-millimeter-wave communication systems.

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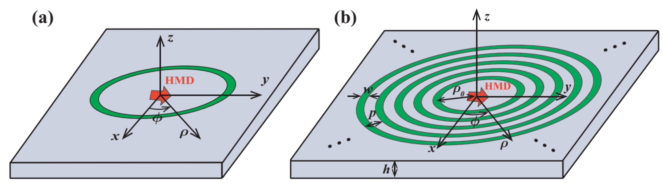

Fig. 1. The reference structure and relevant coordinate system: (a) a single microstrip ring excited by a y-directed horizontal magnetic dipole (HMD). (b) The bull-eye configuration, constituted by N such rings with a common center and a fixed radial spacing.

Over the last two decades, several BE designs have been proposed. Firstly, BE antennas were proposed as an evolution of structures designed to achieve extraordinary transmission and beaming of light through a single subwavelength hole at optical frequencies [12]–[15]. At microwaves, BE antennas are typically realized in printed-circuit technology, by deploying concentric microstrip rings on a grounded dielectric substrate and exciting them by means of a simple source, essentially a surface-wave launcher, placed at the center of the structure [16]–[19]. At higher frequencies, e.g., millimeter waves and terahertz range, entirely metallic structures with annular corrugations have been designed [20]–[22]. Although leaky waves have been shown to play a crucial role in a variety of applications, the BE annular geometry presents fundamental differences from a general 2-D periodic structure. Numerical studies have been carried out to optimize the BE performance (e.g., to achieve directive beaming of light through a sub-wavelength aperture in a plasmonic structure at optical frequencies [23]); however, an analytical model that fully explains its leaky-wave radiation mechanism is not yet available.

In this paper we aim at clarifying the radiation mechanism of BE structures radiating broadside pencil beams. We will focus in particular on printed BE antennas operating in the microwave range (see Fig. 1); if properly excited, these are capable of supporting high-gain broadside pencil beams within a certain range of frequencies (see Fig. 2). BE 2-D LWAs are typically designed by considering a single TM leaky mode supported by their linearized 1-D periodic versions (see, e.g.,

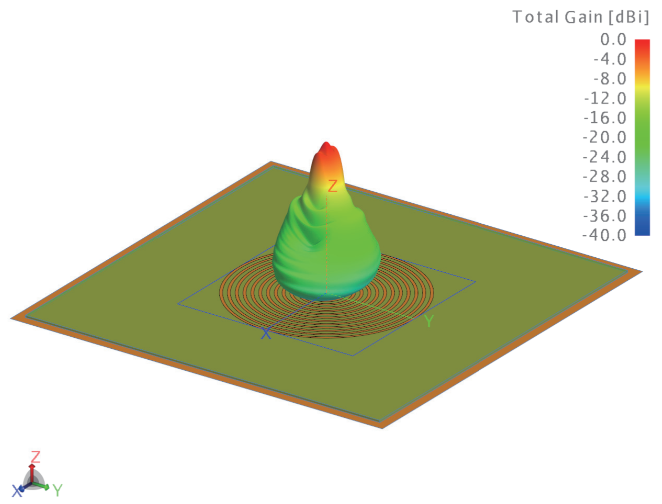

Fig. 2. Full-wave 3-D radiation pattern calculated with FEKO for a BE as in Fig. 1(b), operating at $f = 22$ GHz, with parameters $h = 3.14$ mm, $\varepsilon_{\rm r} = 2.2$, $w = 4$ mm, $p = 10$ mm, $N = 14$ rings, and minimum average ring radius $\rho_0 = 5$ mm.

[16], [18], and [24]). However, a single TM CLW is not able to produce directive radiation at broadside [3]. A novel radiation model is therefore developed and validated in the following, showing that, due to the azimuthal dependence of the field, coupling between the TE and TM polarization is introduced by the ring curvature. In particular, TM and TE leakage with the same radial wavenumber, but different algebraic decay in the principal planes, is responsible for directive radiation in the form of a non-circular pencil beam at broadside. We demonstrate that the CLW responsible for antenna radiation is generally hybrid and that its TE component becomes non-negligible precisely in the frequency ranges where a broadside beam is radiated.

The paper is organized as follows. In Section II, the geometry of the considered structure is described. In Section III, after showing that a single TM CLW is not able to model radiation at broadside, a generalized pencil-of-functions (GPOF) analysis [25] of the near fields is illustrated, unveiling the presence of hybrid TM and TE leaky-wave fields with the same complex wavenumbers, but different radial algebraic decay, around the broadside frequency. In Section IV, general and exact properties of the surface electric currents excited on the microstrip rings by a horizontal magnetic dipole (HMD) are derived, both in the spatial and the spectral domain, in order to establish their hybrid TM/TE nature. The radial periodicity of the actual BE structure is then exploited in Section V, in order to derive an approximate model for the currents excited on the antenna taking into account its modal and hybrid nature. In Section VI, numerical results are presented, based on full-wave simulations of an actual BE antenna, which validate the proposed ring current model as well as the relevant radiation patterns. Finally, conclusions are drawn in Section VII.

II. Structure Description

The basic constituent of the considered structure is an annular microstrip ring of width w printed on a grounded dielectric slab having thickness h and relative permittivity $\varepsilon_{\rm r}$ (see Fig. 1(a), where the relevant reference system is also shown). By arranging N such rings with average radii $\rho_i = \rho_0 + ip$ $(i = 0, \ldots, N-1)$ and a common center at the origin of the reference system, we obtain the BE configuration shown in Fig. 1(a), i.e., a (truncated) radially periodic structure having radial period p.

The BE is assumed here to be excited by a horizontal magnetic dipole (HMD) directed along the y-axis and placed on the z-axis at $z = -h$, i.e., on the ground plane. This idealized source is equivalent to an impressed magnetic current $\mathbf{M}_{\rm dip} = \mathbf{y}_0 M_0 \delta(x)\,\delta(y)\,\delta(z+h)$, where M_0 is a complex amplitude coefficient; it could model practical feeds such as, e.g., a slot etched on the ground plane and excited from below by a microstrip line or a coplanar-waveguide-fed slot dipole [18]. A time-harmonic dependence $\exp(j\omega t)$ is assumed and suppressed.

All the numerical results presented in the next sections are obtained for a structure as in [26], with physical parameters $h = 3.14$ mm, $\varepsilon_{\rm r} = 2.2$, $w = 4$ mm, $p = 10$ mm, $N = 14$ rings, and $\rho_0 = 5$ mm. According to the design guidelines in [18], which are based on the modal analysis of the linearized periodic structure (see Section III below), by the selection of such optimized grating periodicity and fulfilling the condition for the suppression of the TE mode, i.e., $p > 2h\sqrt{\varepsilon_{\rm r} - 1}$, the only perturbed TM mode can be efficiently excited in a leaky regime with radiation occurring over a large frequency bandwidth. Moreover, based on the radiation efficiency condition $\eta = 1 - e^{-2\alpha(Np+p_0)}$, the ring number $N = 14$ would provide more than 90% efficiency at broadside. As shown in Fig. 2, such a structure is capable of producing a directive broadside beam at the frequency $f = 22$ GHz, having different beamwidths in the principal planes and hence a non-circular cross section. Due to the well-known open-stopband effects, only a narrowband operation of 5% is obtained for the reference structure. A wideband broadside fractional bandwidth of about 9.5% or more can be realized with the open-stopband suppression [27], [28].

