Comparative Evaluation of Fuzzy Axiomatic Design and IAMS Comprehensive VIKOR Approaches for Material Selection in Mechanical Design

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Abstract

In the mechanical design, the selection among material alternatives has become a pressing issue due to the progressive growth in the complexity of mechanical systems, in search of continuous increase in performance and the presence of a wide range of possible materials. Moreover, there are many requests for projects, and this makes the choice of material a decisive activity for the success or failure of the project itself. This paper makes a comparative evaluation of two important approaches to identify the best materials alternatives: the C-VIKOR and the Fuzzy Axiomatic Design. The work considers as a case study the selection of the material for the production of the valve seats in a high-performance engine. An evaluation of the results obtained is made to show the peculiarity of each of the two approaches.

Keywords: material selection, Axiomatic Design, MADM, mechanical components, design methods, coating, high-performance engine.

1. INTRODUCTION

Within the industrial environment, the continuous introduction of new materials makes their choice harder and harder. In particular, the material selection discipline needs to be continuously updated and reviewed to know what is the viable algorithm to be considered in any circumstance.

One of the most critical issues is the correct choice of the "selection attributes" because it demands the exact translation of the customer needs into the technical requirements and the attribution of an importance weight to each of them ([1, 2]). In fact, the design team should use these attributes to evaluate the best among a number of possible alternatives for the design. The data about the performance of each of the candidate materials can be obtained by a wide range of sources such as on-line databases, handbooks, datasheet from Suppliers, experimental data, etc. The Multi-Attribute Decision Making (MADM) algorithms are based on the use of this information, the selection attributes and the importance weight of each of them to calculate the ranking of each design alternatives.

A previous work of the authors has developed a model, called Integral Aided Material Selection (IAMS) [3], that use House of Quality approach and one of the reliable MADM algorithm (i.e., the Comprehensive VIKOR algorithm or C-VIKOR [4, 5]). The IAMS method is a good selection process from the translation of the customer needs into the technical requirements and then to obtain the ranking of the design alternatives.

One alternative approach to obtain the ranking of the design alternative is based on Axiomatic Design (i.e., a structured method to identify optimal solution applied in a wide range of scientific and industrial environment [6-11]). Using this method, some specific approaches for MADM algorithm have been developed. In this paper we will consider, in particular, Intuitionistic Fuzzy Axiomatic Design (FAD) and Intuitionistic Weighed Fuzzy Axiomatic Design (WFAD) [12-16].

