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Testing Taylor's Law in Urban Population Dynamics Worldwide with Simultaneous Equation Models

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Abstract: Knowledge of long-term population trends is still incomplete at the global scale. In this perspective, human and animal ecology has intensively studied the relationship between the Mean (M) size and the Variance (V) of specific attributes of subpopulations within a given regional system. One of the best-known relationships between these two attributes suitable to describe long-term population trends is governed by Taylor's law (TL). The present article contributes to the recent literature on population trends worldwide by testing the long-term relationship (1950–2015) between the overall variance and mean in the total population of 1857 metropolitan agglomerations in 155 countries classified into 9 world macro-regions. To estimate the unknown parameter(s) of the V–M relation we made use of a simultaneous equation system using both linear (classical TL) and quadratic specifications, with the aim of ascertaining a wide range of simplified (or more complex) association rules between the two dimensions of demographic change. The empirical results show that TL is verified in all nine cases, although a quadratic relationship provides slightly better results than the classical, linear relationship. More specifically, similar estimates for both linear and quadratic relationships were characteristic of 'new' demographic continents with more recent and intense urbanization processes (the Americas, and African and Asian countries). The predominance of quadratic relationships characterized regions with long-established urbanization processes, such as Europe, Russia, and, partly, China and the Middle East. The relevance of the TL for a refined understanding of urbanization mechanisms worldwide, and the importance of a quadratic term for distinguishing metropolitan systems that have experienced different development paths, were finally discussed.

Keywords: human population; world urbanization perspectives; metropolitan region; urban hierarchy

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1. Introduction

A mix of institutional, political, economic, and cultural factors have influenced long-term urbanization and the underlying population dynamics, two key attributes of the relationship between sustainable development and socio-demographic change in both advanced and emerging economies (Zambon et al. 2017). In this perspective, the overall expansion of cities has been an argument of long-standing interest to social science because of the latent linkage of demographic dynamics with economic growth (Carlucci et al. 2017). Long-term population trends reflect socioeconomic transformations likely better than other territorial factors and/or socioeconomic processes (Cohen 2013). For instance, both short-term and long-term population changes (reflecting, in turn, urban growth or shrinkage at both local and regional scales) were used to identify metropolitan life cycles, defined as time intervals with consolidation (or decline) of a specific urban phase with definite demographic dynamics (Ciommi et al. 2018).

Analysis of metropolitan growth by size class of towns has contributed to understanding the stages of regional development in a given country (Salvati and Serra 2016). Differential growth rates indicate the extent of (internal and international) migration and local changes in fertility and mortality rates governing the natural balance of any population (Gavalas et al. 2014). City size is also regarded as an important attribute influencing sustainable patterns of urbanization worldwide. Given the increased role of large cities in a globalized environment (Carlucci et al. 2018), it is expected that these cities will grow faster than smaller ones. Small and intermediate towns are expected to grow slowly compared to large cities in the early steps of urbanization (Egidi et al. 2020). During late urbanization, small towns are expected to expand more as a result of congestion and crowding in large and intermediate towns (Salvati and Gargiulo Morelli 2014). However, other studies found that cities of different population sizes grow at similar rates (Cohen 2014). In this perspective, relative (urban population) growth gives an indication of demographic shifts at both local and regional scales that is likely better than more accurate socioeconomic indicators in a spatially implicit context (Cohen et al. 2017).

However—while being extensively and successfully applied at both local, regional, and national scales—population and geographical approaches based on a descriptive analysis of socioeconomic and demographic trends have failed sometimes in delineating the intrinsic mechanisms at the base of urbanization processes at the global scale (Cohen 2013). Interpretative frameworks contributing to linking economic theory with historical approaches in a comparative understanding of the role of past cities' structure seem to be more and more relevant in such research fields (Egidi et al. 2020). Being inherently influenced by sustainable development policies, land-use regulations leveraging metropolitan expansion, and (more or less strict) control of immigration, poverty, and social segregation, "urbanization is better understood as a global historical process driven by population dynamics associated with technological and institutional innovations that have substantially improved disease control and food security in urban settlements across the globe. These innovations first emerged in Europe in the eighteenth and nineteenth centuries and were subsequently diffused through colonialism, trade, and international development assistance" (Fox 2012).

Being grounded on a wide-ranging comparison among individual cities and regions, a comprehensive understanding of urbanization patterns and processes relies, to a large extent, on the availability of accurate and consistent information on the spatial distribution, size, and growth of metropolitan areas worldwide (Carlucci et al. 2018). In this perspective, developmental processes can be declined through various definitions and operational approaches (Salvati et al. 2019). The use of official statistics in urban development studies allows differentiated approaches that should be considered for their completeness, diversity, and complexity (Egidi et al. 2020). Production of spatially disaggregated statistics is beginning to be relevant for the analysis of economic systems (Ciommi et al. 2019). These data have demonstrated, under varying socioeconomic contexts, that territorial statistics provide a truly refined and complete key to the interpretation of mechanisms of socioeconomic development (Rozenfeld et al. 2008). In this regard, social sciences have increasingly benefited from big data and non-linear approaches to complexity based on the continuous development of new econometric techniques (Xu and Cohen 2021).

Despite rising interest in the role of long-range urbanization on demographic processes around the world, comparative knowledge of population trends in metropolitan areas is still partial and fragmented (Carlucci et al. 2017). Owing to the intrinsic conditions (e.g., social, cultural, political) of each city, contemporary urban agglomerations give increasing evidence of new, multifaceted relationships between demographic dynamics and the underlying socioeconomic context (Di Feliciantonio and Salvati 2015). For instance, while the highest rates of population growth were observed in the 1960s and the 1970s in advanced economies, more heterogeneous demographic patterns characterize emerging countries and reflect (i) inherent transformations in socio-spatial structures and (ii) a persistent polarization in metropolitan and non-metropolitan areas (Ciommi et al. 2019).