III. GPOF Analysis

The BE radiates via leaky waves excited by the central feed and propagating as cylindrical waves along the radially periodic microstrip arrangement. Since the lack of translational symmetry prevents directly applying the concept of propagation mode, the antenna design is based on the modal analysis of its *linearized* version, i.e., a 1-D periodic metal strip grating (MSG) printed on a grounded dielectric slab [29]. The physical parameters of the 1-D MSG (h, $\varepsilon_{\rm r}$, w, d) are typically selected so that it supports a *single* TM leaky mode; depending on the operation frequency, the corresponding 2-D BE is capable of radiating either conical scanned beams or broadside pencil beams [16], [18].

By symmetry, the cylindrical leaky waves (CLWs) excited by horizontal electric or magnetic dipoles placed inside a

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structure with rotational invariance along the z-axis, such as the BE, have azimuthal order 1. In a uniform (i.e., not radially periodic) structure, the vertical component E_z of the electric field of a TM CLW of azimuthal order 1 can be expressed in the air region $z \geq 0$ as [3]

$$E_z = \frac{\left(k_\rho^{\mathrm{TM}}\right)^2}{2jk_0} \eta_0 C_{\mathrm{TM}} H_1^{(2)}(k_\rho^{\mathrm{TM}}\rho) \cos\phi e^{-jk_z^{\mathrm{TM}} z} \quad (1)$$

where k_0 and η_0 are the free-space wavenumber and characteristic impedance, respectively; k_ρ^{TM} is the complex TM radial propagation wavenumber; $k_z^{\mathrm{TM}} = \sqrt{k_0^2 - \left(k_\rho^{\mathrm{TM}}\right)^2}$ is the relevant vertical wavenumber in air; and C_{TM} is a complex excitation coefficient.

By resorting to the asymptotic expansion of the Hankel function of the second type for large absolute values of their complex argument [30, Eqs. 10.17.6]:

$$H_1^{(2)}(z) \overset{z\to\infty}{\sim} \sqrt{\frac{2j}{\pi z}} e^{-j(z-\pi/2)} \quad (2)$$

close to the BE plane ($z \approx 0$) we have

$$\rho^{1/2} E_z = \frac{\left(k_\rho^{\mathrm{TM}}\right)^{3/2}}{k_0} \eta_0 C_{\mathrm{TM}} \sqrt{\frac{j}{2\pi}} \cos\phi e^{-jk_\rho^{\mathrm{TM}}\rho} \quad (3)$$

Since the BE is not uniform but radially periodic, an infinite number of radial Floquet space harmonics can be expected, hence the whole aperture field can be expressed as a series of complex exponentials, i.e., of cylindrical waves. The wavenumbers and amplitude coefficients of these waves have been estimated by: *i*) sampling the E_z field in the E-plane $\phi = 0°$ above the BE at $z = \lambda/10$, by using a full-wave electromagnetic simulation software (CST Microwave Studio); *ii*) multiplying them by the radial decay factor $\rho^{1/2}$; and *iii*) processing the corrected samples with the GPOF method, which approximates sequences of complex numbers through complex exponentials [25] and thus allows for retrieving the complex wavenumber of the TM wave excited on the BE:

$$\rho^{1/2} E_z(\rho, \phi) = \cos\phi \sum_{m=1}^{M} C_{m,\mathrm{GPOF}}^{\mathrm{TM}} e^{-jk_m^{\mathrm{TM,GPOF}}\rho} \quad (4)$$

The parameters on the right-hand side produced by the GPOF are: M, the numbers of exponentials required to fit the samples to a given accuracy; $C_{m,\mathrm{GPOF}}^{\mathrm{TM}}$, their complex excitations coefficients; and $k_m^{\mathrm{TM,GPOF}}$, their complex wavenumbers.

The BE antenna is designed to radiate through the space harmonic of order -1 in a frequency range where this is radially fast, i.e., whose radial wavenumber $k_{\mathrm{LW}} = \beta_{-1} - j\alpha$ is such that $|\beta_{-1}| < k_0$. Once such wavenumber has been estimated through the GPOF, the relevant far-field patterns in the azimuthal planes $\phi = \mathrm{const}$ can be evaluated analytically by using the radiation formulas in [3]. In Fig. 3 such patterns are reported in the principal planes $\phi = 0°$ (E-plane) and $\phi = 90°$ (H-plane) as well as in the D-plane ($\phi = 45°$) for the reference BE structure operating at the frequency $f = 22$ GHz. It can be seen that they do *not* correctly reproduce the actual antenna patterns, evaluated numerically with the full-wave simulation softwares FEKO and CST. In FEKO,

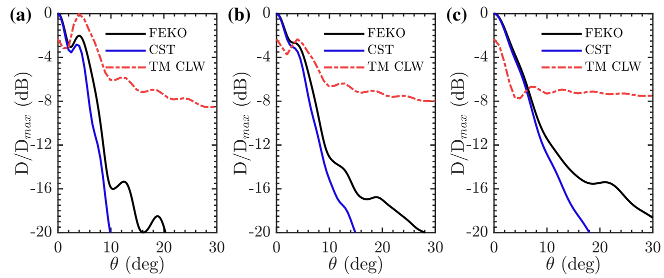

Fig. 3. Comparison between the normalized directivity of the TM CLW and of the full-wave results obtained with FEKO and CST at $f = 22$ GHz over different azimuthal cuts. (a) E-plane. (b) D-plane. (c) H-plane. *Parameters*: as in Fig. 2.

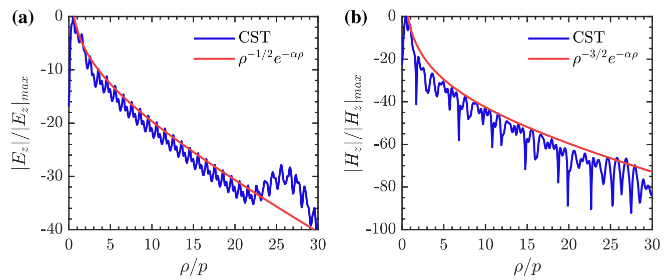

Fig. 4. Radial decay in the principal planes for the vertical components of the fields at $f = 22$ GHz. The fields (a) E_z in the E-plane ($\phi = 0°$) and (b) H_z in the H-plane ($\phi = 90°$) are sampled in CST for a 30-ring BE on the transverse plane $z = \lambda_0/10$. Both fields are normalized by forcing the curves to agree at the mid of the 15-th unit cell. For the E-field, the asymptotic distribution is $\rho^{-1/2}e^{-\alpha\rho}$ and for the H-field it is $\rho^{-3/2}e^{-\alpha\rho}$. For both, the attenuation constant is $\alpha/k_0 = 0.02$, obtained from the MoM simulation of the linearized structure. *Parameters*: as in Fig. 2.

the Method of Moments (MoM) with surface equivalence principle (SEP) is adopted to analyze the antenna, where the surfaces of metal and other media are meshed by triangles. In CST, the frequency domain solver using the finite element method is applied to calculate the normalized directivity. The incapacity to form a sharp broadside beam with a single TM CLW is consistent with previous work in [3].