This work aims to evaluate how two different material selection algorithms behave with respect to the same case study. In particular, the original C-VIKOR results [17] will be compared with the results obtained by the use of Intuitionistic Fuzzy Axiomatic Design (FAD and WFAD).
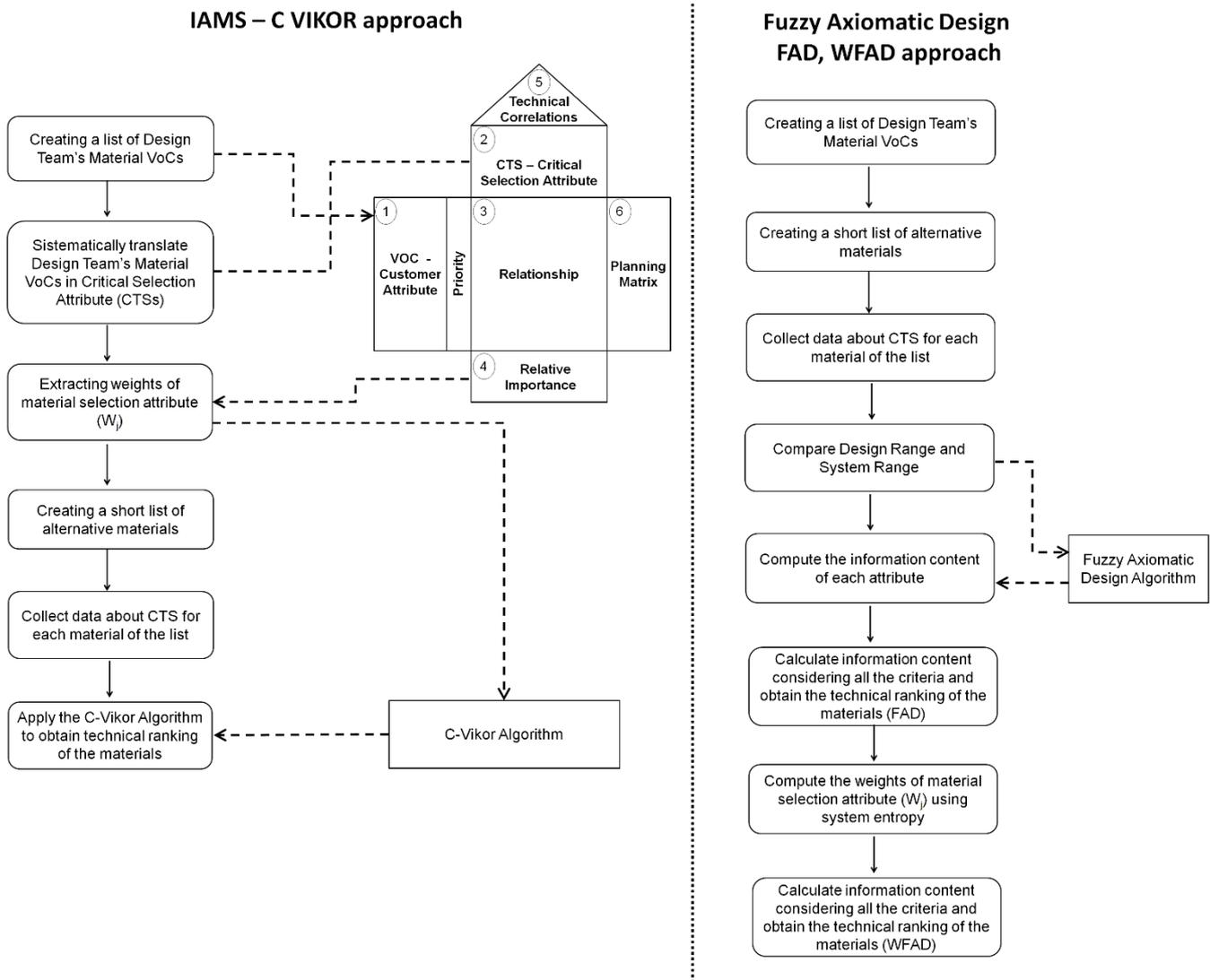
2. THE MATERIALS AND METHODS

As stated in the previous section, this work aims to compare material selection results coming from two different algorithms. Figure 1 shows the logical path to obtain the ranking of the materials when using the two algorithms (IASM-C-VIKOR and FAD-WFAD). The first algorithm considered is C-VIKOR. This algorithm will be considered as a sort of benchmark in this work since its application for the material selection of this case study has been already carried out by [17].

Briefly, the material selection based on the C-VIKOR algorithm is structured in the following steps:

- Definition of the weight of each factor through the generation of a House of Quality (HoQ);
- Then, considering the attributes weight, the C-VIKOR algorithm is applied.

The second algorithm considered, on the contrary, is based on the use of Fuzzy Axiomatic Design, both unweighted (FAD) and weighted (WFAD) [12-15]. The application of this algorithm will be detailed explained in the following of the work.

Fig 1. Comparison between the logical path to obtain the ranking of the materials when using the two algorithms.

From a methodologic perspective, the use of the Fuzzy Axiomatic Design requires less calculation to get the final ranking. For each attribute, a design range is defined by the specification itself. To manage uncertainty, fuzzy logic can be used. In particular, it is possible to choose between triangular, rectangular, and trapezoidal distribution to manage specification uncertainty. For each element considered (in the case study each material), the system range for each attribute shall be defined as well. Even in this case, fuzzy logic can be adopted to manage uncertainty.

Then, by the comparison of design range and common range, the probability of success is calculated. Basing on this information, the Fuzzy Axiomatic Design allows defining an overall parameter suitable to identify the best choice for the considered application. In case of the will to consider even the weight of each parameter, the Fuzzy Axiomatic Design allows us to include it through the evaluation of the system entropy.