Taylor's law (TL), as defined in the following 'Literature and Logical Framework' section, is recognized to be an appropriate specification when studying the inherent variability of human population size over time and space. In other words, the variance around the mean population stock of elementary domains (e.g., cities) calculated over homogeneous regions was frequently interpreted as a measure of departure from a condition of balanced distribution of a given population over space (Carlucci et al. 2018). While contributing operationally to refine demographic assessment over large spatial coverage (namely, countries or continents), the estimation of TL is simplified, requiring only population data at each spatial unit at repeated times (Kendal 2004). With the aim of advancing the use of TL in sustainability science, applied demography, and urban studies, a multi-regional (power) law for the time period 1950–2015 was verified in the present study using both linear and quadratic specifications that investigate the relationship between the average urban population size in more than 1800 cities/metropolitan areas and its variability over space (namely, the variance around the mean) in 9 world geographical areas (i.e., continents or macro-regions). Based on a sample considered to be representative of largely differentiated demographic dynamics, the present work brings insight into the debate over the present and future development of urban agglomerations worldwide.

Literature and Logical Framework

The focus of multi-regional demography is the study of the spatial distribution of human populations at multiple spatial units participating in the same territorial system. Spatial and temporal variability of these sub-populations were investigated considering the evolution of the components of demographic balance within a certain time interval (Rogers 1975). Although population size is considered a key indicator in animal ecology and human demography (Hanski 1999), it was relatively under-investigated as far as (sustainable) patterns of urbanization worldwide are concerned (Dey and Joshi 2006). Investigating the relationship between the variance and the average density of a given non-human population in selected, appropriate spatial partitions, Taylor (1961) developed a formalization later known as Taylor's law, hereafter TL, more recently applied to various fields of study (Taylor et al. 1983).

Quite surprisingly, TL has found relatively few applications in the field of human demography (Cohen et al. 2013a). Although the first test of TL on human population data from the United States Population Census was carried out in the late 1970s (Taylor et al. 1978), only recently TL was more extensively considered in regional demography (Bohk et al. 2015; Cohen 2014; Xu et al. 2017; Benassi and Naccarato 2019). TL describes an exponential relationship between the variance of a non-negative measurement (population, in our case) and its mean level in a given spatial partition. In a given population, partitioned among a number of spatial units, the variance of the population is related to the average population in the same unit as follows:

$$V = \alpha M^{\beta} \tag{1}$$

where V and M are the variance and average of total population size (namely, population stock) within each individual regional unit selected as the elementary domain forming the investigation sample. Adopting logarithmic transformations, Equation (1) becomes:

$$log_{10}V = log_{10}\alpha + \beta log_{10}M \tag{2}$$

and the β parameter represents the effect of the average population (M) on the variability of the same population (V). If the β parameter is positive, it means that an increase in the average population implies a greater variance (namely variability over space) between the elementary analysis' domains. Interest in the scientific debate about the origin and interpretation of TL is great, with reference to both the meaning of the equation coefficients (Ramsayer et al. 2012; Cohen et al. 2013b; Cohen 2014; Giometto et al. 2015; Saitoh and Cohen 2018; Xu and Cohen 2021; Naccarato and Benassi 2022) and the underlying mathematics (Kilpatrick and Ives 2003; Eisler et al. 2008; Fronczak and Fronczak 2010; Kendal and Jørgensen 2011; Cohen and Xu 2015).

2. Methodology

2.1. Empirical Data

The total population in 1857 metropolitan agglomerations with more than 300,000 inhabitants (2014 benchmark) of 155 world countries was derived for every year between 1950 and 2015 from *World Urbanization Prospects* published by the United Nations. We used the last revision of this extensive dataset of official statistics released in 2018 and fully available as downloadable spreadsheets from the webpage of the United Nations Population Division. Based on these data, metropolitan agglomerations were classified into 9 macro-regions (Africa, China, Russia, and other countries historically under Soviet influence, India, other Asian countries and Oceania, Middle Eastern countries, Europe, North America, and Latin America).

Geographical areas were designed with the objective of partitioning the sample of cities into spatially homogeneous groups with similar characteristics, assuming the influence of a common socioeconomic context (Zambon et al. 2018). The number of cities in each geographical area is rather balanced, ranging between 434 agglomerations in China and 108 in Russia. The minimum number of cities included in each macro-region (above 100) complies with the recommended sample size for the calculation of averages and variances (Taylor et al. 1983). A complete analysis of population trends in this sample of cities, based on descriptive statistics, graphs, and correlation inference covering the same time interval of this study, was provided in Egidi et al. (2020).

2.2. Econometric Model

We formalized the TL specification by means of a measure of the dispersion (namely the variance, V) around the arithmetic mean (M) of the population size measured in each city/metropolitan area within the same region or geographical area. Thus, for each geographical area, TL was estimated using the mean population stock that was obtained by averaging the total population of each city/metropolitan area within each region/geographical area at a given year between 1950 and 2015, considering five-interval time steps, e.g., 1950, 1955, 1960, . . . , 2010, 2015, as follows:

$$M_{i,t} = \frac{1}{n_i} \sum_{j=1}^{n_i} p_{ij,t} \tag{3}$$

$$V_{i,t} = \frac{1}{n_i} \sum_{j} \left(p_{ij,t} - M_{i,t}\right)^2 \tag{4}$$

where $p_{ij,t}$ is the population of the j-th city of the geographical area i at time t, and n_i is the number of cities in the i-th geographical area. For each geographical area, Equation (2) becomes:

$$log_{10} V_{i,t} = log_{10} \alpha_i + \beta_i log_{10} M_{i,t} + \varepsilon_{i,t} \tag{5}$$