Trying to understand the reason of such discrepancy, we investigated the role of the TE part of the excited field, associated with the vertical component of the magnetic field H_z. We discovered that the algebraic radial decay factors of the fields E_z and H_z in the E- and H-planes, respectively, are *different*: as shown in Fig. 4, the former has an algebraic decay $\rho^{-1/2}$, whereas the latter decays as $\rho^{-3/2}$. This is a clear indication that the TE part of the field cannot be described by the TE CLW of azimuthal order one that would exist in a uniform structure; in fact, the field H_z of such a wave would be expressed by the dual of (1), hence it would decay as $\rho^{-1/2}$. Therefore, the CLWs of the kind supported by uniform structures cannot directly provide a radiation model for the considered structure.

In the meantime, as reported in Section II, the lowest-order TE_1 mode is suppressed by fulfilling the condition $p > 2h\sqrt{\varepsilon_{\mathrm{r}} - 1}$ [18]. Therefore, the existing in-house MoM approach [29], which is established on the linearized equiva-

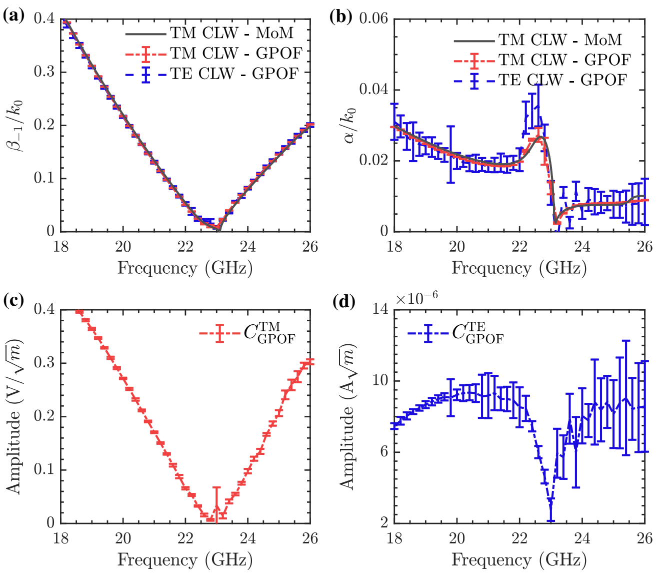

Fig. 5. GPOF analysis of the vertical components of the fields (E_z and H_z): Normalized phase constants β_{-1}/k_0 (a) and attenuation constants α/k_0 (b), compared with results obtained with a MoM modal analysis of the linearized structure; amplitude coefficients (in absolute value) for the TM part (E_z) (c) and the TE part (H_z) (d) of the field. *Parameters*: as in Fig. 2.

lent structure, can only evaluate the behavior for the TM CLW, but not for the TE CLW, which presents a different algebraic decay. For this reason, we have to apply the GPOF method to H_z. To this aim, the following representation is then used

$$\rho^{3/2} H_z(\rho, \phi) = \sin\phi \sum_{m=1}^{M} C^{\mathrm{TE}}_{m,\mathrm{GPOF}} e^{-jk_m^{\mathrm{TE,GPOF}}\rho} \tag{5}$$

where the factor $\rho^{1/2}$ at the left-hand side of (4) has been replaced with the factor $\rho^{3/2}$. The resulting TM and TE wavenumbers and amplitude coefficients (in magnitude) for the radiative space harmonics are reported as a function of frequency in Fig. 5. To reduce the noise, the GPOF method is applied in a certain azimuth angle region on each frequency point ($-30° < \phi < 30°$ for the TM CLW, $60° < \phi < 120°$ for the TE CLW). Then the average values and standard error bars in these regions are presented in Fig. 5. Compared to the TM results, due to the higher-order asymptotic decay nature of the TE CLW, the related field component H_z are relatively small at the sample region, and thus a slightly worse accuracy in CST simulations is inevitable.

As concerns the wavenumbers, in Fig. 5(a) and (b) the normalized phase and attenuation constants retrieved through the GPOF for the leaky TM and TE wavenumbers are reported in a frequency range between 18 and 26 GHz and compared with the phase and attenuation constants of the -1 space harmonic of the TM leaky mode supported by the linearized structure, obtained through a full-wave analysis performed with an in-house code based on the MoM [29]. It can be seen that the GPOF TM and TE wavenumbers are in excellent agreement with the TM MoM wavenumbers. This is a clear indication that the HMD excites on the BE a *single cylindrical leaky mode* having a *hybrid TM/TE nature*.

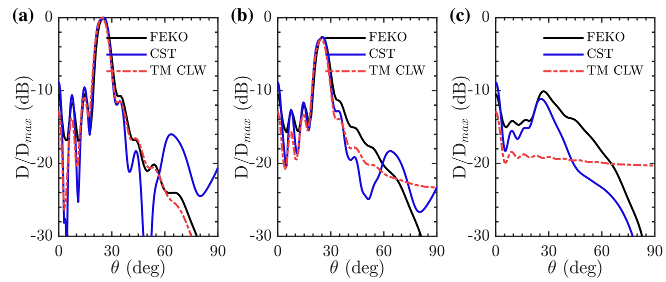

Fig. 6. Comparison between the normalized directivity of the TM CLW and of the full-wave results obtained with FEKO and CST at $f = 18$ GHz over different azimuthal cuts. (a) E-plane. (b) D-plane. (c) H-plane. *Parameters*: as in Fig. 2.

As concerns the amplitudes, it can be seen in Fig. 5(c) that the TM coefficient decays of one order of magnitude from $f = 18$ GHz, when the antenna radiates a scanned beam, to $f = 22$ GHz, when the antenna radiates at broadside; on the other hand, the TE coefficient remains of the same order of magnitude in the same frequency range, as shown in Fig. 5(d); we may conclude that the relative importance of the TE part of the field is *greatly increased* around the broadside frequency.

To further support the previous assumption, theoretical radiation patterns for a pure TM CLW are reported for the reference BE structure operating in the scanned beam case ($\alpha << \beta$). Based on [3], any arbitrary choice of frequency point in the scanned beam can realize a narrow beam with a single TM CLW. For example, at the frequency $f = 18$ GHz in Fig. 6, the antenna generates a scanned beam with maxima in the E-plane at $\theta \simeq 25°$ and negligible radiation in the H-plane. The theoretical patterns correctly reproduce the actual antenna patterns (evaluated with FEKO and CST) in both the E- and D-planes, thus confirming that the TE part of the field is negligible at this frequency. When the frequency increases to the broadside case, as shown before, the expected sharp beam cannot be obtained without the TE contribution. Therefore, the relative contribution of the TE CLW is greatly increased around the broadside frequency.

IV. Ring Currents: Azimuthal Symmetry

In this Section general properties are presented of the surface electric currents excited on a single microstrip ring (see Fig. 1(a)) or multiple concentric microstrip rings (see Fig. 1(b)) by a HMD. The emphasis is on the consequences of the azimuthal symmetry of the configuration, regardless of its radial periodicity, that will be taken into account in the next Section.