The process followed using the Fuzzy Axiomatic Design is quite similar to the one used with the C-VIKOR algorithm,

but there is no need to extract the weight of each attribute. In addition, the list of material can be obtained even without the use of the HoQ, simplifying the process and deriving the weights directly from quantitative information.

Specifically:

• Define the scope of the work collecting the Voice of Customer (VOC) [18-20];

• Create a list of candidates for project satisfaction;

• Collect all the required technical data for the evaluation of each candidate from all the considered perspectives [21];

• Compare, for each attribute design range, and system range, evaluating how each candidate can satisfy each specific attribute;

• By using the Fuzzy Axiomatic Design algorithm, data coming from the design and system range comparison are elaborated to obtain a value that represents how each

attribute is satisfied by each candidate (the lower the number, the better). This value is called information content; there will be as many values as many attributes considered;

- Summing all the information contents, it is possible to calculate the overall information content that can be used to make the final ranking (even in this case, the lower the information content is, the better).

The Fuzzy Axiomatic Design algorithm, as stated above, is based on two main aspects:

- Possibility to manage uncertainty by the use of fuzzy distributions (triangular or trapezoidal);

- Comparison of design range (specification) and system range (candidate) to calculate the probability to satisfy the attribute.

For the case study considered in the following, characteristics have been subdivided into two main categories: data and attributes.

For data characteristics, such as thermal conductivity and Young modulus, design range and system range have been considered as a rectangle, since commonly specifications indicate just a range (and not a distribution).

For attribute characteristics, on the contrary, triangular distributions have been considered for the calculations. Since the design range, in all the cases, is a "larger the-better", asymmetric triangles have been considered. For system range, on the contrary, symmetric triangles centered on the score have been used.

To evaluate what is the best material for the considered application, the information content has been assessed using both the FAD and WFAD approaches [12-15].

Basing on these assumptions, we proceeded to compute the common range as the intersection between design range and system range.

With this data, it is possible to compute the information content of each attribute of each material with the probability of success, assessed as:

$$p_{i,j} = \frac{common\ range}{system\ range} \qquad (1)$$

$$I_{i,j} = -log_2 \frac{1}{p_{i,j}} \qquad (2)$$

Where i is the i-th material (thus, the i-th row) and j is the j-th attribute of the i-th material (thus the j-th column).

By summing the information content related to all the attribute of each material, it is possible to calculate the information content related to the considered material (3):

$$I_i^{FAD} = \sum_j I_{i,j} \qquad (3)$$

This is the algorithm followed to assess the FAD parameter for each material.

As stated in the previous section, also the WFAD parameter has been considered within this work.

With respect to the FAD, the WFAD approach allows us to include in the information content evaluation a weight, to differentiate attribute importance.

To assess the weight, for each attribute of each material, the average value shall be assessed (since we considered the rectangular and triangular design and system ranges). The parameter obtained by this step is called $x_{i,j}$.

At this point, each $x_{i,j}$ shall be normalized (4):

$$f_{i,j} = \frac{x_{i,j}}{\sum_i x_{i,j}} \qquad (4)$$

With this data, the system entropy and its weight coefficient can be calculated using (5) and (6):

$$E_j = -\frac{1}{\ln n} \sum_i f_i \ln f_i \qquad (5)$$

$$w_j = \frac{1-E_j}{\sum_j (1-E_j)} \qquad (6)$$

where n is the number of rows (thus the number of materials considered).

It is important to note that with these expressions, a coefficient for each attribute will be assessed.

Once the weight and the system entropy values have been determined, the information content related to each attribute of each material can be assessed with (7):

$$I_{ij} = \begin{cases} \left[log_2\left(\frac{1}{p_{ij}}\right) \right]^{\frac{1}{w_j}}, 0 \le I_{ij} < 1 \\ \left[log_2\left(\frac{1}{p_{ij}}\right) \right]^{w_j}, \qquad I_{ij} > 1 \\ w_j, \qquad\qquad I_{ij} > 1 \end{cases} \qquad (7)$$

As in the previous case, the global WFAD parameter for each material can be calculated as the sum of all the information content related to all the attributes of the considered material (8).

$$I_i^{WFAD} = \sum_j I_{i,j} \qquad (8)$$

This parameter can then been used to obtain a ranking among the various solutions.