In Equation (5), $\varepsilon_{i,\,t}$ represents the error term of the model for which the usual assumptions of the classical regression model are supposed (Casella and Berger 2002). In the estimation process, we assume that the selected geographical areas constitute a single demographic system (e.g., Gavalas et al. 2014; Chelleri et al. 2015; Ciommi et al. 2018), in which migrations from one continent to another cannot be regarded as completely independent over the selected time interval, namely 14 observation years from 1950 to 2015, as described above. For this reason, TL was estimated for each geographical area selected in the world by means of a Simultaneous Equation Model, hereafter SEM (Zellner 1962), testing the regionally based, explicit relationship between mean and variance population size. The SEM estimation accounts for the dependence structure among the errors of the equations that reflects the dependence structure of the endogenous variable (Lamonica et al. 2020) characteristic of each of the nine geographical areas in the world. In our case, the population size of each of the areas is assumed to implicitly depend on the population size of the other 8 areas, meaning that between the observed variances V_i in the 9 areas, there is a relationship that reflects the correlation structure of the error component of the 9 equations. In other words, SEM estimations assume that demographic patterns characteristic of each geographical area interact through a latent dependency structure made formally explicit. In the SEM model, this structure—represented by the variance and covariance matrix of the error component—is estimated in a first stage, being subsequently adopted for final evaluation, in turn allowing consistent estimates of the unknown parameters. The alternative Ordinary Least Square (OLS) estimation of 9 independent equations, one for each geographical area, was used as a baseline with the aim of evaluating the technical improvements of a SEM. All data were logarithm-transformed prior to analysis.

Data on a log mean and a log variance were tested for linearity comparing classical TL (Equation (5)) with quadratic regression models. In other terms, we compare the results of the estimated Equation (5) with those obtained by estimating a relation in which we have introduced a quadratic term:

$$log_{10}V_{i,t} = log_{10}\alpha_i + \beta_i log_{10}M_{i,t} + \gamma(log_{10}M_{i,t})^2 + \varepsilon_{i,t} \quad (6)$$

This quadratic relation is mathematically identical to Equation (14) of Taylor et al. (1978), as was pointed out in the supplementary information of Lagrue et al. (2015). The model's goodness-of-fit was checked on the basis of (i) the statistical significance of the linear and quadratic regression coefficients (*t*-statistic), (ii) the values of Akaike (1974) Information Criterion, as well as (iii) the adjusted R^2.

3. Results

Our tests indicate that both specifications—linear (Table 1) and quadratic (Table 2)—were highly significant, producing models with adjusted R^2 mostly above 0.98. Differences in the adjusted R^2 between OLS linear and quadratic specifications are relatively weak. Considering the linear TL specification, slope coefficients were positive in all macro-regions, ranging between 1.098 for North American metropolitan agglomerations and 1.832 for India, and 1.805 for Middle East metropolises. Coefficients included in this range were observed for metropolises in the remaining geographical areas. TL models performed highly (adjusted $R^2 > 0.98$) for metropolises in Africa, China, India, Latin America, the Middle East, and North America. Models for European and Russian metropolitan agglomerations totalized a lower R^2, 0.908 and 0.944, respectively.

The use of a quadratic specification significantly improved the goodness-of-fit of regional models in both Europe (adjusted-R^2 = 0.975) and Russia (adjusted-R^2 = 0.976), while impacting positively (but less intensively) the goodness-of-fit of models estimated for the remaining seven macro-regions. Interestingly, models referring to European and Russian metropolises displayed a negative linear coefficient (−37.8 for Europe, −13.1 for Russia) together with a positive quadratic term (6.8 for Europe, 2.7 for Russia). China also showed a similar distribution of coefficients, with coefficient signs fully coherent with Europe and Russia, but with a smaller coefficient intensity. As a matter of fact, the linear coefficient for China was −1.4, and the positive quadratic term was only 0.6. All the other models showed completely different regression signs and coefficient values. Comparable signs and a similar intensity in regression coefficients delineate coherent spatial patterns of Taylor'slLaw in common between Europe, Russia, and China. Based on these data, such macro-regions can be considered as having a millenary history of urban development largely affecting the metropolitan hierarchy reflected in a peculiar, quadratic form of Taylor's law. Interestingly, looking at the coefficients' signs and intensity, a gradient from Europe to Russia and China can be delineated, likely indicating a sort of ordering from 'old demographic' urban systems with a consolidated, long development to systems with slightly more recent development. Based on such results, China is positioned in-between macro-regions with a long tradition in urban development and other regions with a relatively more recent development path. Positive (but weak) coefficients for both linear and quadratic terms were found for African (linear coefficient: 0.698; quadratic coefficient: 0.200) agglomerations. Positive linear coefficients with negative quadratic coefficients were found for North American, Latin American, Middle Eastern, and Asian/Oceanian agglomerations. Compared with evidence for Europe, Russia, and China, these empirical findings may delineate these macro-regions as affected by a more recent urban development and possibly characterized by demographically younger population structures.

Table 1. Linear Taylor's law estimation by geographical area (AIC: Akaike Information Criterion).

Geographical Area	Parameter	Estimated Coefficient	Standard Error	*p*-Value
Africa	$log\alpha$	1.305	0.025	<0.001
	β	1.707	0.010	<0.001
Adjusted R-square	0.998			
AIC	−293.0			
Other Asia and Oceania	$log\alpha$	2.683	0.079	<0.001
	β	1.397	0.028	<0.001
Adjusted R-square	0.975			
AIC	−200.6			
China	$log\alpha$	1.951	0.058	<0.001
	β	1.495	0.023	<0.001
Adjusted R-square	0.983			
AIC	−188.7			
Europe	$log\alpha$	2.023	0.158	<0.001
	β	1.389	0.055	<0.001
Adjusted R-square	0.908			
AIC	−300.7			
India	$log\alpha$	1.167	0.012	<0.001
	β	1.832	0.004	<0.001
Adjusted R-square	0.999			
AIC	−422.5			
Latin America	$log\alpha$	1.775	0.031	<0.001
	β	1.615	0.011	<0.001
Adjusted R-square	0.997			
AIC	−305.1			
Middle East	$log\alpha$	0.990	0.045	<0.001
	β	1.805	0.017	<0.001
Adjusted R-square	0.994			
AIC	−209.9			
North America	$log\alpha$	3.235	0.039	<0.001
	β	1.098	0.013	<0.001
Adjusted R-square	0.990			
AIC	−373.0			
Russia	$log\alpha$	1.837	0.123	<0.001
	β	1.462	0.044	<0.001
Adjusted R-square	0.944			
AIC	−217.4			

Source: our elaboration on United Nations Population Division data.