The primary field excited by the HMD, i.e., the field in the absence of the microstrip rings, can be obtained through standard spectral domain formulas as

$$\begin{aligned} H_\rho(\rho,\phi,z) &= M_0 \int_{-\infty}^{+\infty} g^{\mathrm{HM}}_{\rho y}(k_\rho;\rho,\phi,z)\, \mathrm{d}k_\rho \\ H_\phi(\rho,\phi,z) &= M_0 \int_{-\infty}^{+\infty} g^{\mathrm{HM}}_{\phi y}(k_\rho;\rho,\phi,z)\, \mathrm{d}k_\rho \end{aligned} \tag{6}$$

where the integrals extend along the extended Sommerfeld integration path (see, e.g., [31, p. 515]) and the Green's functions in the integrands are

$$\begin{aligned} g_{\rho y}^{\mathrm{HM}}\left(k_\rho;\rho,\phi,z\right) &= -\frac{1}{4\pi\rho}\,H_1^{(2)}\left(k_\rho\rho\right)I_V\left(k_\rho;z\right)\sin\left(\phi\right) \\ g_{\phi y}^{\mathrm{HM}}\left(k_\rho;\rho,\phi,z\right) &= -\frac{1}{4\pi}\,k_\rho H_1^{(2)'}\left(k_\rho\rho\right)I_V\left(k_\rho;z\right)\cos\left(\phi\right) \end{aligned} \tag{7}$$

Here $H_1^{(2)}(\cdot)$ is the Hankel function of the first order and second kind, the apex $'$ indicates a derivative with respect to the argument, and I_V is the current due to a unit-amplitude voltage generator placed at $z=-h$ in the transverse equivalent network associated with the grounded dielectric slab [32].

The primary magnetic field on the plane $z=0$ (the antenna aperture plane) has thus the form

$$\mathbf{H}^{\mathrm{in}}\left(\rho,\phi\right) = h_\rho\left(\rho\right)\sin\left(\phi\right)\boldsymbol{\rho}_0 + h_\phi\left(\rho\right)\cos\left(\phi\right)\boldsymbol{\phi}_0 \tag{8}$$

and the azimuthal symmetry of the microstrip rings then implies that the surface electric current $\mathbf{J}_{\mathrm{s}}$ on each ring has the form

$$\mathbf{J}_{\mathrm{s}}\left(\rho,\phi\right) = j_\rho\left(\rho\right)\cos\left(\phi\right)\boldsymbol{\rho}_0 + j_\phi\left(\rho\right)\sin\left(\phi\right)\boldsymbol{\phi}_0 \tag{9}$$

In order to ascertain the TM, TE, or hybrid character of the surface current, these can be Fourier-transformed with respect to x, y, thus obtaining the spectral surface current $\mathbf{J}_{\mathrm{s}}\left(k_x,k_y\right)$. By using polar spectral coordinates k_ρ, Φ, the TM and TE parts of the spectral currents can readily be obtained by projecting them onto the spectral radial and azimuthal unit vectors ($\mathbf{u}_0$ and $\mathbf{v}_0$, respectively) [32].

By Fourier-transforming (9), after some algebraic manipulations (reported in Appendix A) the spectral surface current is obtained in the form

$$\tilde{\mathbf{J}}_{\mathrm{s}}\left(k_\rho,\Phi\right) = \tilde{j}_u\left(k_\rho\right)\cos\left(\Phi\right)\mathbf{u}_0 + \tilde{j}_v\left(k_\rho\right)\sin\left(\Phi\right)\mathbf{v}_0 \tag{10}$$

where the radial spectral functions $\tilde{j}_u\left(k_\rho\right)$, $\tilde{j}_v\left(k_\rho\right)$ are expressed by suitable linear combinations of Hankel transforms of the radial spatial functions $j_\rho\left(\rho\right)$, $j_\phi\left(\rho\right)$. In particular, by letting $\tilde{j}_p^{(q)}$ be the Hankel transform of order q of the radial spatial function $j_p\left(\rho\right)$ ($p=\rho,\phi$)), it results

$$\begin{aligned} \tilde{j}_u\left(k_\rho\right) &= \pi\left[\tilde{j}_\rho^{(0)}\left(k_\rho\right) - \tilde{j}_\phi^{(0)}\left(k_\rho\right) - \tilde{j}_\rho^{(2)}\left(k_\rho\right) - \tilde{j}_\phi^{(2)}\left(k_\rho\right)\right] \\ \tilde{j}_v\left(k_\rho\right) &= \pi\left[\tilde{j}_\rho^{(0)}\left(k_\rho\right) - \tilde{j}_\phi^{(0)}\left(k_\rho\right) + \tilde{j}_\rho^{(2)}\left(k_\rho\right) + \tilde{j}_\phi^{(2)}\left(k_\rho\right)\right] \end{aligned} \tag{11}$$

These formulas show that both TM (u) and TE (v) components are generally present in the surface current. In particular, this is guaranteed if the azimuthal component of the surface current is absent or negligible, as shown in Appendix B. [Note that, in contrast, if the primary field (and hence also the ring currents and scattered field) were *azimuthally invariant*, then a purely radial current would be purely TM and a purely azimuthal current would be purely TE; these would be the cases of a microstrip ring excited by centered vertical electric or magnetic dipoles [24]].

Conversely, on the basis of the formal identity between (9) and (10), the radial spatial functions can be expressed in terms of the radial spectral functions using the very same formulas, except for the factor $1/\left(2\pi\right)^2$ that occurs in the inverse Fourier transform. Hence, by letting $j_p^{(q)}$ be the inverse Hankel transform of order q of the radial spectral function $\tilde{j}_p\left(k_\rho\right)$ ($p=u,v$), it results

$$\begin{aligned} j_\rho\left(\rho\right) &= \frac{1}{4\pi}\left[j_u^{(0)}\left(\rho\right) - j_v^{(0)}\left(\rho\right) - j_u^{(2)}\left(\rho\right) - j_v^{(2)}\left(\rho\right)\right] \\ j_\phi\left(\rho\right) &= \frac{1}{4\pi}\left[j_u^{(0)}\left(\rho\right) - j_v^{(0)}\left(\rho\right) + j_u^{(2)}\left(\rho\right) + j_v^{(2)}\left(\rho\right)\right] \end{aligned} \tag{12}$$

From these one can see that both the radial and azimuthal components are present in a purely TM current (i.e., with $\tilde{j}_v=0$) as well as in a purely TE current (i.e., with $\tilde{j}_u=0$). However, the radial and azimuthal components of a purely TM (or TE) current are not independent, in the sense that, given one, the other is uniquely determined.

V. Ring Currents: Radial Periodicity and Radiation Model

In this Section the radial periodicity of the actual BE configuration will be exploited, along with the properties derived in the previous section, in order to build an approximate model for the current excited on the microstrip rings and hence a radiation model for the BE antenna.

A. Current Model

As is known, the lack of translation symmetry of the considered annular geometry prevents from performing a direct modal analysis of the BE antenna. Therefore a linearization procedure is usually adopted, wherein the radially periodic annular structure becomes an infinite linear equispaced array of straight infinite microstrips, i.e., a MSG on a grounded dielectric slab. The radial direction then becomes the direction orthogonal to the strips (say, the ξ-axis), whereas the azimuthal direction becomes the direction parallel to the strips (say, the η-axis).

Thanks to the invariance of the fields along the η-axis, the Bloch modes of the MSG decouple into purely TM$^\xi$ and purely TE$^\xi$ modes, with the modal surface currents being directed longitudinally for TM modes and transversely for TE modes [29]. The MSG is typically dimensioned so that, at the operating frequency, it supports a *single, weakly attenuated TM leaky mode* which radiates through the -1 space harmonic having wavenumber $k_{\mathrm{LW}}=\beta_{-1}-j\alpha$. The currents excited on the microstrip rings by the HMD can then be expected to have a *dominant radial component* (this would be the only nonzero component if the structure were excited by an omnidirectional source such as a VED placed along the z-axis).