3. CASE STUDY

As previously stated, in this paper, the comparison between the results obtained by the application of two different algorithms for the material selection of a case study is carried out. The case study consists of the choice of the coating for the valve seats of a high-performance engine (Figure 2).

Valve seats are a critical component of a combustion chamber since it needs to carry out two main functions.

- Guarantee the chamber tightness ensuring the contact with the valve;

- Dissipate the heats absorbed by the valve, allowing the

engine to work at higher temperatures and increasing the thermodynamic efficiency.
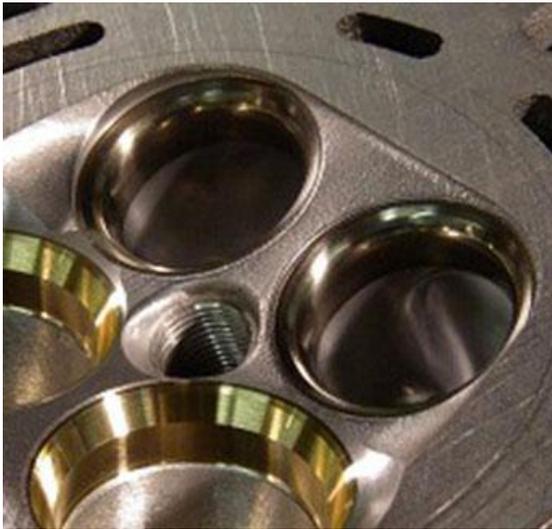
Fig 2. Example of valve seats of a high-performance engine.

Since the more recent high-performance engines come with aluminum structures (to allow an important mass reduction), the necessity to choose the right material for the valve seats is fundamental for the engine life. Aluminum, in fact, is not able to withstand thermo-mechanical stresses produced by the engine working; therefore it is essential to realize the considered component in a different material. Materials evaluated in this paper and the attribute value are listed in Table 1. The critical direction of improvement has been defined as Larger-The-Better (LTB), attributes that require a target value (Target), and Smaller-The-Better (STB).

Before to move on, an explanation of how data that are not affected by uncertainty have been used is necessary. Since the Fuzzy Axiomatic Design approach is based on the probability of success evaluation, exact data are not processable with this kind of approach. In fact, the probability computed as per above for an attribute expressed by a single value is always zero (since the associate range is 0), regardless of the position with respect to the design range. Therefore, to make the Fuzzy Axiomatic Design approach applicable to these data (enabling a direct result comparison), exact data have been translated is small ranges (±5% in most of the cases, for the hardness ±0.8%).

Table 1. Completed selection matrix.

	K [W/mK]		TEC [10⁻⁶/K]		E [GPa]		Hardness [HV]		Abrasion Resistance [null]		Hot corrosion resistance [null]		Rupture strength [MPa]		Density [g/cm³]		Machinability [null]		Cost [€/kg]	
Optimization type	LTB		Target		Target		Target		LTB		LTB		LTB		STB		LTB		LTB	
Best Value			22		80		270		5		5		5				5			
Materials	min	MAX	min	MAX	min	MAX	min	MAX	min	MAX	min	MAX	min	MAX	min	MAX	min	MAX	min	MAX
1. CuBe	105	118	16.7	17	130	132	318	382	4	4	2	3	1030	1310	8.26	8.41	4	4	20.8	25.2
2. CuBe2	105	130	16.7	17	131	134	353	413	4	5	2	3	1140	1380	8.25	8.36	5	5	23.5	25.9
3. Alloy 3	240	240	17.6	17.6	138	138	195	250	2	3	3	3	690	900	8.83	8.83	3	4	21.9	24.1
4. Alloy 310₂	235	235	17.6	17.6	135	135	234	260	2	3	3	3	720	820	8.8	8.8	3	4	22.5	23.6
5. C18150	280	324	17	17	117	120	130	155	1	2	2	2	380	520	8.89	8.9	4	5	7.6	7.84
6. C18000	185	225	16.2	17.5	114	130	185	195	2	2	2	3	585	605	8.75	8.84	4	4	8.73	10
7. SS 410	24.9	24.9	9.9	9.9	200	200	339	410	4	5	5	5	985	1310	7.74	7.8	1	2	1	13
8. 50CrV4	46.6	46.6	12.2	12.2	205	205	309	350	4	5	3	4	1020	1145	7.83	7.85	2	3	0.7	0.8
9. x38CrMoV51	18	21	11	11.8	207	215	551	632	5	5	3	4	1835	2100	7.8	7.8	2	3	4.24	5