Table 2. Quadratic Taylor's law estimation by geographical area.

Geographical Area	Parameter	Estimated Coefficient	Standard Error	p-Value
Africa	$log\alpha$	2.552	0.137	<0.001
	β	0.698	0.110	<0.001
	γ	0.200	0.022	<0.001
Adjusted R-square	0.999			
AIC	−346.7			
Other Asia and Oceania	$log\alpha$	−5.483	0.209	<0.001
	β	7.251	0.150	<0.001
	γ	−1.041	0.027	<0.001
Adjusted R-square	0.999			
AIC	−411.9			
China	$log\alpha$	5.618	0.360	<0.001
	β	−1.377	0.281	<0.001
	γ	0.554	0.054	<0.001
Adjusted R-square	0.994			
AIC	−251.4			
Europe	$log\alpha$	58.078	2.146	<0.001
	β	−37.775	1.485	<0.001
	γ	6.838	0.257	<0.001
Adjusted R-square	0.975			
AIC	−386.8			
India	$log\alpha$	1.950	0.106	<0.001
	β	1.246	0.079	<0.001
	γ	0.108	0.015	<0.001
Adjusted R-square	0.999			
AIC	−461.9			
Latin America	$log\alpha$	−0.806	0.179	<0.001
	β	3.514	0.132	<0.001
	γ	−0.346	0.024	<0.001
Adjusted R-square	0.999			
AIC	−399.5			
Middle East	$log\alpha$	−1.618	0.163	<0.001
	β	3.936	0.133	<0.001
	γ	−0.427	0.026	<0.001
Adjusted R-square	0.999			
AIC	−315.5			
North America	$log\alpha$	−0.987	0.680	0.152
	β	4.005	0.468	<0.001
	γ	−0.499	0.080	<0.001
Adjusted R-square	0.994			
AIC	−402.5			
Russia	$log\alpha$	21.689	2.104	<0.001
	β	−13.131	1.545	<0.001
	γ	2.674	0.283	<0.001
Adjusted R-square	0.976			
AIC	−273.6			

Source: our elaboration on United Nations Population Division data.

Although with modest differences, the results suggest how quadratic regression models perform better than linear regression ones. On the quadratic TL, tests were performed to verify the hypotheses of the absence of heteroscedasticity and autocorrelation of the residuals. The evaluation of the second of the two hypotheses appears particularly relevant since the temporal nature of the observations suggests the presence of an autoregressive component that cannot be ignored. The results reported in Table 3 demonstrated heteroskedastic and autocorrelated residuals at least up to the third order. It is therefore necessary to use a generalized estimator that involves the variance and covariance matrix of the residuals in the estimation procedure of the unknown parameters separately for each macro-region.

Table 3. Heteroscedasticity White test and Autocorrelation Ljung–Box test for quadratic TL estimated model, by geographical area.

Geographical Area	Heteroscedasticity	Autocorrelation of Order 1	Autocorrelation of Order 2	Autocorrelation of Order 3
Africa	p-value = $P(\chi^2_{(4)} > 13.2) = 0.01$	p-value = $P(F_{(1,62)} > 697.1) = 1.98 \times 10^{-35}$	p-value = $P(F_{(2,61)} > 444.6) = 4.27 \times 10^{-37}$	p-value = $P(F_{(3,60)} > 312.1) = 1.50 \times 10^{-36}$
Other Asia and Oceania	p-value = $P(\chi^2_{(4)} > 31.7) = 2.16 \times 10^{-6}$	p-value = $P(F_{(1,62)} > 1010.2) = 4.38 \times 10^{-40}$	p-value = $P(F_{(2,61)} > 708.6) = 5.97 \times 10^{-43}$	p-value = $P(F_{(3,60)} > 481.2) = 6.58 \times 10^{-42}$
China	p-value = $P(\chi^2_{(4)} > 34.2) = 6.69 \times 10^{-7}$	p-value = $P(F_{(1,62)} > 191.6) = 1.25 \times 10^{-20}$	p-value = $P(F_{(2,61)} > 96.7) = 1.22 \times 10^{-19}$	p-value = $P(F_{(3,60)} > 64.1) = 1.06 \times 10^{-18}$
Europe	p-value = $P(\chi^2_{(4)} > 13.4) = 0.009$	p-value = $P(F_{(1,62)} > 744.2) = 3.05 \times 10^{-36}$	p-value = $P(F_{(2,61)} > 416.7) = 2.69 \times 10^{-36}$	p-value = $P(F_{(3,60)} > 291.0) = 1.07 \times 10^{-35}$
India	p-value = $P(\chi^2_{(4)} > 13.7) = 0.008$	p-value = $P(F_{(1,62)} > 539.7) = 2.68 \times 10^{-32}$	p-value = $P(F_{(2,61)} > 289.9) = 7.06 \times 10^{-32}$	p-value = $P(F_{(3,60)} > 207.7) = 1.23 \times 10^{-31}$
Latin America	p-value = $P(\chi^2_{(4)} > 13.5) = 0.009$	p-value = $P(F_{(1,62)} > 545.0) = 2.05 \times 10^{-32}$	p-value = $P(F_{(2,61)} > 309.1) = 1.20 \times 10^{-32}$	p-value = $P(F_{(3,60)} > 215.1) = 4.63 \times 10^{-32}$
Middle East	p-value = $P(\chi^2_{(4)} > 20.4) = 0.0004$	p-value = $P(F_{(1,62)} > 1939.9) = 1.70 \times 10^{-48}$	p-value = $P(F_{(2,61)} > 3780.0) = 1.12 \times 10^{-64}$	p-value = $P(F_{(3,60)} > 2482.6) = 7.47 \times 10^{-63}$
North America	p-value = $P(\chi^2_{(4)} > 28.1) = 1.21 \times 10^{-5}$	p-value = $P(F_{(1,62)} > 1392.7) = 3.40 \times 10^{-44}$	p-value = $P(F_{(2,61)} > 2436.5) = 6.46 \times 10^{-59}$	p-value = $P(F_{(3,60)} > 1649.8) = 1.40 \times 10^{-57}$
Russia	p-value = $P(\chi^2_{(4)} > 24.2) = 7.29 \times 10^{-5}$	p-value = $P(F_{(1,62)} > 1032.6) = 2.31 \times 10^{-40}$	p-value = $P(F_{(2,61)} > 544.1) = 1.29 \times 10^{-39}$	p-value = $P(F_{(3, 60)} > 369.6) = 1.25 \times 10^{-38}$

Source: our elaboration on United Nations Population Division data.