By the Floquet theorem, the longitudinal profile of the modal current on each strip of the MSG is the same except for a complex exponential propagation factor $\exp\left(-jk_{\mathrm{LW}}\xi\right)$. In the annular bull-eye structure, on the other hand, the Floquet theorem cannot be invoked; nevertheless, we will assume that the radial dependence of both radial and azimuthal components of the surface current is expressed by *profile functions* $f_\rho\left(\rho\right)$, $f_\phi\left(\rho\right)$ common to all rings, multiplied however by suitable *modulation functions* $m_\rho(\rho)$, $m_\phi(\rho)$ that take into account the cylindrical spreading of the waves along the xy plane in the annular geometry.

By using the HMD excitation, the BE structure can only support the $n=1$ leaky wave which varies as $\cos\phi$ or $\sin\phi$. A TM$_z$ leaky wave having ϕ symmetry can be expressed in terms of the magnetic vector potential in the air region as:

$$\mathbf{A} = \mathbf{u}_z \frac{1}{2} H_1^{(2)}(k_\rho \rho)\cos\phi e^{-jk_z z}, z>0 \tag{13}$$

From the equivalence principle and image theory, we have:

$$\begin{aligned} J_\rho^{\mathrm{TM}} &= k_\rho H_1^{(2)'}(k_\rho\rho)\cos\phi \\ J_\phi^{\mathrm{TM}} &= -\frac{1}{\rho} H_1^{(2)}(k_\rho\rho)\sin\phi \end{aligned} \tag{14}$$

While for the TE$_z$ leaky wave, the electric vector potential is taken to be:

$$\mathbf{F} = \mathbf{u}_z \frac{\omega\mu_0}{2k_z} H_1^{(2)}(k_\rho\rho)\sin\phi e^{-jk_z z}, z>0 \tag{15}$$

Similarly, the induced current components are:

$$\begin{aligned} J_\rho^{\mathrm{TE}} &= \frac{1}{\rho} H_1^{(2)}(k_\rho\rho)\cos\phi \\ J_\phi^{\mathrm{TE}} &= -k_\rho H_1^{(2)'}(k_\rho\rho)\sin\phi \end{aligned} \tag{16}$$

Let us assume a TM$_z$ CLW of amplitude C$^{\mathrm{TM}}$, and a TE$_z$ CLW of amplitude C$^{\mathrm{TE}}$ are excited by the HMD. Naturally, the total current is a linear combination with coefficients C$^{\mathrm{TM}}$ and C$^{\mathrm{TE}}$ of the TM and TE equivalent currents associated with the $n=1$ TM and TE CLWs. This results in modulation functions as

$$\begin{aligned} m_\rho(\rho) &= C^{\mathrm{TM}} J_\rho^{\mathrm{TM}} + C^{\mathrm{TE}} J_\rho^{\mathrm{TE}} \\ m_\phi(\rho) &= C^{\mathrm{TM}} J_\phi^{\mathrm{TM}} + C^{\mathrm{TE}} J_\phi^{\mathrm{TE}} \end{aligned} \tag{17}$$

Furthermore, for the current on the i-th ring, the developed model should incorporate the correct behavior of the current in the unit cell. In this case, on the edge, the normal component of the current should have a zero of order $1/2$ whereas the parallel component should have an algebraic singularity of order $1/2$. Therefore, we may assume the normalized profiles:

$$\begin{aligned} f_\rho(\rho) &= \frac{2}{w}\sqrt{(\rho+w/2)(w/2-\rho)} \\ f_\phi(\rho) &= \frac{w}{2}\frac{1}{\sqrt{(\rho+w/2)(w/2-\rho)}} \end{aligned} \tag{18}$$

Finally, the current on the i-th ring can be modeled as

$$\begin{aligned} j_\rho &= f_\rho(\rho-\rho_i) m_\rho(\rho) \\ j_\phi &= f_\phi(\rho-\rho_i) m_\phi(\rho) \end{aligned} \tag{19}$$

B. Radiated Fields

The far electric field radiated by the surface current (19),

$$\mathbf{E}^{\mathrm{ff}}(r,\theta,\phi) = j\eta_0 k_0 \frac{e^{-jk_0 r}}{4\pi r}\left[N_\theta(\theta,\phi)\boldsymbol{\theta}_0 + N_\phi(\theta,\phi)\boldsymbol{\phi}_0\right], \tag{20}$$

can be obtained, by reciprocity, calculating the reaction of the surface currents flowing on the microstrip rings with the field produced by a uniform plane wave incident from the direction (θ,ϕ) with amplitude $E_0 = j\eta_0 k_0 \exp(-jk_0 r)/(4\pi r)$. It results

$$\begin{aligned} N_\theta(\theta,\phi) &= \cos(\theta)\left(1+\Gamma^{\mathrm{TM}}\right)\tilde{J}_{\mathrm{s},u}\left(k_\rho = k_0\sin(\theta), \Phi=\phi\right) \\ N_\phi(\theta,\phi) &= \left(1+\Gamma^{\mathrm{TE}}\right)\tilde{J}_{\mathrm{s},v}\left(k_\rho = k_0\sin(\theta), \Phi=\phi\right) \end{aligned} \tag{21}$$

where $\tilde{J}_{\mathrm{s},u/v}$ are the TM and TE components of the spectral current, whereas $\Gamma^{\mathrm{TM/TE}}$ are the TM/TE plane-wave reflection coefficients at the air-substrate interface:

$$\begin{aligned} \Gamma^{\mathrm{TM}} &= \frac{2k_{z1}}{k_{z1} - j\varepsilon_{\mathrm{r}} k_{z0}\cot(k_{z1}h)} \\ \Gamma^{\mathrm{TE}} &= \frac{2k_{z0}}{k_{z0} - jk_{z1}\cot(k_{z1}h)} \end{aligned} \tag{22}$$

with $k_{z0} = k_0\cos(\theta)$, $k_{z1} = k_0\sqrt{\varepsilon_{\mathrm{r}} - \sin^2(\theta)}$.

The spectral currents in (21) can be calculated through (10), (11), where the Hankel transforms of order 0 and 2 of the radial functions (19) are evaluated numerically.

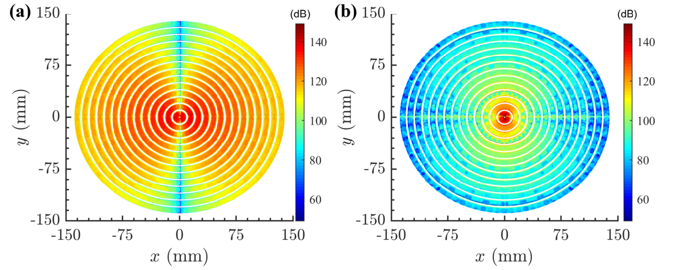

Fig. 7. Amplitude colormaps of the components (a) J_ρ and (b) J_ϕ of the surface current density excited on the metal strips at $f = 22$ GHz. *Parameters*: as in Fig. 2.

VI. Model Validation

The model developed in the previous sections will now be validated through comparisons with numerical full-wave simulations.

In Fig. 7 colormaps are reported for the radial and azimuthal components J_ρ and J_ϕ of the surface currents excited on the microstrip rings. It can be seen that the radial component is largely predominant, and hence the contribution of azimuthal components J_ϕ is trivial compared to the radial components J_ρ; as shown in Section IV, the nature of the excited field is necessarily hybrid.