4. RESULT

Proceeding with the calculation as per above, we obtained the following results concerning the evaluation of the unweighted information content (Table 2) and the Ranking of solution technical optimality based considering FAD algorithm (Table 3).

For what concerns the weighted approach (WFAD), the probabilities are the same as in the previous case; the difference consists in considering weight factors calculated using (6). The results are reported in Table 4 and the ranking in Table 5.

The results obtained by the application of the C-VIKOR algorithm used as reference in this work [17] are listed in table 6 for comparison.

Table 2: Selection matrix with the value of the parameters I^{FAD}.

	K	TEC [10^{-6}/°C]	E [GPa]	Hardness [HV]	Abrasion Resistance [null]	Hot corrosion resistance [null]	Rupture strength [MPa]	Density [kg/m^3]	Machinability [null]	Cost [€/kg]	
System Range	160	13	90	120	2.5	2.5	1150	2	2.5	25	
Material	p	p	p	p	P	p	p	p	p	p	I^{FAD}
1. CuBe	1	1	1	0.66	0.78	0.08	1	1	0.78	1	4.89
2. CuBe2	1	1	1	0.12	0.92	0.08	1	1	1.96	0.625	6.49
3. Alloy 3	1	1	1	0.18	0.08	0.28	1	1	0.56	1	8.71
4. Alloy 310$_2$	1	1	1	0.77	0.08	0.28	1	1	0.56	1	6.63
5. C18150	0	1	1	0	0	0	0	1	0.92	1	$+\infty$
6. C18000	1	1	1	0	0	0.08	0	1	0.78	1	$+\infty$
7. SS 410	0	0	0.5	0.30	0.92	1.96	1	1	0	1	$+\infty$
8. 50CrV4	0	1	0.26	1.00	0.92	0.56	1	1	0.08	1	$+\infty$
9. x38CrMoV51	0	0	0	0	1.96	0.56	0	1	0.08	1	$+\infty$

Table 3: Ranking of solution technical optimality based on FAD algorithm.

Place	Ranking based on I_{FAD}	I^{FAD}
		STB
1st	CuBe (Materion Alloy 165 temper TF00)	4.89
2nd	CuBe2	6.49
3th	Alloy 310 (Materion Alloy 310 temper TF00)	6.63
4th	Alloy 3 (Materion Alloy 3 temper TF00)	8.71
N.A.	C18150	$+\infty$
N.A.	C18000	$+\infty$
N.A.	50CrV4 (AISI 6150 steel oil quenched, 540°C tempering, D=50mm)	$+\infty$
N.A.	SS 410 martensitic stainless steel	$+\infty$
N.A.	x38CrMoV51 (AISI Type H11)	$+\infty$

Table 4: Selection matrix with the value of the parameters I^{WFAD}.

	K	TEC [10^{-6}/°C]	E [GPa]	Hardness [HV]	Abrasion Resistance [null]	Hot corrosion resistance [null]	Rupture strength [MPa]	Density [kg/m³]	Machinability [null]	Cost [€/kg]	
System Range	**160**	**13**	**90**	**120**	**2.5**	**2.5**	**1150**	**2**	**2.5**	**25**	
w	0.279	0.020	0.028	0.084	0.071	0.033	0.087	0.002	0.050	0.347	
Material	p	p	p	p	p	P	p	p	p	p	I^{WFAD}
1. CuBe	1	1	1	0.66	0.78	0.08	1	1	0.78	1	1.05
2. CuBe2	1	1	1	0.12	0.92	0.08	1	1	1.96	0.625	2.52
3. Alloy 3	1	1	1	0.18	0.08	0.28	1	1	0.56	1	3.22
4. Alloy 310$_2$	1	1	1	0.77	0.08	0.28	1	1	0.56	1	2.14
5. C18150	0	1	1	0	0	0	0	1	0.92	1	$+\infty$
6. C18000	1	1	1	0	0	0.08	0	1	0.78	1	$+\infty$
7. SS 410	0	0	0.5	0.30	0.92	1.96	1	1	0	1	$+\infty$
8. 50CrV4	0	1	0.26	1	0.92	0.56	1	1	0.08	1	$+\infty$
9. x38CrMoV51	0	0	0	0	1.96	0.56	0	1	0.08	1	$+\infty$