The nine Equations were estimated using a generalized estimation scheme with the aim of (i) accounting for residuals' heteroscedasticity and autocorrelation and (ii) considering a dependence structure across geographies. The Three Stage Least Square estimator (3SLS)—which considers both the variance and covariance matrix of the residuals in each individual area as well as that between different areas — provides consistent and efficient estimates of the unknown parameters of the nine equations. Models referring to European and Russian metropolitan hierarchies displayed negative linear coefficients (−40.3 for Europe, −15.9 for Russia), slightly higher than those observed with the standard econometric estimation reported in Table 2, together with a positive quadratic term (7.3 for Europe, 3.2 for Russia). China also showed a similar coefficient distribution, with coefficient signs fully coherent with Europe and Russia (linear term: −1.2; quadratic term: 0.5), but with a comparatively smaller coefficient intensity. The other models revealed different regression signs and coefficient values, as observed in Table 2. Comparable signs and similar intensity in regression coefficients confirm coherent spatial patterns of Taylor's law in common between Europe, Russia, and China, as already mentioned when discussing the econometric results illustrated in Table 2. Positive (but relatively weak) coefficients for both linear and quadratic terms were found for African (linear coefficient: 0.75; quadratic coefficient: 0.19) agglomerations. Positive linear coefficients with negative quadratic coefficients were found for North American, Latin American, Middle Eastern, and Asian/Oceanian agglomerations. Referring to the SEM estimation, the results reported in Table 4 finally suggest the relevance of a dependency structure among the nine equations, which justifies the estimation of a simultaneous equations model.

Table 4. 3SLS econometric estimation of quadratic Taylor's law.

Geographical Area	Parameter	Estimated Coefficient	Standard Error	p-Value
Africa	$log\alpha$	2.498	0.099	<0.001
	β	0.745	0.079	<0.001
	γ	0.190	0.016	<0.001
Adjusted R-square	0.999			
Other Asia and Oceania	$log\alpha$	−5.625	0.165	<0.001
	β	7.354	0.118	<0.001
	γ	−1.060	0.021	<0.001
Adjusted R-square	0.999			
China	$log\alpha$	5.475	0.255	<0.001
	β	−1.248	0.198	<0.001
	γ	0.526	0.526	<0.001
Adjusted R-square	0.997			
Europe	$log\alpha$	61.716	2.505	<0.001
	β	−40.281	1.747	<0.001
	γ	7.270	0.305	<0.001
Adjusted R-square	0.975			
India	$log\alpha$	1.876	0.071	<0.001
	β	1.303	0.053	<0.001
	γ	0.098	0.010	<0.001
Adjusted R-square	0.999			
Latin America	$log\alpha$	−0.945	0.135	<0.001
	β	3.615	0.099	<0.001
	γ	−0.364	0.018	<0.001
Adjusted R-square	0.999			
Middle East	$log\alpha$	−1.798	0.134	<0.001
	β	4.085	0.108	<0.001
	γ	−0.457	0.022	<0.001
Adjusted R-square	0.998			
North America	$log\alpha$	−1.366	0.511	<0.001
	β	4.278	0.351	<0.001
	γ	−0.548	0.060	<0.001
Adjusted R-square	0.993			
Russia	$log\alpha$	25.472	1.395	<0.001
	β	−15.903	1.021	<0.001
	γ	3.182	0.187	<0.001
Adjusted R-square	0.975			

Source: our elaboration on United Nations Population Division data.

The variance–covariance matrix between the residuals of the equations estimated separately (Table 5) shows non-zero correlations, and the Breusch and Pagan (1979) heteroscedasticity test (χ^2 = 764.8, p < 0.0001) leads to the rejection of the hypothesis of residuals' homoscedasticity for the nine equations. The values above the main diagonal in Table 5 show that the residuals of the different equations are significantly correlated. These results confirm the initial hypothesis that the variability of population size in a region depends on what happens in the remaining areas, justifying the use of Structural Equation Modelling.

Table 5. Variance–covariance estimated matrix of SEM residuals (correlation above diagonal).

	Africa	Other Asia and Oceania	China	Europe	India	Latina America	Middle East	North America	Russia
Africa	0.00028	(0.027)	(0.555)	(0.748)	(0.843)	(−0.606)	(−0.043)	(0.854)	(0.686)
Other Asia and Oceania	4.63×10^{-6}	0.00011	(0.071)	(0.280)	(−0.274)	(0.410)	(0.842)	(0.308)	(0.349)
China	0.00032	2.56×10^{-5}	0.0012	(0.861)	(0.466)	(−0.625)	(−0.047)	(0.416)	(0.747)
Europe	0.00016	3.60×10^{-5}	0.00038	0.00016	(0.682)	(−0.479)	(0.292)	(0.765)	(0.942)
India	9.96×10^{-5}	-1.98×10^{-5}	0.00011	6.02×10^{-5}	4.95×10^{-5}	(−0.512)	(−0.031)	(0.778)	(0.673)
Latin America	−0.00011	4.74×10^{-5}	−0.00025	-6.78×10^{-5}	-4.06×10^{-5}	0.00013	(0.627)	(−0.298)	(−0.235)
Middle East	1.54×10^{-5}	0.00018	-3.53×10^{-5}	7.86×10^{-5}	-4.63×10^{-5}	0.00015	0.00046	(0.400)	(0.478)
North America	0.00016	3.51×10^{-5}	0.00016	0.00011	6.07×10^{-5}	-3.73×10^{-5}	9.52×10^{-5}	0.00012	(0.778)
Russia	0.00034	0.00011	0.00078	0.00035	0.00014	-7.91×10^{-5}	0.00031	0.00026	0.00089

Source: our elaboration on United Nations Population Division data.