Fig. 8 reports a comparison for the normalized absolute value of the radial current $|J_\rho|$ between our model and full-wave results in the E-plane ($\phi = 0°$) and D-plane ($\phi = 45°$) at $f = 22$ GHz obtained with FEKO and CST (due to the term $\cos\phi$, $|J_\rho| = 0$ in the H-plane $\phi = 90°$). An excellent agreement is observed. It should be noted that, due to the singularity of the Hankel functions when $|k_\rho\rho| \to 0$, the modeling of the surface current on the first microstrip ring might be inaccurate. To solve this problem, the singular parts have been removed from the Hankel functions, retaining the dominant regular term. With these regularized Hankel functions, an excellent agreement is obtained also for the first ring.

To quantitatively assess the achieved agreement between the far-field radiation generated by the surface current and the measured beam, we finally report in Fig. 9(a)-(c) the 1-D cuts of the far-field pattern in the three principal planes at $f = 22$ GHz. Compared to Fig. 3, the model provides better agreement with full-wave results obtained with FEKO and CST, especially in the E-plane. The mild disagreement in the D-plane and H-plane is due to higher order effects [18] not taken into account in the proposed model.

To test the rigorousness of the proposed model, additional analysis results for another structure with different geometrical and physical parameters ($h = 1.27$ mm, $\varepsilon_{\rm r} = 10.2$, $w = 1$ mm, $p = 8$ mm, $N = 9$) as in [33] are presented in Fig. 10. It should be noted that the frequency point $f = 19.8$ GHz is very close to the well-known open-stopband region (around 20 GHz). This may raise some unpredictable effects and lead to a bad agreement in Fig. 10(c). Even so, at least as concerns the main lobe of the radiation pattern, the proposed model with the hybrid TM/TE CLW provides better accuracy than the traditional CLW theory with only the single TM CLW, thus confirming the generality of the adopted approach.

We note that the phenomenon of extraordinary transmission and directive beaming at optical frequencies through a subwavelength aperture placed on the center of a BE can be characterized by a model similar to that used in this paper for describing pencil-beam radiation at broadside by a HMD. The plasmonic BE structure usually consists of radially periodic concentric annular grooves around a subwavelength aperture in a thin silver film [13], [14]. At optical frequencies, due to the negative permittivity of the medium, a cylindrical plasmonic leaky wave propagates radially along the structure, with characteristics equivalent to the hybrid TM/TE CLW depicted here.

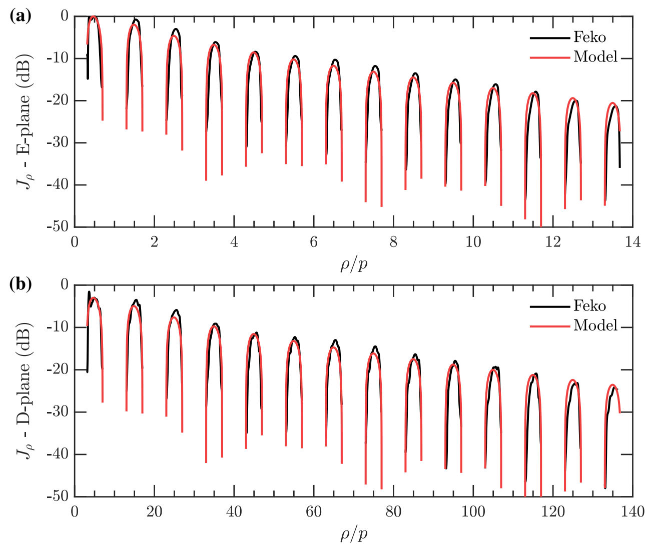

Fig. 8. Comparison between the surface current distributions of the full-wave FEKO simulation (black lines) and our model (red lines) at $f = 22$ GHz. Amplitudes of the current distribution on each strip are sampled in (a) the E-plane and (b) the D-plane. Wavenumbers and amplitudes of TM and TE parts of the currents are $k_\rho^{\rm TM}/k_0 = -0.059 - j0.02, k_\rho^{\rm TE}/k_0 = -0.057 - j0.04$, $\mathrm{C^{TM}/C^{TE}} = k_\rho^{\rm TM}/k_\rho^{\rm TE}$. *Parameters*: as in Fig. 2.

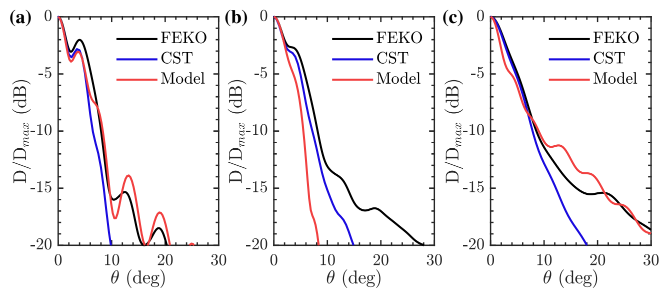

Fig. 9. Radiation patterns at $f = 22$ GHz: comparison between full-wave FEKO simulations (black lines), full-wave CST simulations (blue lines) and our model (red lines) : (a) E-plane; (b) D-plane; (c) H-plane. Wavenumbers and amplitudes of TM and TE CLWs are the same as in Fig. 8. *Parameters*: as in Fig. 2.

VII. Conclusion

Electromagnetic radiation in the microwave range from a class of two-dimensional radially periodic structures, commonly known as bull-eye antennas, has been deeply investigated. The considered structures consist of a finite number of equally spaced, concentric microstrip rings printed on a grounded dielectric slab. When excited by a horizontal magnetic dipole placed at their center, they are capable of radiating a directive narrow pencil beam pointing at broadside through a cylindrical leaky wave propagating radially.

Such antennas, usually designed on the basis of the modal analysis of their linearized counterpart, have been demonstrated to support both frequency-scanning conical beams and broadside pencil beams. In particular, the linearized periodic structure is dimensioned to support only a single TM leaky Bloch mode, thus allowing TM CLWs to describe the antenna radiation off broadside. However, leaky-wave theory reveals that a single TM CLW is not able to predict and describe directive radiation at broadside, thus creating a sort of paradox.

To solve this paradox, an original model has been developed that is able to describe the nature of broadside radiation from the bull-eye antennas. Due to the azimuthal variation of the primary and scattered fields, the hybrid TM/TE nature of the currents on the microstrip rings is rigorously established, while only a purely TM leaky modal field is supported by the linearized 1-D periodic version of the BE structure. A generalized pencil-of-function analysis of the near fields generated on the antenna aperture explicitly proves the hybrid nature of the radiation, showing that both TM and TE leaky fields are excited with the same complex wavenumbers but having different algebraic decays in the E- and H-planes. Furthermore, the presence of the TE leaky component of the field is found to be non-negligible only around the frequency where broadside radiation is obtained, whereas the TM component of the field is mainly responsible for radiation at angles different from broadside. Comparisons with full-wave results for both the surface currents and the radiated far fields have validated the

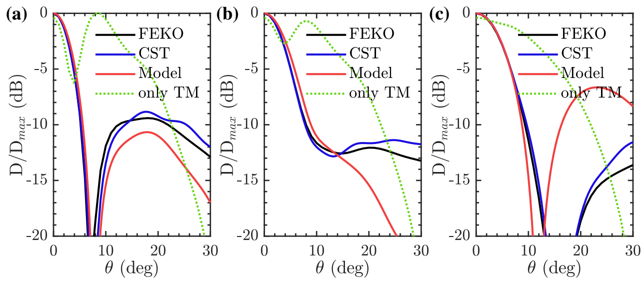

Fig. 10. Radiation patterns at $f = 19.8$ GHz for the structure in [33], with parameters $h = 1.27$ mm, $\varepsilon_r = 10.2$, $w = 1$ mm, $p = 8$ mm, $N = 9$: comparison between full-wave FEKO simulations (black lines), full-wave CST simulations (blue lines), our model (red lines) and the single TM CLW (green lines): (a) E-plane; (b) D-plane; (c) H-plane. Wavenumbers and amplitudes of TM and TE parts of the currents are $k_\rho^{\mathrm{TM}}/k_0 = -0.011 - j0.11$, $k_\rho^{\mathrm{TE}}/k_0 = -0.019 - j0.2$, $\mathrm{C}^{\mathrm{TM}}/\mathrm{C}^{\mathrm{TE}} \approx k_\rho^{\mathrm{TM}}/k_\rho^{\mathrm{TE}}$.

proposed model.