Table 5: Ranking of solution technical optimality based on WFAD algorithm.

Place	Ranking based on I_{WFAD}	I^{WFAD} STB
1st	CuBe (Materion Alloy 165 temper TF00)	1.05
2nd	CuBe2	2.52
3th	Alloy 310 (Materion Alloy 310 temper TF00)	2.14
4th	Alloy 3 (Materion Alloy 3 temper TF00)	3.22
N.A.	C18150	$+\infty$
N.A.	C18000	$+\infty$
N.A.	50CrV4 (AISI 6150 steel oil quenched, 540°C tempering, D=50mm)	$+\infty$
N.A.	SS 410 martensitic stainless steel	$+\infty$
N.A.	x38CrMoV51 (AISI Type H11)	$+\infty$

Table 6: Ranking of solution technical optimality based on IAMS- C-VIKOR algorithm [17].

Place	Ranking based on Q_i	Q_i STB
1st	CuBe2	0.00
2nd	CuBe (Materion Alloy 165 temper TF00)	0.310
3th	Alloy 310 (Materion Alloy 310 temper TF00)	0.372
4th	Alloy 3 (Materion Alloy 3 temper TF00)	0.399
5th	C18150	0.815
6th	C18000	0.852
7th	50CrV4 (AISI 6150 steel oil quenched, 540°C tempering, D=50mm)	0.935
8th	SS 410 martensitic stainless steel	0.991
9th	x38CrMoV51 (AISI Type H11)	1.140

By the comparison of the results coming from the application C-VIKOR and Fuzzy Axiomatic Design approaches (considering both FAD and WFAD approach) it is possible to observe several aspects:

- C-VIKOR is able to produce a ranking with all the candidates, regardless of their ability to fulfill the requirements. The algorithm is able to discriminate between all of the various solutions;

- Fuzzy Axiomatic Design, using both FAD and WFAD, gives an indication only if the candidate is able to satisfy all the requirements in some way (i.e. the probability shall be never zero considering each of the selection attributes). Otherwise, the information content becomes $+\infty$ with even one criterion not met. From a practical standpoint, it means that Fuzzy Axiomatic Design algorithms are useful in defining what is the best solution among a pool of good candidates (optimization) but is not good in discerning among non-optimal and very bad candidates;

- For the candidates able to satisfy all the attributes, C-VIKOR and Fuzzy Axiomatic Design give quite aligned results.3

4. CONCLUSION

Summing up results obtained in this work, it is evident that the two algorithms have different behaviors.

C-VIKOR returns a complete ranking for all the candidates, allowing, on the one hand, to distinguish between the non-optimal and the worst. On the other hand, C-VIKOR does not highlight the difference between the candidates that fulfill all the attributes and who is, in some cases, is out of design range.

FAD and WFAD approaches are exactly the opposite. Being based on the probability of satisfying the design specification, as soon as a candidate is out of design, the information content became $+\infty$. Therefore, it is impossible to distinguish between who has just one attribute out of design from who has many attributes out of design. On the other hand, it is immediate, with the Fuzzy Axiomatic Design, to see whether a candidate fulfills all the attributes or not.

In conclusion, Fuzzy Axiomatic Design can be effectively used in an optimization phase, when either non-optimal candidates have already been removed by the selection list or when the design ranges have been re-defined. C-VIKOR, on the contrary, can also be used in a screening phase, to define what are the worst choices and remove them from the next selection iterations.

When candidates are reasonable, C-VIKOR and Fuzzy Axiomatic Design demonstrated to give aligned results.

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