When the quadratic TL is found to be the best model, it is appropriate to consider a measure of the overall (mean) effect of the mean $M_{i,t}$ on the variance $V_{i,t}$ (Naccarato and Benassi 2018). To do this we have considered the antilogarithm of Equation (6)

$$V_{i,t} = \alpha_i {M_{i,t}}^{\beta_i + \gamma_i log_{10} M_{i,t}} \tag{7}$$

from which it is evident that in the case of TL (1), the effect of $M_{i,t}$ on the temporal variability of $V_{i,t}$ is constant, while it is a linear function of the logarithm of the mean in the case of quadratic TL. By indicating $\overline{log_{10} M_{it}}$ as the mean of $log_{10} M_{it}$, we can write the overall effect as:

$$\widetilde{\beta}_i = \hat{\beta}_i + \hat{\gamma}_i \overline{log_{10} M_{i,t}} \tag{8}$$

Table 6 shows $\widetilde{\beta}_i$ values for each of the nine geographical areas. Values delineate a strong divergence between 'old settlement' world regions with intense population aging, such as Europe and Russia, and demographically younger regions with positive coefficients, such as Asia, Oceania, the Americas, and, in part, India and Africa. The position of China in-between the two groups was finally noted.

Table 6. Taylor's law estimated parameter $\widetilde{\beta}$, by macro-region.

Geographical Area	$\widetilde{\beta}$
Africa	1.26
Other Asia and Oceania	4.22
China	0.17
Europe	−19.14
India	1.58
Latin America	2.56
Middle East	2.87
North America	2.64
Russia	−6.92

Source: our elaboration on United Nations Population Division data.

In line with the empirical results presented below in this paper, tipping points of the quadratic model for both mean (x) and variance (F(x)) were calculated from the parametric estimation of the two econometric models presented in Table 2 (standard quadratic) and Table 4 (3SLS) by macro-region and illustrated in Figure 1, distinguishing results (plot (a) and (b)) for the two models. The two graphs illustrate substantially stable positions of world macro-regions as far as the tipping points of the quadratic Taylor's law applied to the diachronic analysis of population dynamics in metropolitan regions. Both scatterplots delineate results fully compatible with the empirical evidence outlined above in previous descriptive and econometric analysis, distinguishing three groups of macro-regions (Europe, Russia, and China; Africa and India; the Americas, Oceania, and Asian countries except China as well as the Middle East). In other words, tipping points of the quadratic specification of Taylor's law summarize—likely better than other indicators—the demographic differentiation at the base of consolidated and emerging metropolitan hierarchies all over the world, separating urban systems with a long history, such as Europe, Russia and, in some way, China, from 'demographically young' systems consolidating more recently, basically in the last one-to-two centuries.

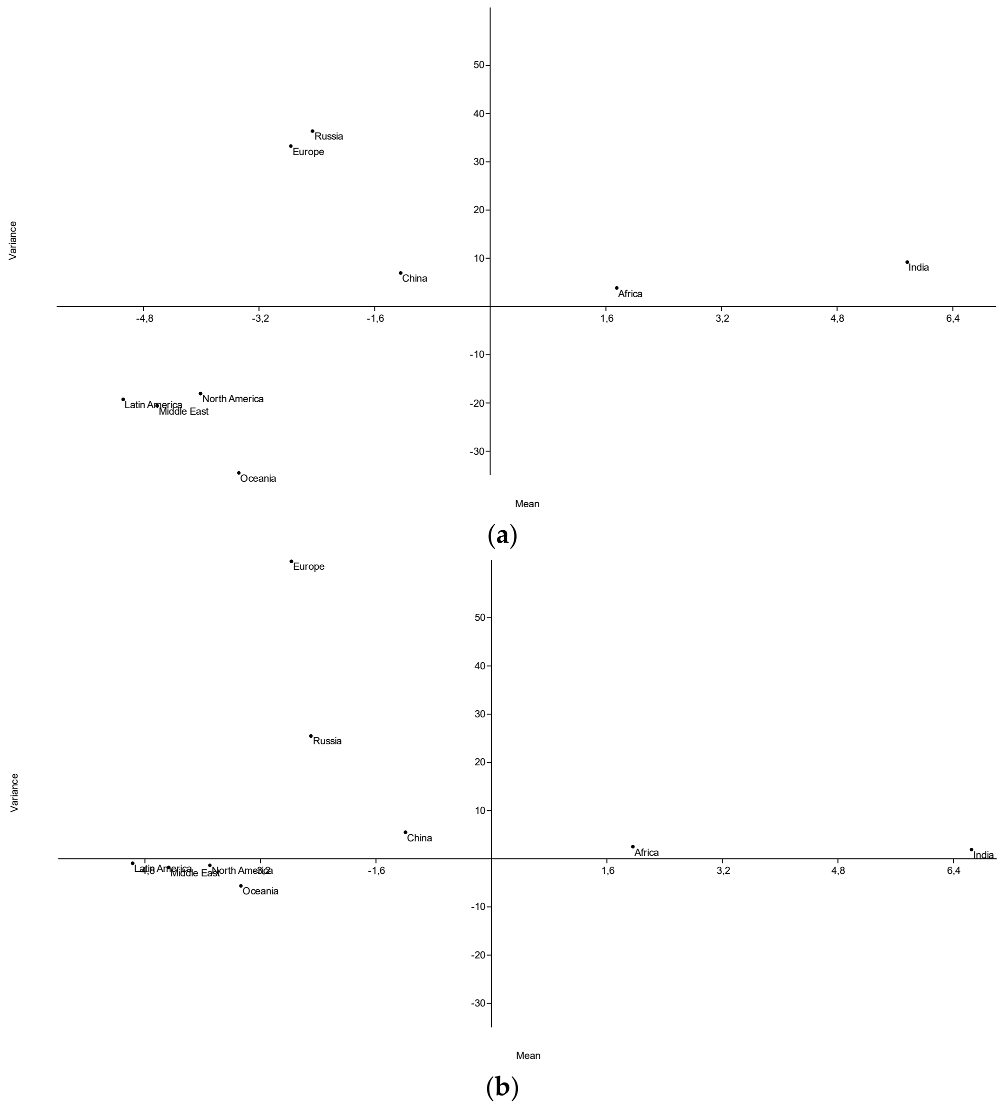

Figure 1. Scatterplots illustrating the position of world macro-regions based on tipping points for both mean (x) and variance (F(x)) of the quadratic model (see results in Tables 2 and 4) by econometric specification (plot (**a**): results from the standard quadratic form; plot (**b**): results from the 3SLS estimation). Oceania includes other Asian countries except China (source: our elaboration on United Nations Population Division data).