This work demonstrates that there are aspects in the radiation mechanism of bull-eye antennas that cannot be captured by a modal analysis of their linearized counterpart, although such analysis typically provides the basis for their design. Our analysis shows that the result comes from the curvature of the rings; therefore, we believe that similar conclusions also apply to other radiators belonging to the same class of azimuthally invariant, radially periodic structures, such as the metal corrugated bull-eyes operating in the millimeter- and sub-millimeter-wave ranges and the plasmonic bull-eye screens used at optical frequencies for achieving extraordinary transmission and directive beaming from a single subwavelength aperture.

Appendix A
Evaluation of the Spectral Currents

In this Appendix the spectral ring currents, i.e., the Fourier transform of the spatial ring currents (9) will be calculated. To this aim, let us consider the rectangular components:

$$\begin{aligned} J_{\mathrm{s},x} &= \cos(\phi)\, J_\rho - \sin(\phi)\, J_\phi \\ &= j_\rho(\rho)\cos^2(\phi) - j_\phi(\rho)\sin^2(\phi) \\ J_{\mathrm{s},y} &= \sin(\phi)\, J_\rho + \cos(\phi)\, J_\phi \\ &= \left[j_\rho(\rho) + j_\phi(\rho)\right]\sin(\phi)\cos(\phi) \end{aligned} \tag{23}$$

By Fourier transforming these with respect to x, y and then changing coordinates from rectangular k_x, k_y to polar k_ρ, Φ:

$$\begin{aligned} \tilde{J}_{\mathrm{s},x} &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} J_{\mathrm{s},x} e^{j(k_x x + k_y y)}\,\mathrm{d}x\,\mathrm{d}y \\ &= \int_0^{+\infty}\int_{-\pi}^{+\pi} J_{\mathrm{s},x} e^{jk_\rho\rho\cos(\Phi-\phi)}\rho\,\mathrm{d}\phi\,\mathrm{d}\rho \\ &= \int_0^{+\infty} \rho j_\rho(\rho)\, I_{\mathrm{c}}(\rho)\,\mathrm{d}\rho - \int_0^{+\infty}\rho j_\phi(\rho)\, I_{\mathrm{s}}(\rho)\,\mathrm{d}\rho \\ \tilde{J}_{\mathrm{s},y} &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} J_{\mathrm{s},y} e^{j(k_x x + k_y y)}\,\mathrm{d}x\,\mathrm{d}y \\ &= \int_0^{+\infty}\int_{-\pi}^{+\pi} J_{\mathrm{s},y} e^{jk_\rho\rho\cos(\Phi-\phi)}\rho\,\mathrm{d}\phi\,\mathrm{d}\rho \\ &= \int_0^{+\infty} \rho\left[j_\rho(\rho) + j_\phi(\rho)\right] I_{\mathrm{sc}}(\rho)\,\mathrm{d}\rho \end{aligned} \tag{24}$$

where

$$\begin{aligned} I_{\mathrm{c}}(\rho) &= \int_{-\pi}^{+\pi}\cos^2(\phi)\, e^{jk_\rho\rho\cos(\Phi-\phi)}\,\mathrm{d}\phi \\ I_{\mathrm{s}}(\rho) &= \int_{-\pi}^{+\pi}\cos^2(\phi)\, e^{jk_\rho\rho\cos(\Phi-\phi)}\,\mathrm{d}\phi \\ I_{\mathrm{sc}}(\rho) &= \int_{-\pi}^{+\pi}\sin(\phi)\cos(\phi)\, e^{jk_\rho\rho\cos(\Phi-\phi)}\,\mathrm{d}\phi \end{aligned} \tag{25}$$

Let us consider $I_{\mathrm{c}}(\rho)$:

$$\begin{aligned} I_{\mathrm{c}}(\rho) &= \frac{1}{2}\int_{-\pi}^{+\pi}\left[1+\cos(2\phi)\right] e^{jk_\rho\rho\cos(\Phi-\phi)}\,\mathrm{d}\phi \\ &= \frac{1}{2}\int_{-\pi}^{+\pi} e^{jk_\rho\rho\cos(\Phi-\phi)}\,\mathrm{d}\phi \\ &\qquad + \frac{1}{2}\int_{-\pi}^{+\pi}\cos(2\phi)\, e^{jk_\rho\rho\cos(\Phi-\phi)}\,\mathrm{d}\phi \\ &= \frac{1}{2}\int_{-\pi}^{+\pi} e^{jk_\rho\rho\cos(\phi')}\,\mathrm{d}\phi' \\ &\qquad + \frac{1}{2}\int_{-\pi}^{+\pi}\cos\left[2(\phi'+\Phi)\right] e^{jk_\rho\rho\cos(\phi')}\,\mathrm{d}\phi' \end{aligned} \tag{26}$$

from which, using the addition formula for the cosine function and a standard integral representation [30, Eq. 10.9.2] for the Bessel functions $J_n(\cdot)$, we obtain

$$I_{\mathrm{c}}(\rho) = \pi J_0(k_\rho\rho) - \pi\cos(2\Phi)\, J_2(k_\rho\rho) \tag{27}$$

With similar steps one finds:

$$\begin{aligned} I_{\mathrm{s}}(\rho) &= \pi J_0(k_\rho\rho) + \pi\cos(2\Phi)\, J_2(k_\rho\rho) \\ I_{\mathrm{sc}}(\rho) &= -\pi\sin(2\Phi)\, J_2(k_\rho\rho) \end{aligned} \tag{28}$$

By inserting (27) and (28) in (24) we have

$$\begin{aligned} \tilde{J}_{\mathrm{s},x} &= \pi\tilde{j}_\rho^{(0)}(k_\rho) - \pi\cos(2\Phi)\,\tilde{j}_\rho^{(2)}(k_\rho) \\ &\qquad - \pi\tilde{j}\phi^{(0)}(k_\rho) - \pi\cos(2\Phi)\,\tilde{j}_\phi^{(2)}(k_\rho) \\ \tilde{J}_{\mathrm{s},y} &= -\pi\sin(2\Phi)\left[\tilde{j}_\rho^{(2)}(k_\rho) + \tilde{j}_\phi^{(2)}(k_\rho)\right] \end{aligned} \tag{29}$$

Finally, by evaluating the polar components of the spectral current from the rectangular components given in (29), equations (10) and (11) are obtained.

Appendix B
Hybrid Nature of Purely Radial Currents

In this Appendix we show that ring currents as in (9) with $j_\phi(\rho) = 0$ are necessarily hybrid, i.e., they comprise both a TM and a TE component.