4. Discussion

A comprehensive, comparative investigation of urban patterns and processes of population expansion may benefit from an extended analysis of the demographic dynamics characteristic of individual cities and metropolitan systems worldwide, considering together the intrinsic mechanisms of growth and change (Ciommi et al. 2019). Despite the importance of population density in demography, empirical studies have occasionally focused on the law existing between spatial variability and average size in human populations partitioned over appropriately selected, reference locations. In this perspective, Taylor's law was applied—for the first time to our knowledge—to long-term population dynamics in metropolitan areas, classifying cities worldwide into nine geographical regions assumed to be homogeneous as far as the socioeconomic and normative context at large is concerned (Egidi et al. 2020). The empirical results of our study document how metropolitan growth was far from being a homogeneous process over space and time (Carlucci et al. 2017), since different types of population dynamics influenced long-term urbanization patterns, shaping the overall spatial structure of metropolitan regions and their socioeconomic role in the global arena and altering the potential for sustainable development of individual cities (Rozenfeld et al. 2008).

TL was verified using a linear specification for all cases (Cohen 2013). However, a quadratic regression showed a better performance than a linear specification, especially in specific regions (e.g., Cohen et al. 2013a). The statistical results were interpreted considering the regression slopes and the adjusted R^2 jointly (Carlucci et al. 2020). Considering the TL assumption (linear specification), regression slopes were always positive and significant, highlighting that an increase in the average population size implies greater heterogeneity between the metropolises of each region in terms of population size (e.g., Cohen et al. 2013b). Taylor et al. (1978) estimated a TL slope for the USA equal to 2.04 ± 0.01. Similar results were reported by Cohen (2013) in the case of Norway. In our case, considering only urban sub-populations, the TL slope was found to be a bit lower. This can be justified with the assumption that we are studying more homogeneous sub-populations (metropolitan agglomerations > 300,000 inhabitants) than the total population by country. All these results are, however, congruent with those achieved in earlier studies confirming the ability of TL to describe multifaceted spatial distributions of human populations (Newman 2005).

Considering the adjusted R^2, linear specifications are satisfactory for all regions except for Europe and Russia, since the gain in the adjusted R^2 when using quadratic specifications instead of the linear specification is <1% for all regions, >4% for Russia, and >8% for Europe. Moreover, the <1% gain for the seven macro-regions can be compensated by the fact that the quadratic model is less parsimonious than the linear model. However, the results of the quadratic models can be considered for all regions in order to provide additional indications as far as the spatial distribution of urban population is concerned (Gavalas et al. 2014). Quadratic models for European and Russian metropolises had a negative linear coefficient and a positive quadratic coefficient, representing a convex curve, similar to what was observed for Chinese metropolises (Cohen 2014). By contrast, the straight line for the remaining regions showed a much lower slope than that of Europe, evidencing an estimation structure with both positive linear and quadratic coefficients. This indicates that the influence of changing population size on population size variability grows more for Europe than for other regions, as the average levels increase (Egidi et al. 2020).

Therefore, as the average levels grow, population size was more heterogeneous among European cities than in African cities (Marquet et al. 2005). An inverse structure, with a positive linear coefficient and negative quadratic coefficient, provides a concave curve, outlining how the increase in average levels caused a decrease in variability. This indicates that, as average levels increase, the population size of North American, Latin American, and Middle Eastern countries tends to be more uniform. These findings consolidate the initial assumption of Cohen (2013, p. 39) that TL could be used as an empirically tested baseline against which to evaluate population projections at varied spatial scales. These also foster the test of TL in other geographical and temporal contexts (Xu et al. 2017) and/or with other analyses' spatial scales (Salvati and Zitti 2009).

These results are, in our view, of outstanding interest because, as Cohen (2013, p. 39) states, they "could offer a new empirical regularity in human demography and a useful empirically tested baseline or standard against which to evaluate population projections at varying spatial scales. In this case, TL could be added to the ensemble of demographic techniques and models (like the exponential model and age-structured population models) shared by demographers of human and non-human populations". In this regard, regression slopes are an indicator of the degree of homogeneity of the spatial distribution of a given population (Salvati et al. 2019), being bigger when reducing the distance from a homogeneous territorial distribution of the population (Rozenfeld et al. 2008; Salvati et al. 2013; Ciommi et al. 2019). The SEM model chosen for the parameters' estimation at the regional level may confirm the implicit relationship between population size in the nine world regions (Egidi et al. 2020).

In summary, the statistical results of our study delineate salient differences between metropolitan systems in the nine world macro-regions. Based on a comparative perspective, European and neighboring Russian cities displayed the same behavior as far as Taylor's law is concerned (Cohen 2014). China provides similar, but relatively more mixed results. The homogeneity of the results may underline the intrinsic characteristics of millenary urbanization processes in these three regions, as reflected in the most recent (1950-onwards) population dynamics (Xu and Cohen 2021). The other world regions can be split into two groups. The former, where linear and quadratic relationships gave similar results, include 'new' demographic continents, with more recent and intense urbanization processes (African and Asian countries) and positive quadratic coefficients. The latter includes regions with long-established urbanization processes, such as North American and Latin American metropolises, experiencing an intense but spatially balanced population increase reflected in concave curves (Cohen et al. 2013a). The use of regression coefficients and goodness-of-fit estimates to discriminate urban systems (i.e., metropolitan hierarchies) into homogeneous regions is an interesting by-product of the TL analysis that can be appropriately applied to several contexts at different spatio-temporal scales (Cohen et al. 2013b). A comparative analysis using longer time series and refined indicators will provide the necessary validation of the results presented here under improved technical conditions (e.g., econometric estimation) and for broader socioeconomic contexts (Xu and Cohen 2021).

The empirical findings of our study suggest how novel approaches in urban studies attempting to combine theoretical perceptions behind scale and agglomeration effects with a definite focus on demographic effects within cities may inspire a more comprehensive analysis of metropolitan systems. Going beyond linear interpretations of urbanization as a sequential process of growth (Salvati and Gargiulo Morelli 2014), theoretically oriented and empirically grounded analyses inspired by Taylor's law allow a detailed investigation of urbanization trends worldwide, contributing to clarifying the interplay between long-term demographic dynamics and short-term (and medium-term) urban transitions (Kroll and Kabisch 2012).

Going beyond the economic polarization in advanced and emerging countries, increasingly mixed patterns of demographic growth were observed for urban agglomerations in both advanced and emerging countries (Cohen et al. 2013a). Urban demographic dynamics highlighted here fundamentally distinguish the old world (Europe and Russia) from the new world (Africa, America, the remaining part of Asia, and Oceania), with some transitional regions in-between (India and the Middle East). Moving beyond a traditional 'developmental' model separating affluent countries from emerging economies, this polarization may clarify the importance of socio-demographic mechanisms regulating individual urban development pathways. In the context of low fertility and the more volatile contribution of migration to metropolitan growth, these results provide basic knowledge for refined (and, possibly, heterodox) studies in urban sustainability and local development of metropolitan regions (Naccarato and Benassi 2018).

These findings also demonstrate that alternative approaches to urban growth may enrich the economic explanation of demographic transitions, highlighting the legacy of historical, cultural, institutional, and religious factors (Carlucci et al. 2017). Economic globalization and the rising availability of digital data allow increasingly advanced statistical approaches to the study of complex socioeconomic systems at different spatial scales (Zambon et al. 2017), in turn contributing to delineating the most appropriate development policies at the required territorial level (Saitoh and Cohen 2018). A refined analysis of such data should reveal the effect of the geographical scale—which is increasingly mixed at the traditional levels of analysis typical of applied economics, spatial econometrics, and regional statistics, basically coinciding with administrative units (Ramsayer et al. 2012). This may give fragmented results at disaggregated levels of analysis (Cohen 2013). Indeed, spatially explicit investigations of economic growth and relevant socio-demographic implications are becoming increasingly important when examining complex and non-linear growth processes, social demographic dynamics, and the related impact of economic cycles (Ciommi et al. 2019). To cope with this deserving research issue, a consistent set of global urbanization projections that cover long time horizons and span a full range of uncertainty is clearly required, possibly improving the existing data sources (Xu et al. 2017). Extending the projections from the United Nations Population Division—that provide a single demographic scenario over the next few decades—with a long-term, new global set of urbanization projections at country (and regional) level is a particularly urgent research and operational task with significant impact on sustainable development studies (Bohk et al. 2015).

Taken together, the empirical evidence of our study is important both from a positive perspective and from a normative point of view. Operationally speaking, the difficulty of defining urban, suburban, and rural areas complicates the empirical verification of Taylor's law using macro-demographic data (Benassi and Naccarato 2019). Outlining homogeneous areas means, at least indirectly, adhering to a specific model of settlement distribution (e.g., mono-centric, polycentric, scattered), which does not always adapt to multiple and complex metropolitan forms (Salvati and Serra 2016). At the same time, together with improvements in official statistics and demographic indicators, our study suggests how the analysis of the impact of developmental policies (e.g., enhancing socioeconomic conditions, reducing poverty, controlling social segregation, and ameliorating housing conditions) should increasingly consider the inherent role of spatial heterogeneity in the individual communities forming metropolitan regions (Rozenfeld et al. 2008). The positive (or negative) impact of any possible strategy mentioned above is intrinsically bonded to the spatial complexity of socioeconomic systems at both local (urban) and metropolitan (regional) scales (Egidi et al. 2020). Multi-scale approaches to socio-spatial complexity are an appropriate tool for responding to such research and policy challenges in both advanced economies and emerging countries.

5. Conclusions

Global urbanization will continue at high speed, and the world's urban population is projected to increase by more than 3 billion people between 2010 and 2050. While cities maintain a key role in shaping population trends worldwide, different developmental paths emerge depending on the rate of urbanization, possibly indicating that future demographic dynamics will be less sequential and more unpredictable and heterogeneous over space. A refined analysis of worldwide metropolitan dynamics based on Taylor's law (and other conceptual frameworks relating average changes in population size and the intrinsic variability in homogeneous regions) plays a decisive role in the refined understanding of globalized economic systems. The results of such analyses may clarify the role of the demographic dimension of urban growth in the sustainable development of cities, as clearly outlined in the United Nations 2030 Sustainability strategy—namely in the Sustainable Development Goal 11 (Sustainable Cities and Communities). The empirical findings of our study go exactly in such an interpretative direction, demonstrating the demographic divergence in 'old settlement' world regions (such as Europe, Russia, and, in part, the Middle East and China) and in 'new settlement' world regions (the Americas, Asian countries, Oceania) as far as the relationship between average population size and its spatial variability over large areas is concerned.

In light of this policy target, supra-national and country-specific developmental policies may reveal inadequacies or inefficiencies in promoting sustainable urban development in both advanced and emerging countries and should consider the specificity of local contexts and the intrinsic demographic dynamics of individual cities more seriously. To provide policy-makers with advanced knowledge of socio-demographic mechanisms underlying urban growth, scholars should refine demographic data sets to include spatial factors as well as city-scale vital rates and make significant improvements to forecasting methods currently in use. A comparative analysis of spatially detailed and updated population data is also required to assess newly emerging demographic patterns associated with complex socioeconomic processes acting at vastly different spatial scales. Dealing together with future urbanization patterns and demographic dynamics worldwide, innovative approaches for large-scale modeling of global demographic processes are vital to the study of urban growth in light of sustainable development dynamics.

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