To this aim we use (11), which in this case becomes

$$\begin{aligned}\tilde{j}_u(k_\rho) &= \pi\left[\tilde{j}_\rho^{(0)}(k_\rho) - \tilde{j}_\rho^{(2)}(k_\rho)\right]\\ \tilde{j}_v(k_\rho) &= \pi\left[\tilde{j}_\rho^{(0)}(k_\rho) + \tilde{j}_\rho^{(2)}(k_\rho)\right]\end{aligned} \tag{30}$$

Let us shown first that a TM component must be present, i.e., that $\tilde{j}_u(k_\rho) \neq 0$. In fact, if $\tilde{j}_u(k_\rho) = 0$ then from (30) $\tilde{j}_\rho^{(0)}(k_\rho) - \tilde{j}_\rho^{(2)}(k_\rho) = 0$, i.e.,

$$\begin{aligned}0 &= \int_0^{+\infty} j_\rho(\rho)\left[J_0(k_\rho\rho) - J_2(k_\rho\rho)\right]\rho\,\mathrm{d}\rho\\ &= 2\int_0^{+\infty} j_\rho(\rho)\,J_1'(k_\rho\rho)\,\rho\,\mathrm{d}\rho\\ &= \frac{2}{k_\rho}\Bigg\{\left[\rho j_\rho(\rho)\,J_1(k_\rho\rho)\right]_0^{+\infty}\\ &\qquad - \int_0^{+\infty}\frac{\mathrm{d}}{\mathrm{d}\rho}\left[\rho j_\rho(\rho)\right]J_1(k_\rho\rho)\,\mathrm{d}\rho\Bigg\}\end{aligned} \tag{31}$$

having used $2J_1'(z) = J_0(z) + J_2(z)$ [30, Eq. 10.6.1], where the apex ($'$) indicates a derivative w.r.t. the argument, and having then integrated by parts. Now, since $j_\rho(\rho)$ is finitely supported, the finite term from integration by parts vanishes. The remaining term is the Hankel transform of order 1 of $\frac{\mathrm{d}}{\mathrm{d}\rho}\left[\rho j_\rho(\rho)\right]$; hence (31) implies that $\frac{\mathrm{d}}{\mathrm{d}\rho}\left[\rho j_\rho(\rho)\right] = 0$, i.e., $j_\rho(\rho) = C/\rho$, where C is a constant. But this is impossible, since again $j_\rho(\rho)$ is finitely supported. We conclude that $\tilde{j}_u(k_\rho) \neq 0$, as claimed.

Similarly, we may show that the TE component must be present as well, i.e., that $\tilde{j}_v(k_\rho) \neq 0$. In fact, if $\tilde{j}_v(k_\rho) = 0$ then from (30) $\tilde{j}_\rho^{(0)}(k_\rho) + \tilde{j}_\rho^{(2)}(k_\rho) = 0$, i.e.,

$$\begin{aligned}0 &= \int_0^{+\infty} j_\rho(\rho)\left[J_0(k_\rho\rho) + J_2(k_\rho\rho)\right]\rho\,\mathrm{d}\rho\\ &= \frac{2}{k_\rho\rho}\int_0^{+\infty} j_\rho(\rho)\,J_1(k_\rho\rho)\,\rho\,\mathrm{d}\rho\end{aligned} \tag{32}$$

having used $(2/z)\,J_1(z) = J_0(z) + J_2(z)$ [30, Eq. 10.6.1]. Now (31) shows that the Hankel transform of order 1 of $j_\rho(\rho)$ is zero, hence $j_\rho(\rho) = 0$, which would imply having a ring current that is identically zero. We conclude that $\tilde{j}_v(k_\rho) \neq 0$, as claimed.

Appendix C
Implementation of GPOF

In this Appendix, the details on the adopted GPOF method are provided. To this aim, let us consider the TM case:

i) Samples of the near E_z field above the BE at constant intervals along a line $z = \mathrm{const}$, $\phi = \mathrm{const}$ are obtained by using the full-wave electromagnetic simulation software;

ii) By multiplying the samples by the radial decay factor $\rho^{1/2}$, the sample field can be expressed as the sum of complex exponentials:

$$\rho^{1/2}E_z(\rho = n\triangle x, \phi) = \cos\phi\sum_{m=1}^{M} C_{m,\mathrm{GPOF}}^{\mathrm{TM}}e^{-jk_m^{\mathrm{TM,GPOF}}\rho} \tag{33}$$

The parameters on the right-hand side produced by the GPOF are: M, the numbers of exponentials required to fit the samples to a given accuracy; $C_{m,\mathrm{GPOF}}^{\mathrm{TM}}$, their complex excitations coefficients; and $k_m^{\mathrm{TM,GPOF}}$, their complex wavenumbers along the azimuth angle of propagation ϕ on the structure.

iii) Following the procedure for the GPOF method [25], two matrices of field samples can be defined as:

$$\begin{aligned}\mathbf{G}_1 &= (\bar{\mathbf{y}}_1\ \bar{\mathbf{y}}_2\ \bar{\mathbf{y}}_3\ \cdots\ \bar{\mathbf{y}}_P)\\ \mathbf{G}_2 &= (\bar{\mathbf{y}}_2\ \bar{\mathbf{y}}_3\ \bar{\mathbf{y}}_4\ \cdots\ \bar{\mathbf{y}}_{P+1})\end{aligned} \tag{34}$$

and

$$\bar{\mathbf{y}}_i = \rho^{1/2}(E_i\ E_{i+1}\ E_{i+2}\ \cdots\ E_{N-P+i-1})^T \tag{35}$$

where P is the pencil parameter. Let us consider the eigenvalues z_m of the generalized eigenvalue problem:

$$\mathbf{G}_2\bar{e}_m = z_m\mathbf{G}_1\bar{e}_m \tag{36}$$

where $\bar{e}_m$ are the generalized eigenvectors of the pencil $(\mathbf{G}_2 - z_m\mathbf{G}_1)$.

iv) By using the singular value decomposition $\mathbf{G}_1 = \mathbf{U}\mathbf{D}\mathbf{V}^H$, it can be expressed as:

$$\mathbf{D}^{-1}\mathbf{U}^H\mathbf{G}_2\mathbf{V}\mathbf{V}^H\bar{e}_m = z_m\mathbf{V}^H\bar{e}_m \tag{37}$$

where $\mathbf{U}$ and $\mathbf{V}^H$ are unitary matrices associated with left and right singular vectors, respectively, and $\mathbf{D}$ is a diagonal matrix with singular values of $\mathbf{G}_1$ on the diagonal in descending order.

v) Therefore, z_m are the eigenvalues of the matrix $\mathbf{D}^{-1}\mathbf{U}^H\mathbf{G}_2\mathbf{V}\mathbf{V}^H$. Once z_m are obtained, the complex propagation wavenumbers can be calculated as:

$$k_m^{\mathrm{TM,GPOF}} = \beta_m^{\mathrm{TM,GPOF}} - j\alpha_m^{\mathrm{TM,GPOF}} = j\frac{\ln z_m}{\triangle x} \tag{38}$$

vi) By using $k_m^{\mathrm{TM,GPOF}}$ and solving an overdetermined system of equations by using a least-squares fit, the complex amplitude components $C_{m,\mathrm{GPOF}}^{\mathrm{TM}}$ can be obtained.

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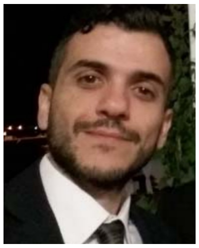

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